Choose the following roles for yourselves. If you only have three people, combine Reporter and Process Analyst.

| **Team Roles** | **Team Member** |
|---|---|
| **Manager** Reads the questions out loud, keeps track of time, and makes sure everyone contributes appropriately. | |
| **Reporter** talks to the instructor and other teams. | |
| **Quality Control** records all answers and questions and provides team reflection to team and instructor. | |

## Objectives

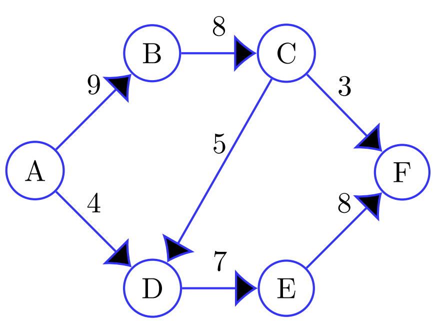
We are going to learn an algorithm for determining the network flow capacity of a graph. In the following graphs, the edge weights represent capacity. There are four sections: spend 10–15 minutes on each one.

## Graph $G1$

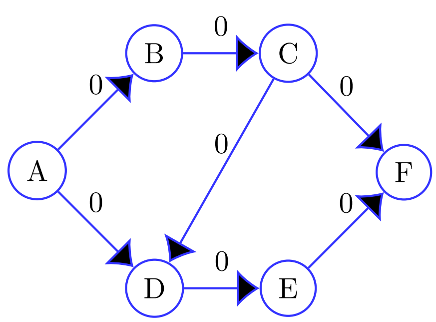
We are given as input graph $G1$.

We create two new graphs: a *flow graph* $F$ and a *residual graph* $R$.

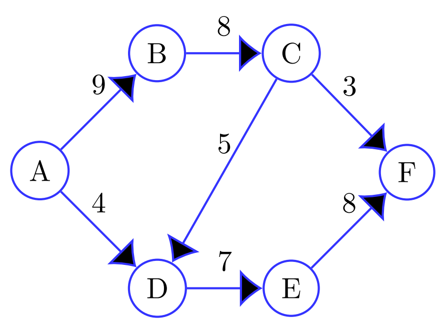
Graph $G$




Flow Graph $F$




Residual Graph $R$

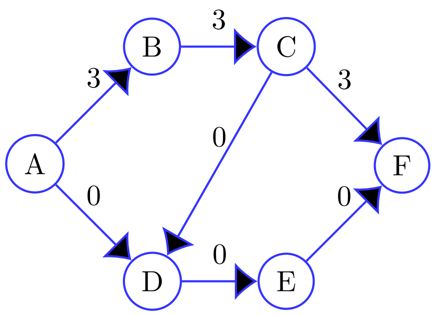



**Problem 1.**

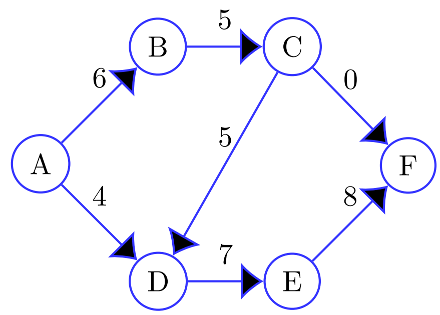
The algorithm works by selecting paths from the residual graph $R$. The first path selected is $A \to B \to C \to F$ in graph $R$. This path's flow capacity is 3. What do you think determines the flow capacity?

The algorithm uses the path to modify graphs $F$ and $R$. Here is the result.

Graph $F$




Graph $R$




**Problem 2.**

Examine the new versions of $F$ and $R$ above. What is being done with the path selected from $R$ to modify these graphs?

**Problem 3.**

The next path selected was $A \to D \to E \to F$ in graph $R$. What is the flow capacity of that path?

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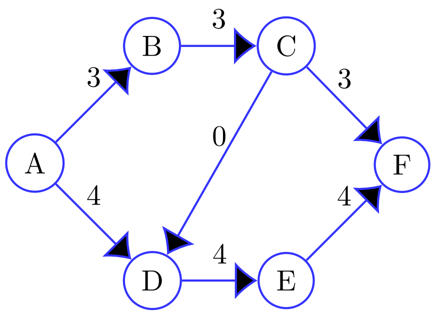
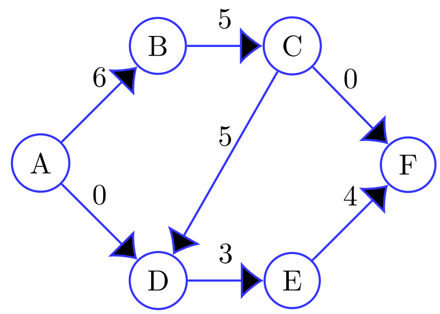
The resulting working graphs are these:

Graph $F$




Graph $R$




**Problem 4.**

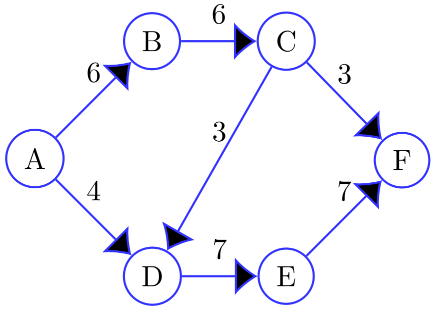
We select path $A \to B \to C \to D \to E \to F$. What is the flow capacity of that path?

**Problem 5.**

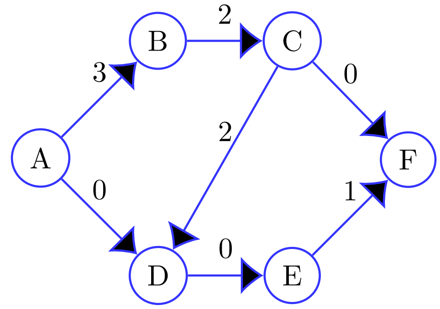
The paths selected always start from node $A$ and end with node $F$. What is different about these nodes compared to the others?

Here are the final working graphs $F$ and $R$.

Graph $F$




Graph $R$




**Problem 6.**

At this point, the algorithm is finished. How can we know the algorithm is done by examining graph $R$?

**Problem 7.**

For nodes $B$, $C$, $D$, and $E$, what is the relationship between the in-flows and the out-flows? Why does that relationship have to exist?

**Problem 8.**

Using the final flow graph $F$ above, determine the maximum flow of graph $G1$.

**Problem 9.**

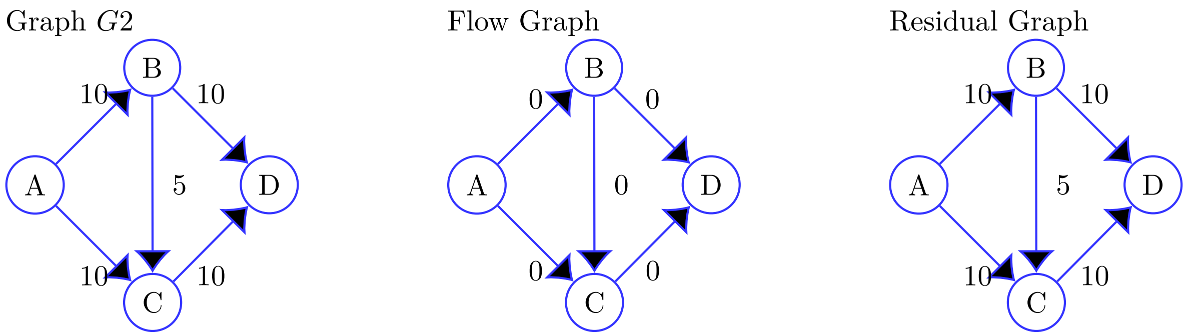
In graph $F$, the outflow of $A$ is equal to the inflow of $F$. Should that always be the case?

**Problem 10.**

Node $A$ is called a *source node* and node $F$ is called a *sink node*. Would this technique work if there were multiple source and sink nodes? Why or why not?

