#23: BTrees
March 11, 2018 · *Fagen-Ulmschneider, Zilles*

$BTree_m$

Goal: Build a tree that uses ___________________ /node!
...optimize the algorithm for your platform!

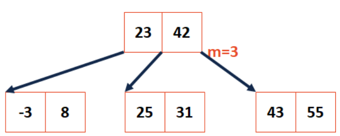
A **BTree of order m** is an m-way tree where:

1. All keys within a node are ordered.

BTree Insert, using m=5

...when a BTree node reaches **m** keys:

BTree Insert, m=3:



Great interactive visualization of BTrees:

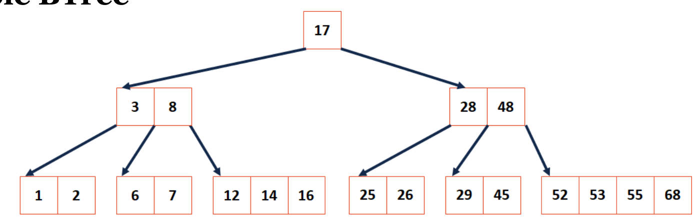
https://www.cs.usfca.edu/~galles/visualization/BTree.html

BTree Properties

For a BTree of order **m**:

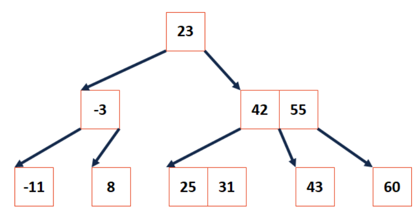
1. All keys within a node are ordered.
2. All leaves contain no more than **m-1** keys.
3. All internal nodes have exactly **one more child than keys**.
4. Root nodes can be a leaf or have **[2, m]** children.
5. All non-root, internal nodes have **[ceil(m/2), m]** children.
6. All leaves are on the same level.

Example BTree



What properties do we know about this BTree?

BTree Search



BTree.hpp

```
bool Btree<K, V>::_exists(BTreeNode & node, const K & key) {
  unsigned i;
  for (i=0; i < node.keys_ct_ && key < node.keys_[i]; i++) { }

  if ( i < node.keys_ct_ && key == node.keys_[i] ) {
    return true;
  }

  if ( node.isLeaf() ) {
    return false;
  } else {
    BTreeNode nextChild = node._fetchChild(i);
    return _exists(nextChild, key);
  }
}
```

BTree Analysis

The height of the BTree determines maximum number of _____________ possible in search data.

...and the height of our structure:

Therefore, the number of seeks is no more than: ____________.

...suppose we want to prove this!

BTree Proof #1

In our AVL Analysis, we saw finding an **upper bound** on the height (**h** given **n,** aka **h = f(n)**) is the same as finding a **lower bound** on the keys (**n** given **h,** aka $\mathbf{f^{-1}(h)}$).

Goal: We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

BTree Strategy:

1. Define a function that counts the minimum number of nodes in a BTree of a given order.
   a. Account for the minimum number of keys per node.
2. Proving a minimum number of nodes provides us with an upper-bound for the maximum possible height.

Proof:

1a. The minimum number of <u>nodes</u> for a BTree of order **m** at each level is as follows:

root:

level 1:

level 2:

level 3:

...

level h:

1b. The minimum total number of <u>nodes</u> is the sum of all levels:

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2. The minimum number of keys:

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3. Finally, we show an upper-bound on height:

CS 225 – Things To Be Doing:
1. Programming Exam B starts on Tuesday **2.** MP4 is due tonight by 11:59pm; MP5 released Tuesday **3.** lab_btree released on Wednesday **4.** Daily POTDs are ongoing!