CS 225

Data Structures

March 12 – BST Balance

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BST Analysis

Therefore, for all BST:

Lower bound: h >= O(lg(n))

U**pper bound: h <= O(n)**

BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

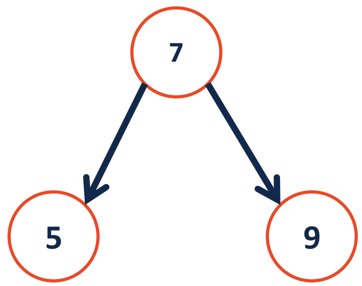
Q: What is the average height of all the arrangements?

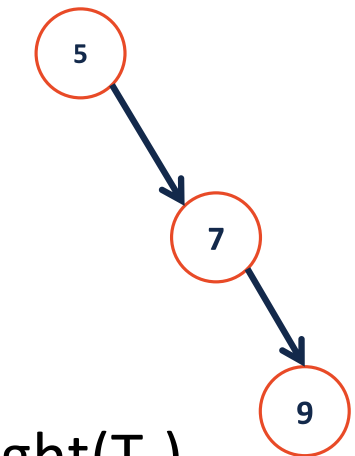
BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
find				
insert				
delete				
traverse				

Height-Balanced Tree

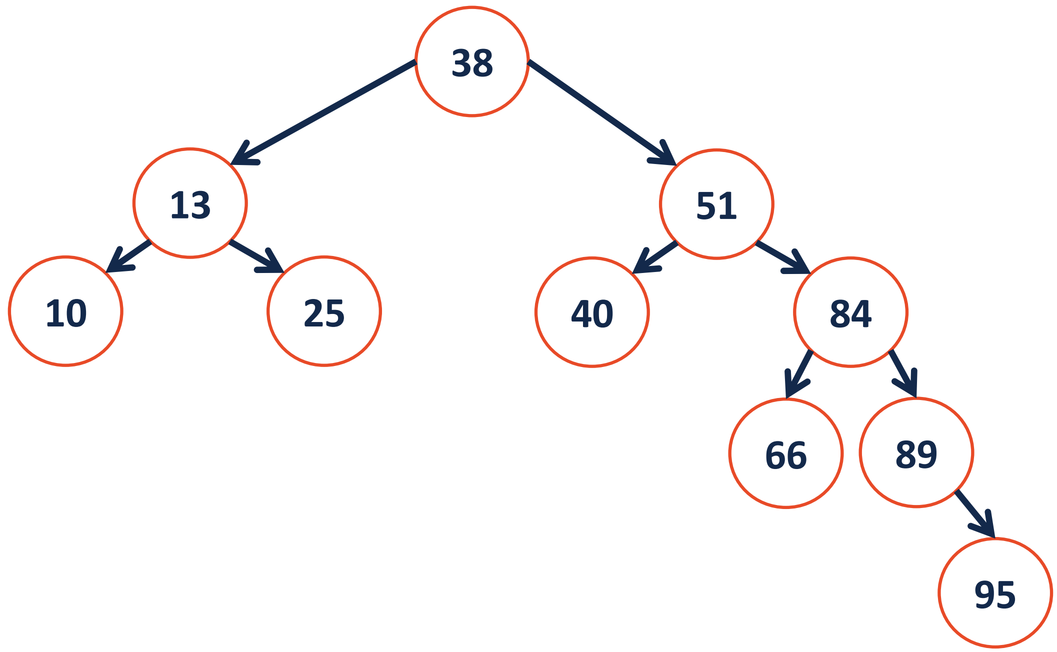
What tree makes you happier?





