CS 225

Data Structures

April 26 – Dijkstra's Algorithm Analysis

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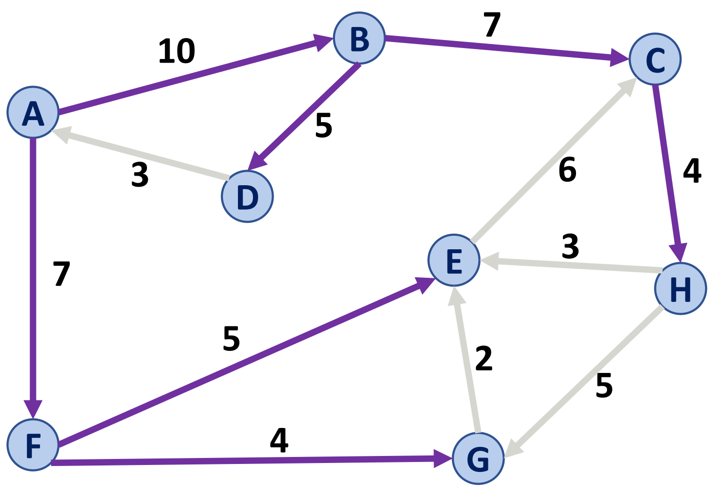
Dijkstra’s Algorithm (SSSP)



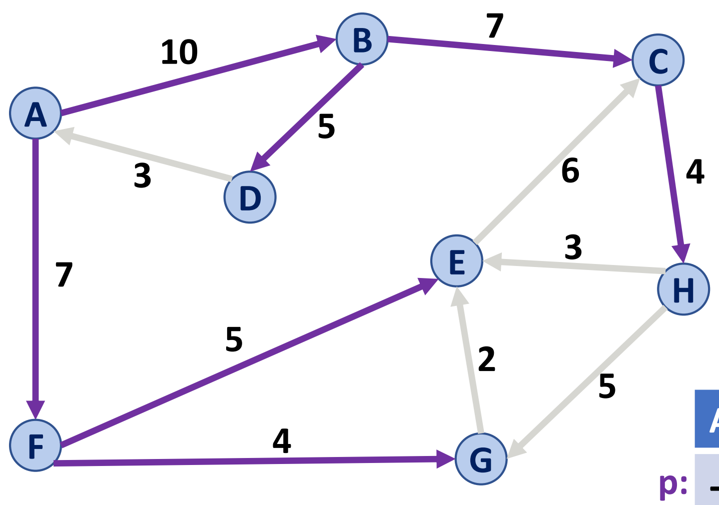
```
DijkstraSSSP(G, s):
  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T         // "labeled set"

  repeat n times:
    Vertex u = Q.removeMin()
    T.add(u)
    foreach (Vertex v : neighbors of u not in T):
      if cost(u, v) + d[u] < d[v]:
        d[v] = cost(u, v) + d[u]
        p[v] = m
```

Dijkstra’s Algorithm (SSSP)

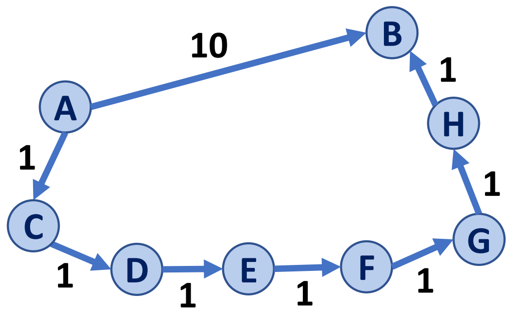
Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	--	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

Dijkstra’s Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



Dijkstra’s Algorithm (SSSP)

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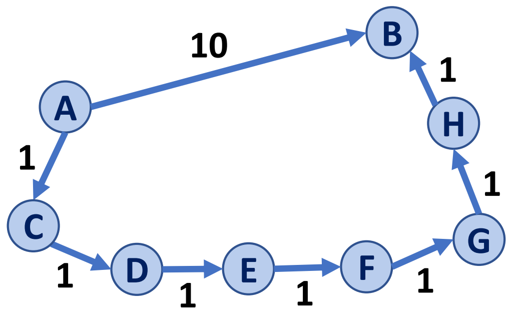
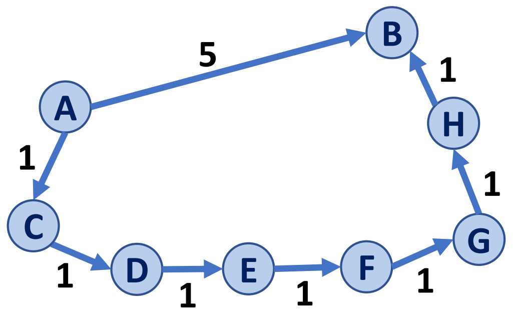

$$\begin{tikzcd}
A \arrow[r, "10"] \arrow[d, "1"] & & & & B \\
C \arrow[r, "1"] & D \arrow[r, "1"] & E \arrow[r, "1"] & F \arrow[r, "1"] & G \arrow[r, "1"] & H \arrow[ull, "1"]
\end{tikzcd}$$

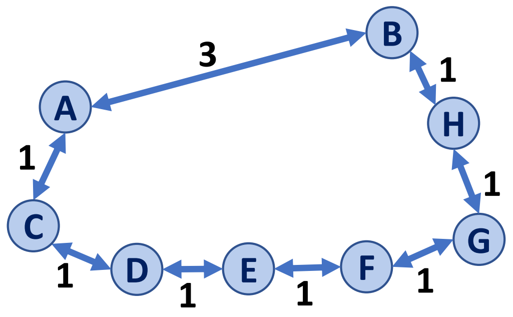
Dijkstra’s Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?


$$\begin{tikzcd}
 & & & & & B \\
A \arrow[urrrrr, "5"] \arrow[d, "1"'] & & & & & H \arrow[u, "1"'] \\
C \arrow[r, "1"'] & D \arrow[r, "1"'] & E \arrow[r, "1"'] & F \arrow[r, "1"'] & G \arrow[ur, "1"'] &
\end{tikzcd}$$

Dijkstra’s Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



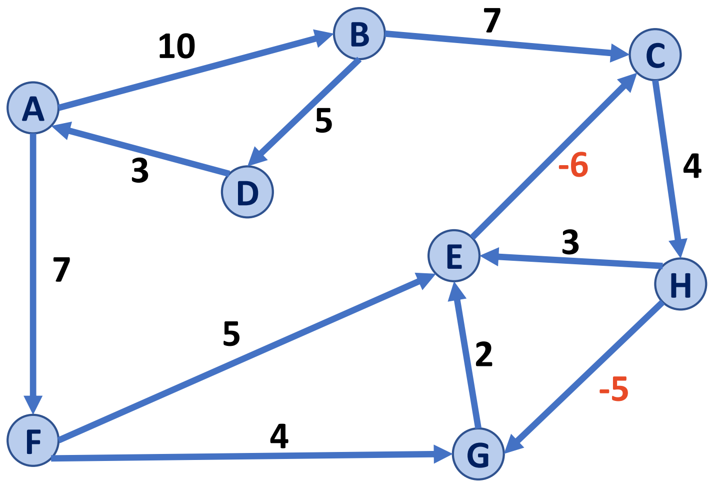
Final Exam Review

- Practice Final Theory Exams Available Now
  - Just like Theory Exam C, infinite exams with random questions
  - **Over 500 questions so far** – lots of practice, lots of variations, lots of ways to ensure you know DS!

- TA Review Session IN LECTURE on Wednesday
  - Final lecture is Monday, April. 29

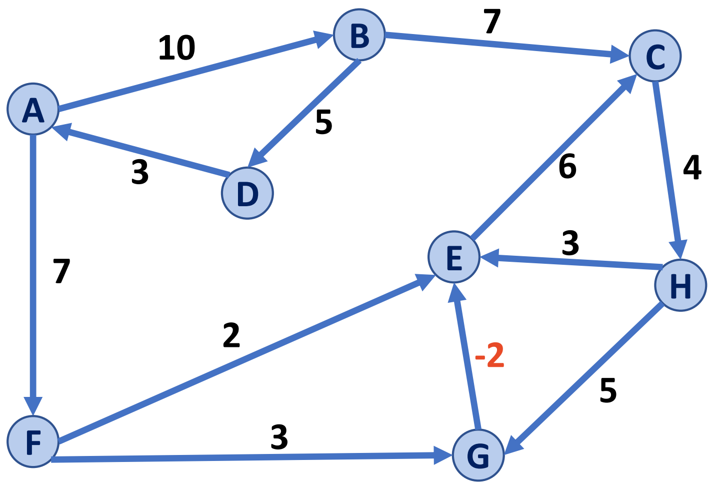
Dijkstra’s Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges, without a negative weight cycle?



Dijkstra's Algorithm (SSSP)

What is Dijkstra's running time?

```
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        d[v] = cost(u, v) + d[u]
        p[v] = m

  return T
```
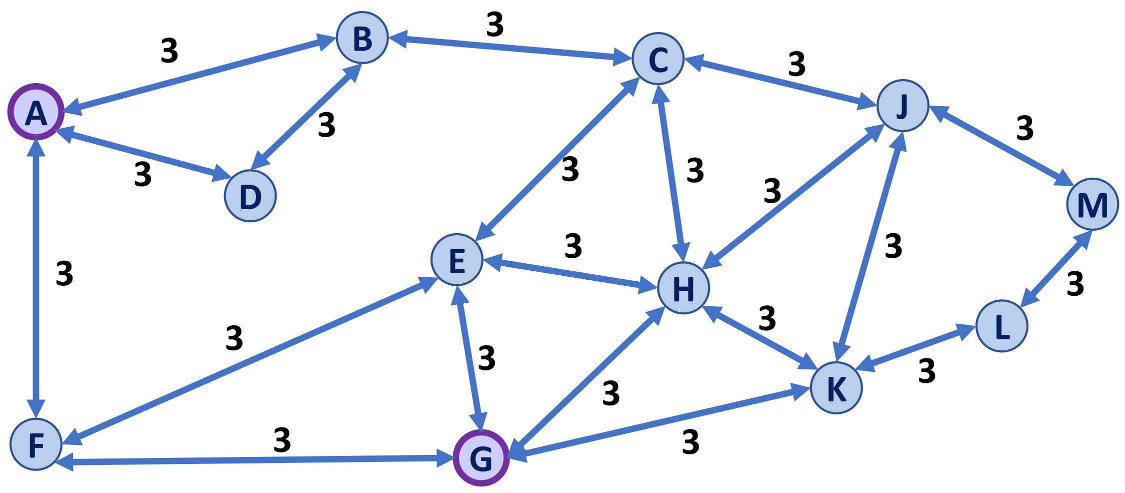

Landmark Path Problem

Suppose you want to travel from **A** to **G**.

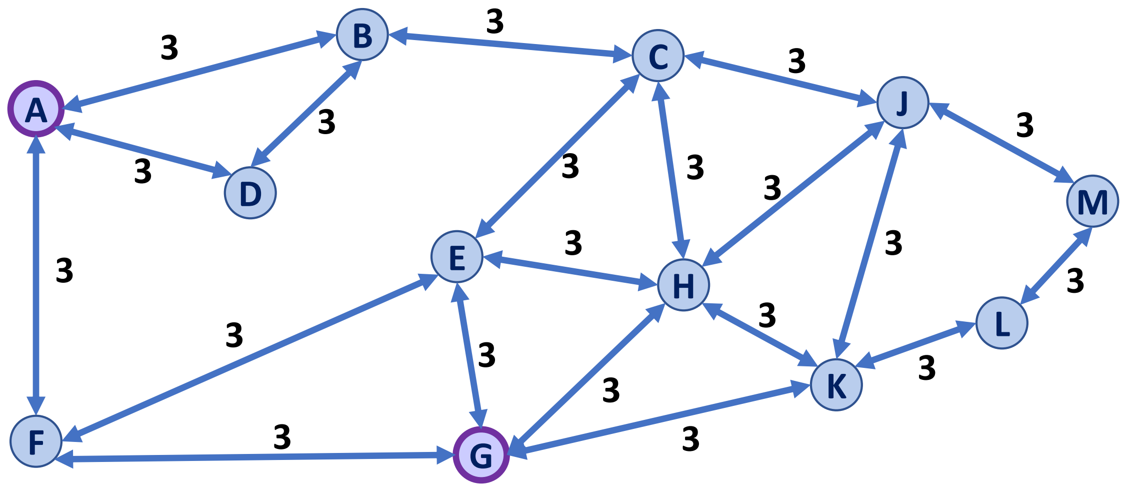
Q1: What is the shortest path from **A** to **G**?



Landmark Path Problem

Suppose you want to travel from **A** to **G**.

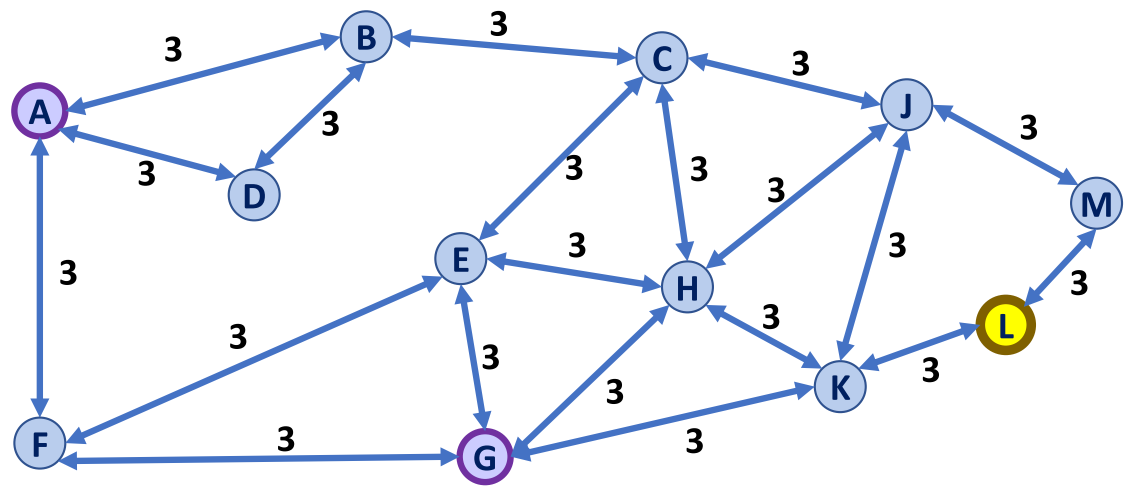
Q2: What is the fastest algorithm to use to find the shortest path?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

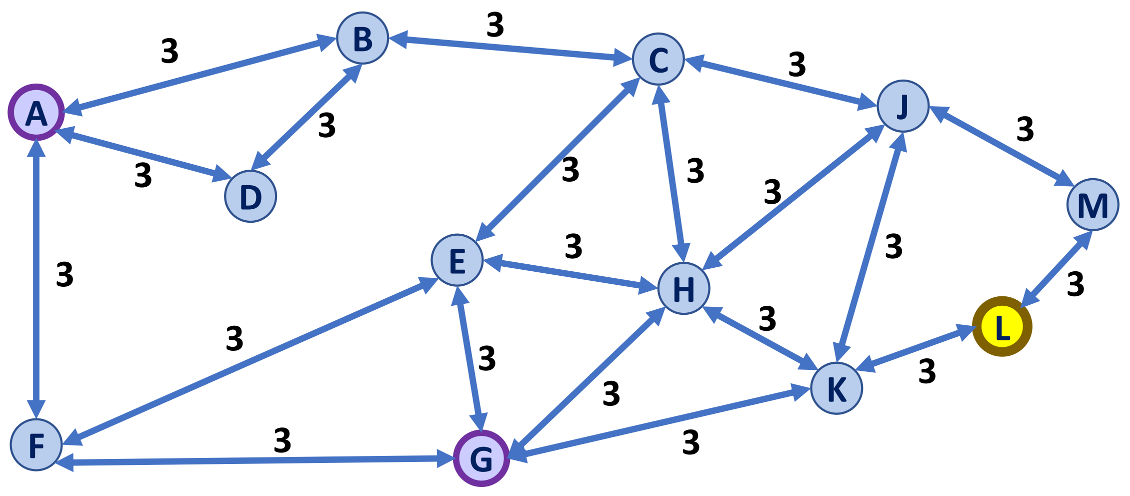
Q3: What is the shortest path from **A** to **G** that visits **L**?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

Q4: What is the fastest algorithm to find this path?
Q5: What are the specific call(s) to this algorithm?



ICES

Thank you for an amazing semester!