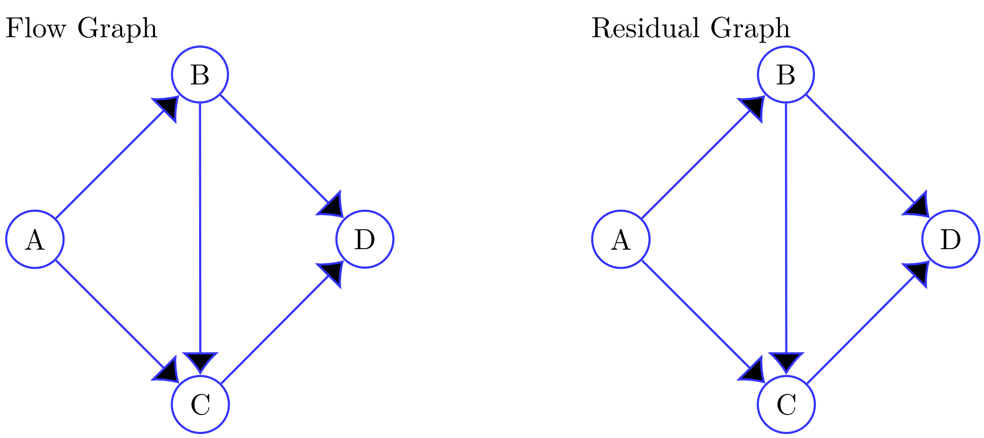
## Graph $G2$

Now we are going to look at a case that messes up the algorithm.




**Problem 11.**
The algorithm picks path $A \to B \to C \to D$. What is the capacity of that path?

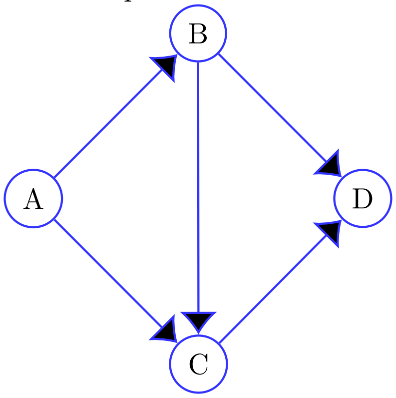
**Problem 12.**
Update the flow and residual graphs as a result of selecting this path.



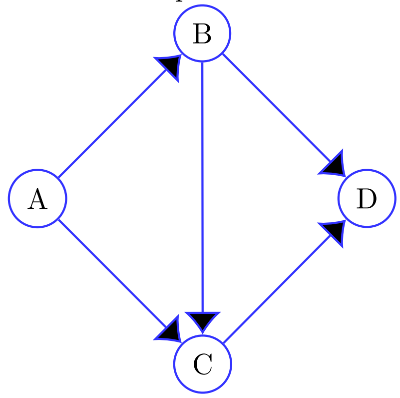

**Problem 13.**
Select path $A \to B \to D$ from the above residual graph. What is the capacity of that path?

**Problem 14.**
Update the flow and residual graphs as a result of selecting this path.

Flow Graph



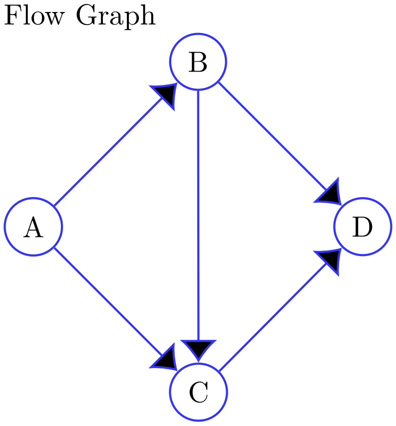

Residual Graph



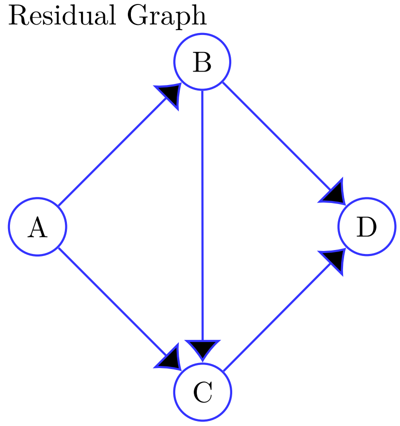

**Problem 15.**
Select path $A \to C \to D$. What is the capacity of that path?

**Problem 16.**
Update the flow and residual graphs as a result of selecting this path.

Flow Graph




Residual Graph




At this point, the algorithm is finished.

**Problem 17.**
What is the maximum network flow of $G2$, according to the algorithm?

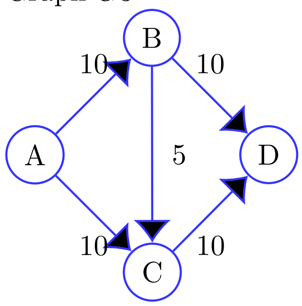
**Problem 18.**
Is this number correct? Why or why not? Examine $G2$ to verify your answer.

**Problem 19.**
Suppose a maximum flow is running through $G2$. What would be the flow on edge $B \to C$ in this situation? Would it change the total flow of $G2$ if we deleted $B \to C$?

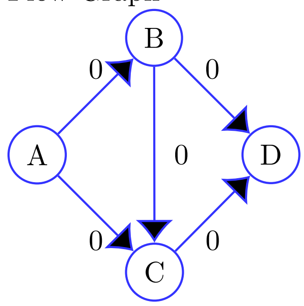
## Graph $G3$

We are going to modify the algorithm. Starting again with the previous graph, we make a new kind of residual graph. The dotted edges are added, and are legal edges to be traversed in the residual graph.

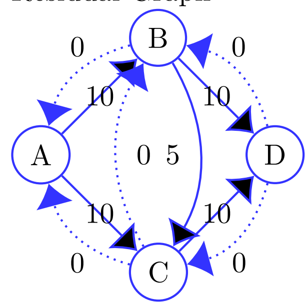
Graph $G3$




Flow Graph




Residual Graph

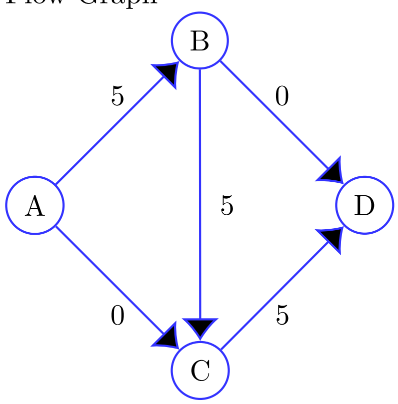



**Problem 20.**

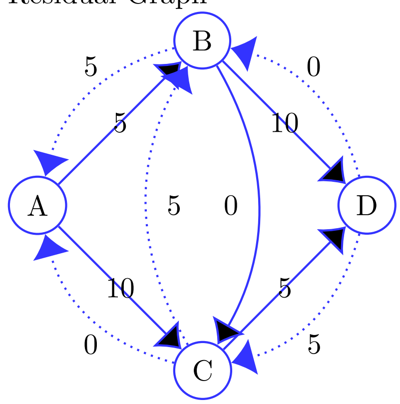
Select path $A \to B \to C \to D$. What is the capacity of that path?

Here are the updated flow and residual graphs:

Flow Graph

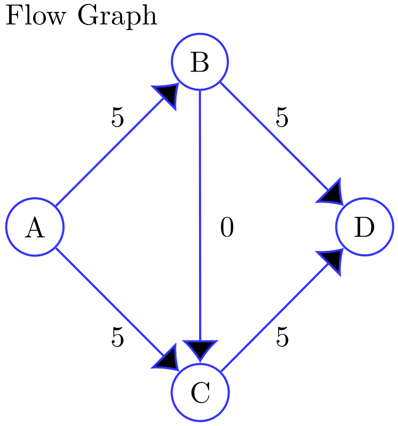



Residual Graph

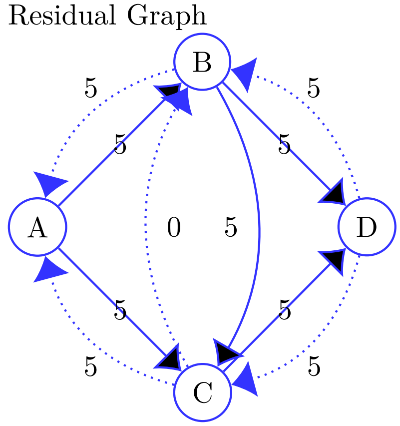



Now we select path $A \to C \to B \to D$.
Here are the updated flow and residual graphs:

Flow Graph




Residual Graph



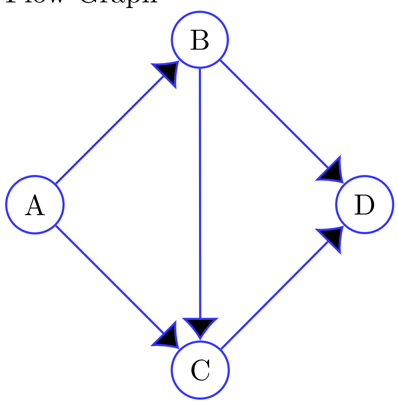
**Problem 21.**

Our algorithm adds capacity to the reverse edges when we update the residual graph. In your own words, can you explain what the reverse edges represent?

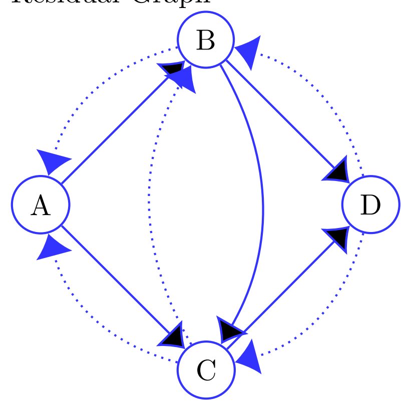
**Problem 22.**

Select path $A \to B \to C \to D$. (Yes, we are repeating this path.) What are the resulting flow and residual graphs?

Flow Graph




Residual Graph

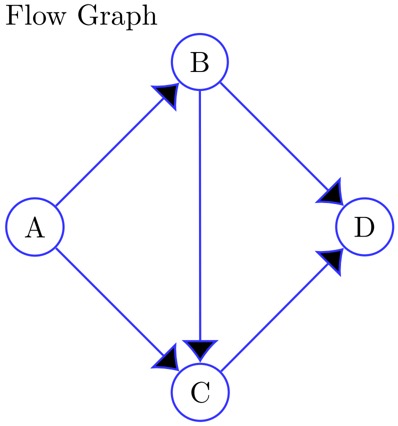



**Problem 23.**

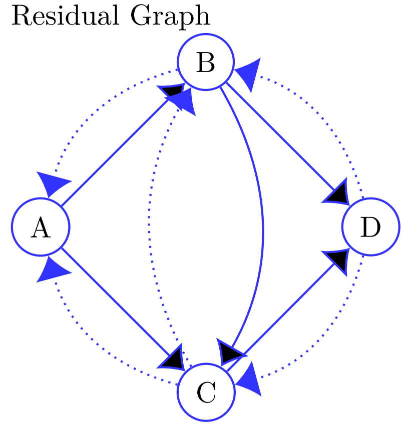
Now we select path $A \to C \to B \to D$.

What are the updated flow and residual graphs?

Flow Graph




Residual Graph




**Problem 24.**

At this point, the algorithm should be done. Is the final network flow accurate now?

## Reflection

Congratulations! You have discovered the Ford-Fulkerson method of determining the maximum flow of a network!

**Problem 25.**
What was the strongest aspect of your performance as a team?

**Problem 26.**
How could your team perform even more effectively on the next activity?

**Problem 27.**
What insights did you have working on this activity?