Height balance: $b = \text{height}(T_R) - \text{height}(T_L)$

A tree is height balanced if: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89
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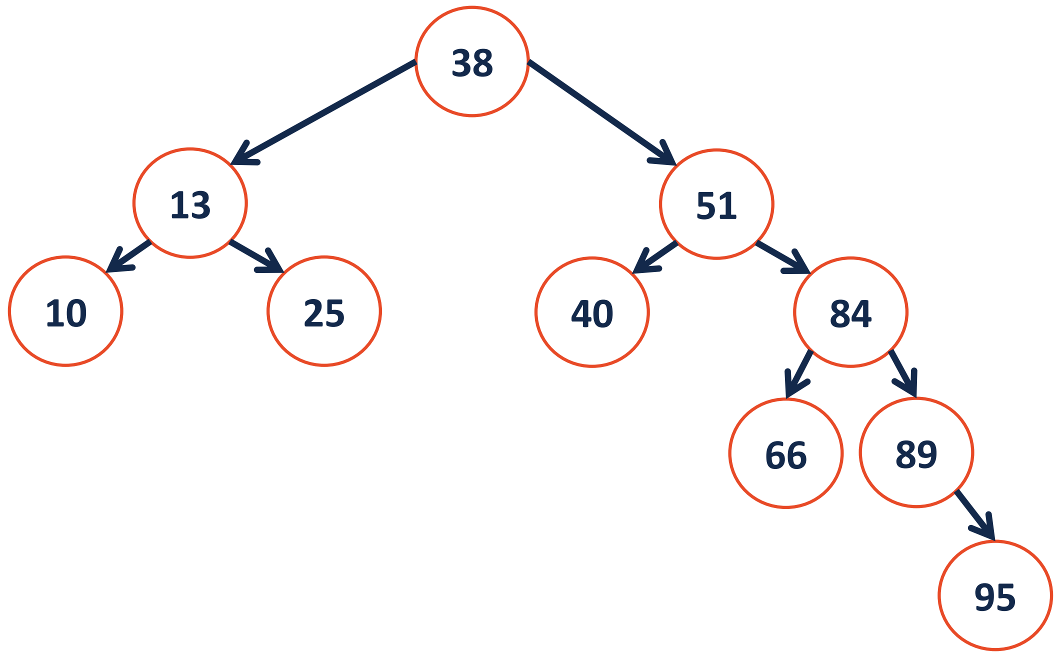
BST Rotation

We will perform a rotation that maintains two properties:

1.

2.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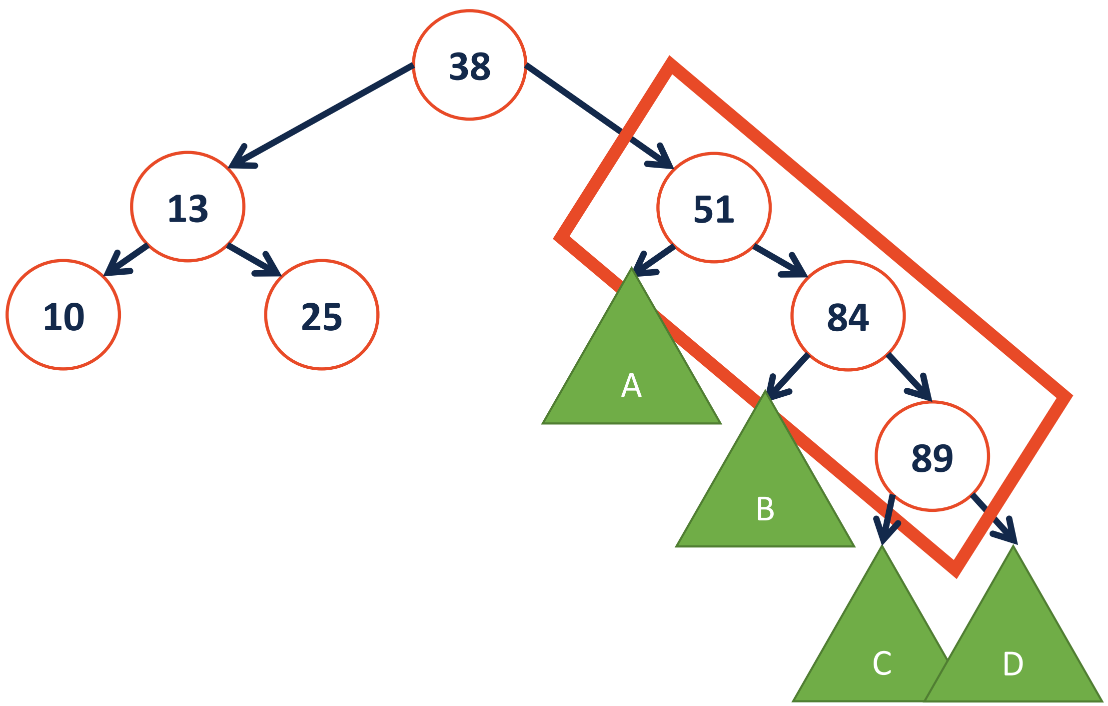
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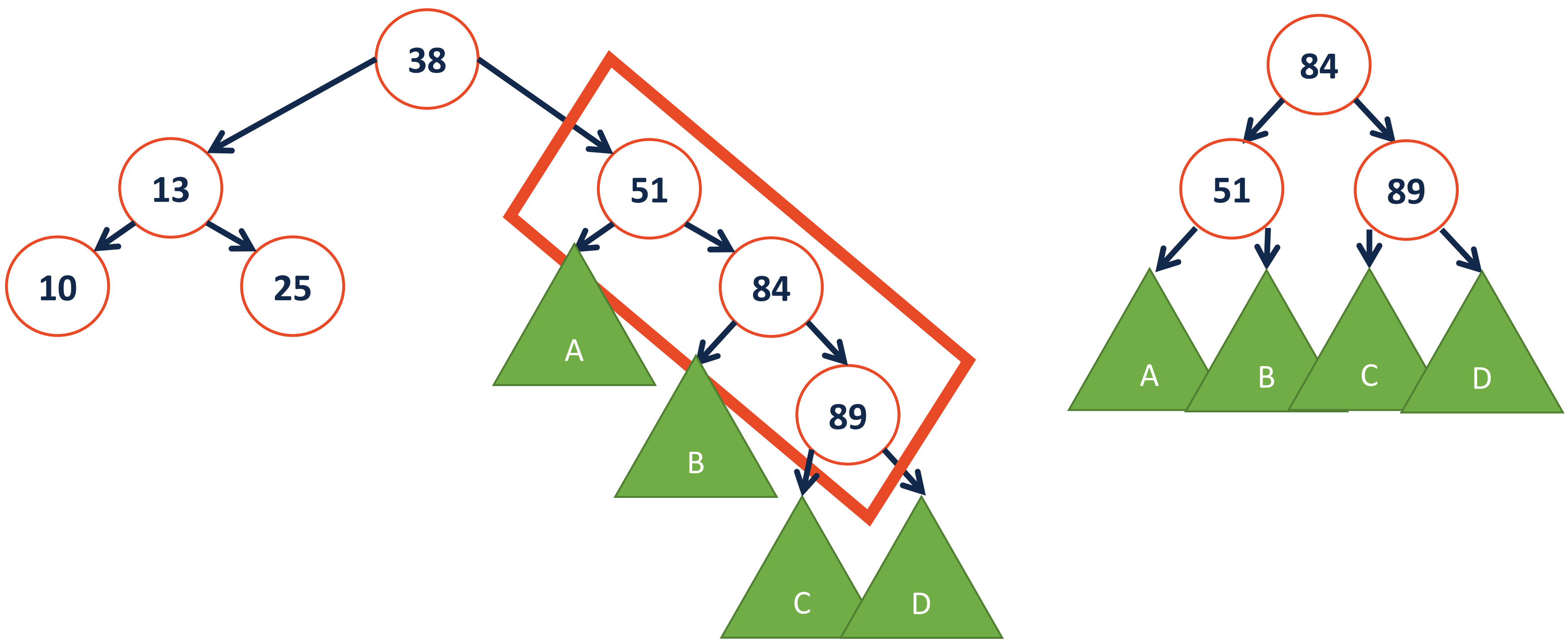
A
B
C
D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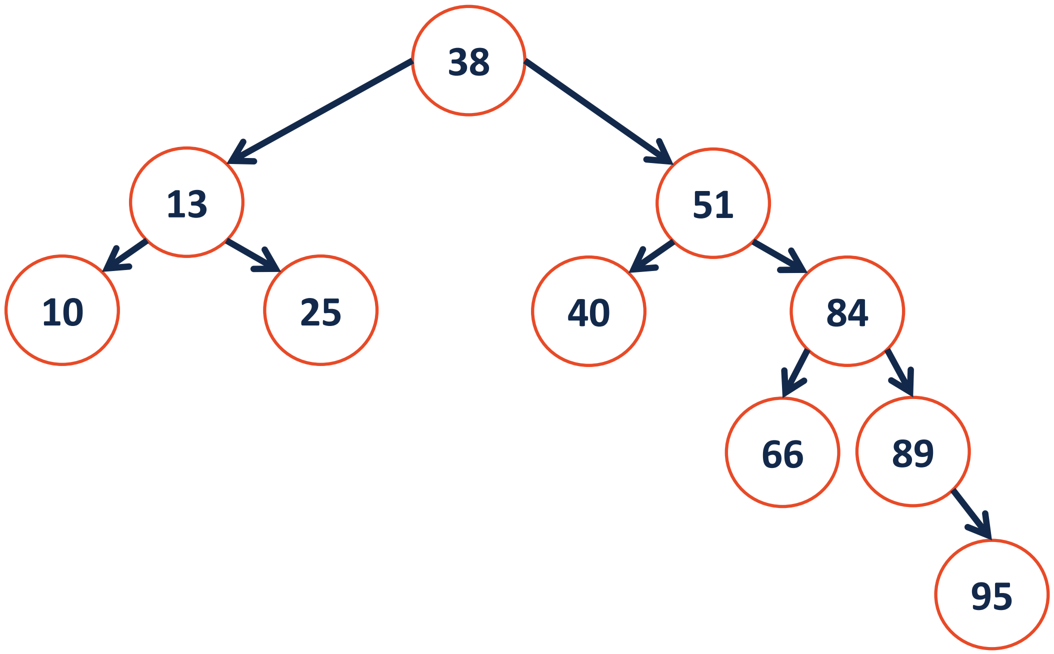
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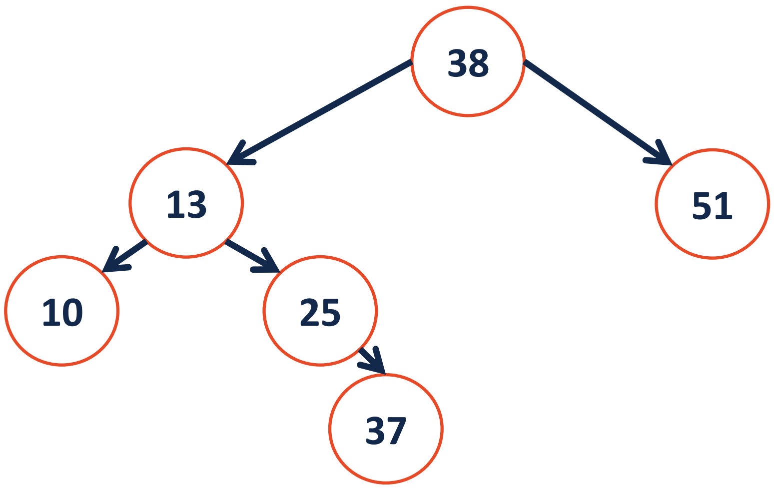
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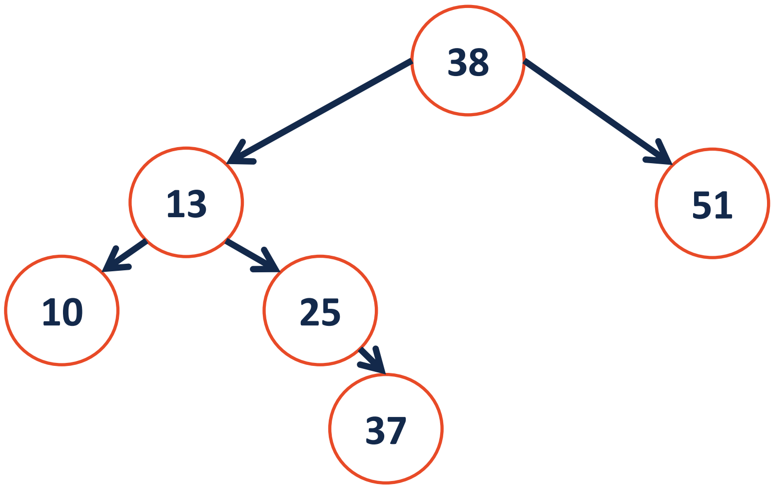
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BST Rotation Summary

- Four kinds of rotations (L, R, LR, RL)
- All rotations are local (subtrees are not impacted)
- All rotations are constant time: O(1)
- BST property maintained

GOAL:

We call these trees:

AVL Trees

Three issues for consideration:

- Rotations
- Maintaining Height
- Detecting Imbalance