March 15 – AVL Trees
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BST Rotation Summary

- Four kinds of rotations (L, R, LR, RL)
- All rotations are local (subtrees are not impacted)
- All rotations are constant time: O(1)
- BST property maintained

**GOAL:**

We call these trees:
AVL Trees

Three issues for consideration:
- Rotations
- Maintaining Height
- Detecting Imbalance
AVL Tree Rotations

Four templates for rotations:
Finding the Rotation on Insert

**Theorem:**
If an insertion occurred in subtrees $t_3$ or $t_4$ and a subtree was detected at $t$, then a __________ rotation about $t$ restores the balance of the tree.

We gauge this by noting the balance factor of **$t$->right** is ______.
Theorem:
If an insertion occurred in subtrees $t_2$ or $t_3$ and a subtree was detected at $t$, then a __________ rotation about $t$ restores the balance of the tree.

We gauge this by noting the balance factor of $t$->right is ______.
Insertion into an AVL Tree

```c
struct TreeNode {
    T key;
    unsigned height;
    TreeNode *left;
    TreeNode *right;
};
```

`_insert(6.5)`
**Insertion into an AVL Tree**

**Insert (pseudo code):**
1: Insert at proper place
2: Check for imbalance
3: Rotate, if necessary
4: Update height

```c
struct TreeNode {
    T key;
    unsigned height;
    TreeNode *left;
    TreeNode *right;
};
```

`_insert(6.5)`
template <typename K, typename V>
void AVL<K, D>::_insert(const K & key, const V & data, TreeNode * & cur) {
    if (cur == NULL)         { cur = new TreeNode(key, data);   }
    else if (key < cur->key) { _insert( key, data, cur->left ); }  
    else if (key > cur->key) { _insert( key, data, cur->right );}  
    _ensureBalance(cur);  
}
template <typename K, typename V>
void AVL<K, D>::_ensureBalance(TreeNode * & cur) {
  // Calculate the balance factor:
  int balance = height(cur->right) - height(cur->left);

  // Check if the node is current not in balance:
  if (balance == -2) {
    int l_balance =
      height(cur->left->right) - height(cur->left->left);
    if (l_balance == -1) { _____________________________; }
    else { _____________________________; }
  } else if (balance == 2) {
    int r_balance =
      height(cur->right->right) - height(cur->right->left);
    if (r_balance == 1) { _____________________________; }
    else { _____________________________; }
  }
  _updateHeight(cur);
};
Height-Balanced Tree

Height balance: \( b = \text{height}(T_R) - \text{height}(T_L) \)
AVL Tree Analysis

We know: insert, remove and find runs in: ____________.

We will argue that: $h = ____________$. 
AVL Tree Analysis

Definition of big-O:

...or, with pictures: