



CS 225

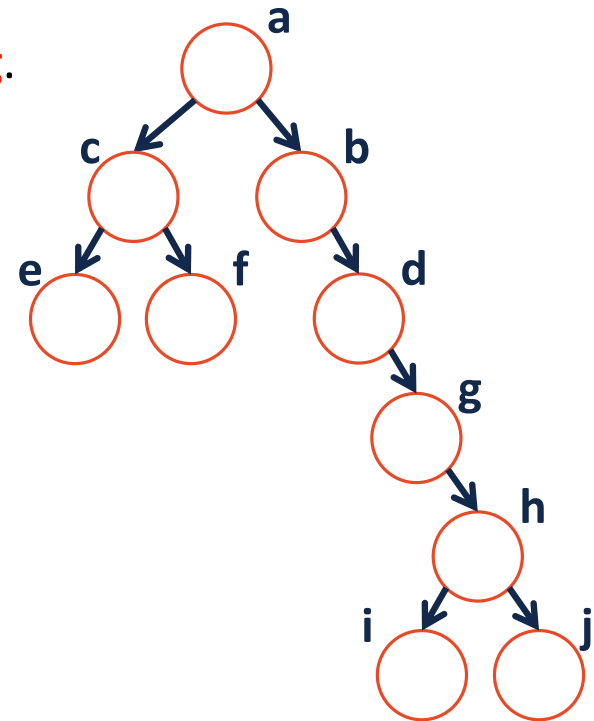
Data Structures

February 15 – Tree Proof

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Tree Terminology

- Find an **edge** that is not on the longest **path** in the tree. Give that edge a reasonable name.
- One of the vertices is called the **root** of the tree. Which one?
- Identify the vertices that have a **parent** but no **sibling**.
- How many parents does each vertex have?
- Which vertex has the fewest **children**?
- Which vertex has the most **ancestors**?
- Which vertex has the most **descendants**?
- List all the vertices in b's left **subtree**.
- List all the **leaves** in the tree.



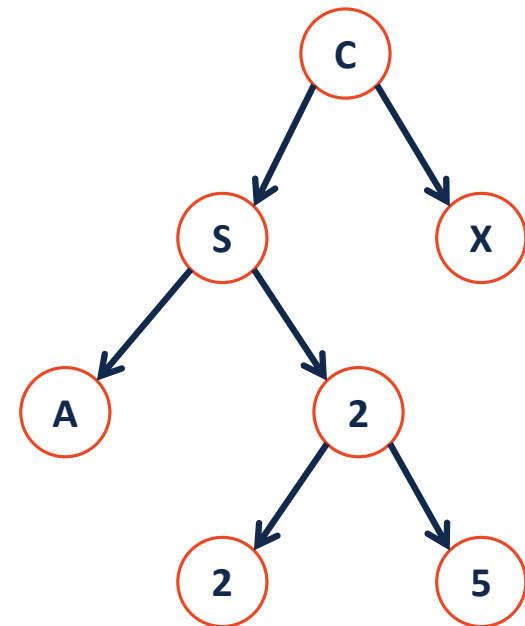
Binary Tree – Defined

A binary tree T is either:

-

OR

-

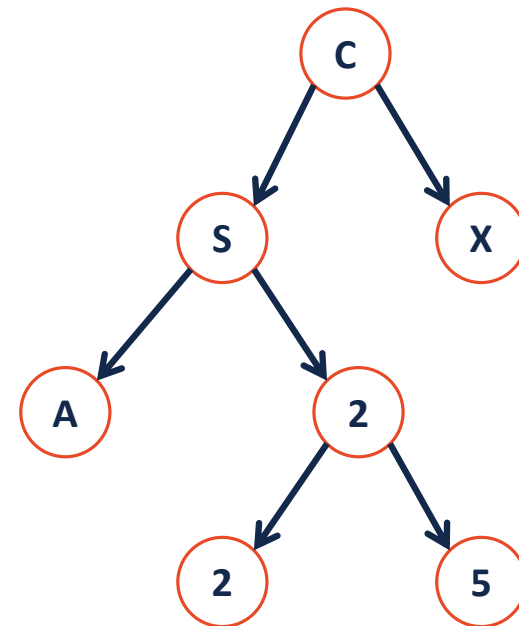


Tree Property: height

height(T): length of the longest path from the root to a leaf

Given a binary tree T:

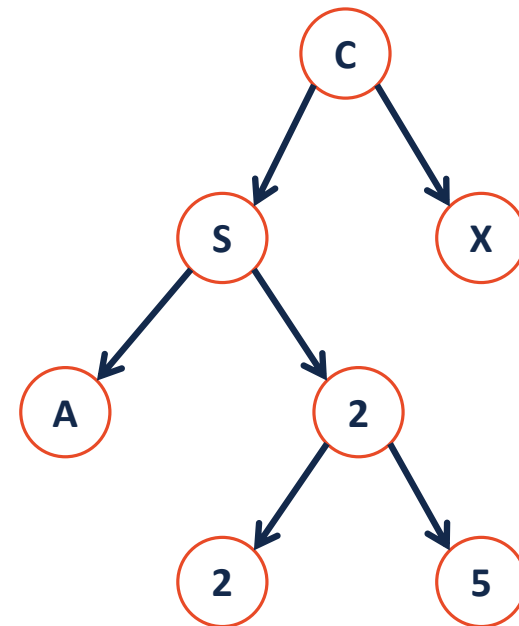
height(T) =



Tree Property: full

A tree F is **full** if and only if:

- 1.
- 2.

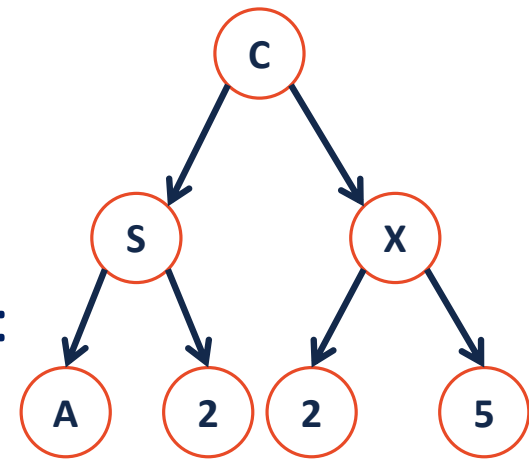


Tree Property: perfect

A **perfect** tree P is defined in terms of the tree's height.

Let P_h be a perfect tree of height h , and:

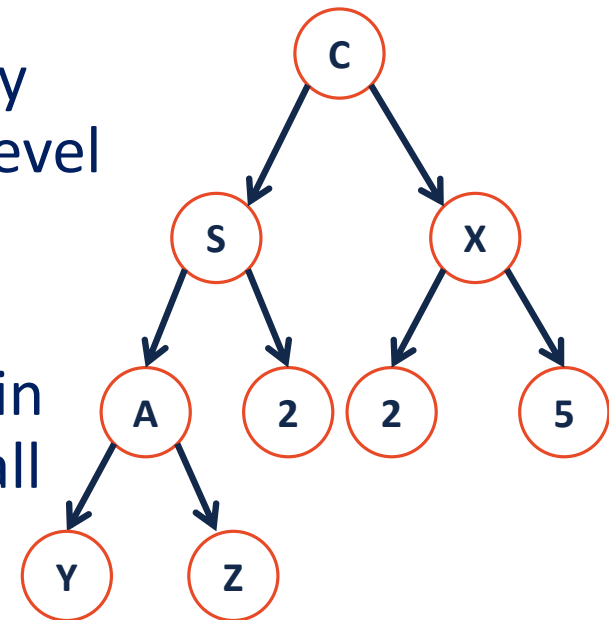
- 1.
- 2.



Tree Property: complete

Conceptually: A perfect tree for every level except the last, where the last level is “pushed to the left”.

Slightly more formal: For all levels k in $[0, h-1]$, k has 2^k nodes. For level h , all nodes are “pushed to the left”.



Tree Property: complete

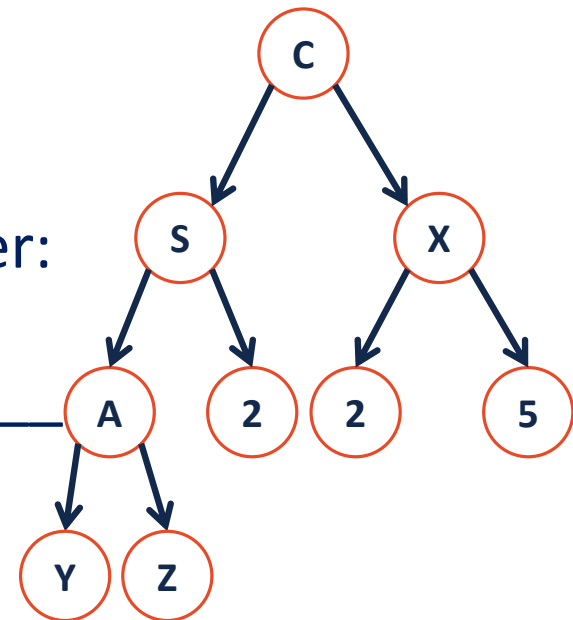
A **complete** tree **C** of height **h**, C_h :

1. $C_{-1} = \{\}$
2. C_h (where $h > 0$) = $\{r, T_L, T_R\}$ and either:

T_L is _____ and T_R is _____

OR

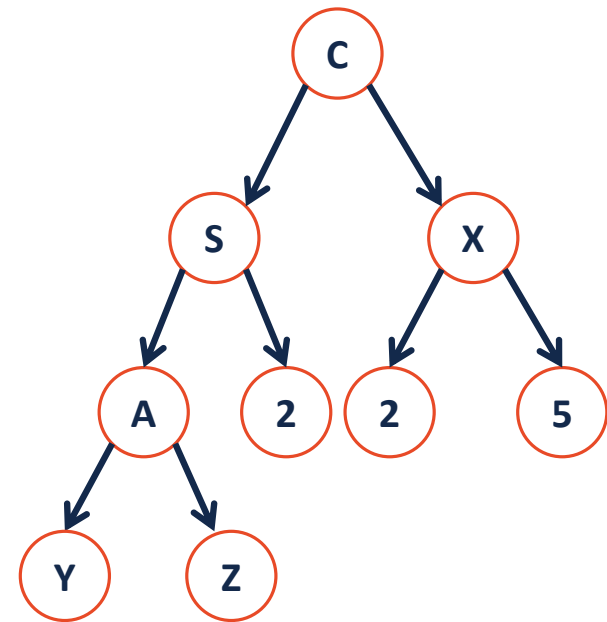
T_L is _____ and T_R is _____



Tree Property: complete

Is every **full** tree **complete**?

If every **complete** tree **full**?





Open Office Hours



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CS 225 has **over 50 hours of open office hours each week**, lots of time to get help!



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1. Understand the problem, don't just give up.
 - "I segfaulted" is not enough. *Where? Any idea why?*



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2. Your topic must be specific to one function, one test case, or one exam question.

- Helps us know what to focus on before we see you!
- Helps your peers to ensure all get questions answered!



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3. Get stuck, get help – not the other way around.
 - If you immediately re-add yourself, you're setting yourself up for failure.



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4. Be awesome.



Tree ADT



Tree ADT

insert, inserts an element to the tree.

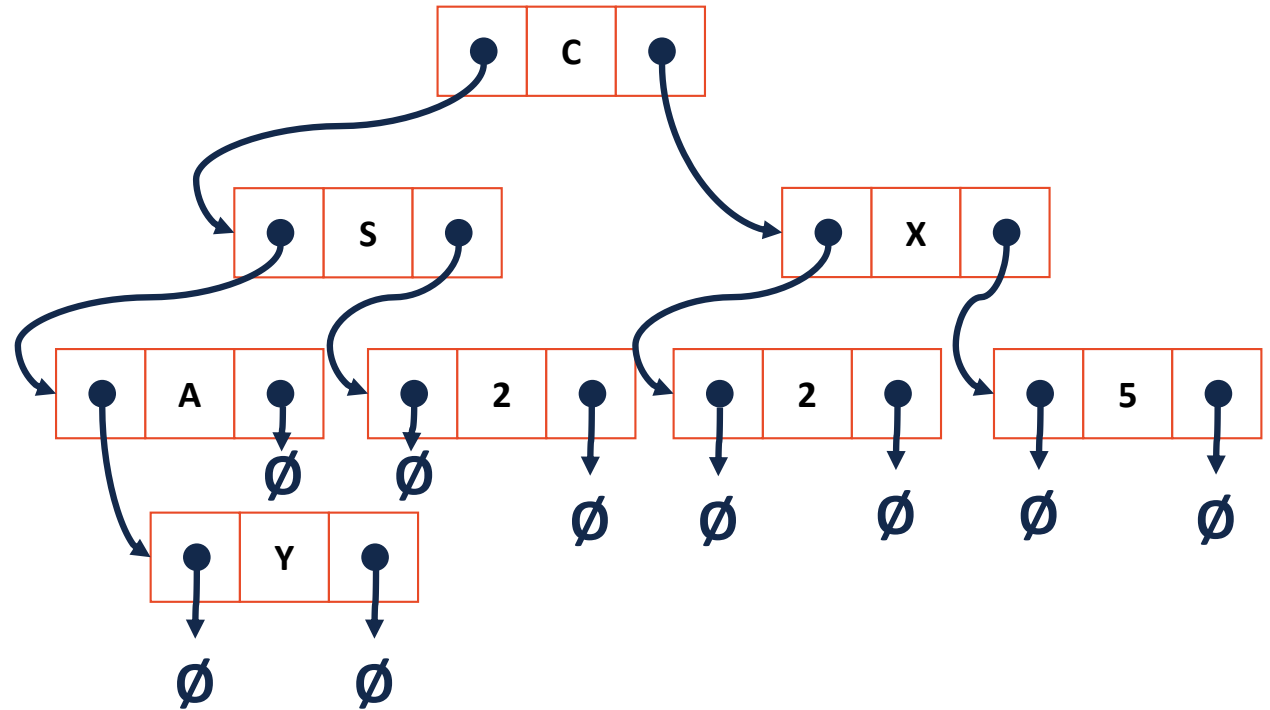
remove, removes an element from the tree.

traverse,

BinaryTree.h

```
1 #pragma once
2
3 template <class T>
4 class BinaryTree {
5     public:
6         /* ... */
7
8     private:
9
10
11
12
13
14
15
16
17
18
19 };
```

Trees aren't new:





How many NULLs?

Theorem: If there are n data items in our representation of a binary tree, then there are _____ NULL pointers.



How many NULLs?

Base Cases:

$n = 0$:

$n = 1$:

$n = 2$:



How many NULLs?

Induction Hypothesis:



How many NULLs?

Consider an arbitrary tree T containing n data elements: