CS 225

Data Structures

April 11 – Graphs

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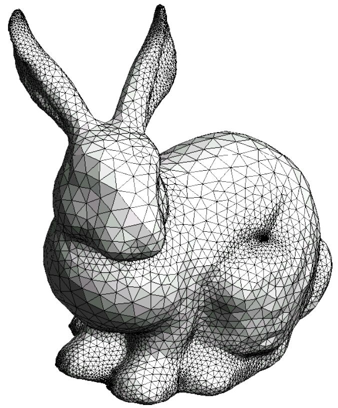
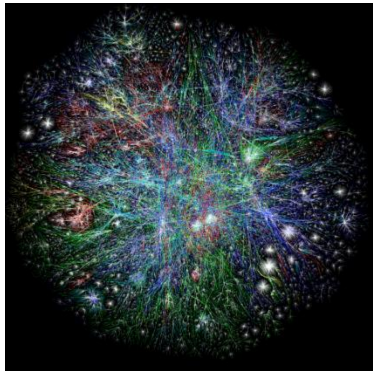
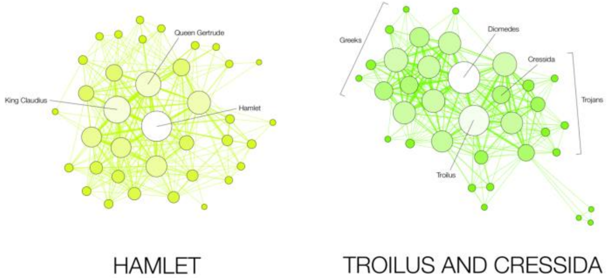
Graphs

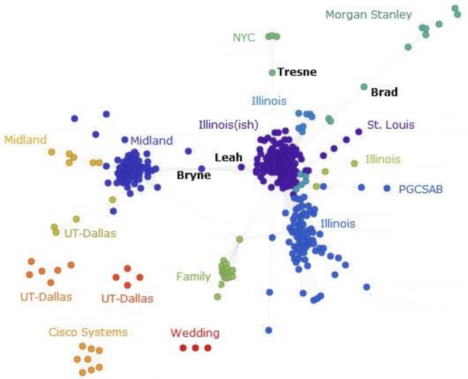


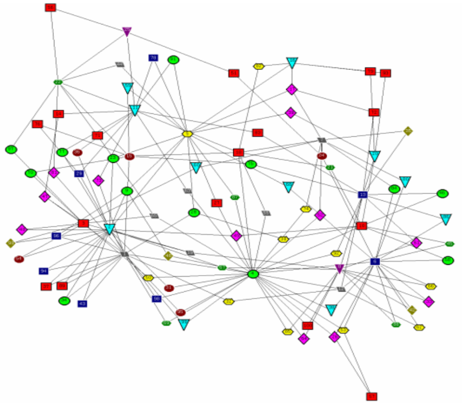


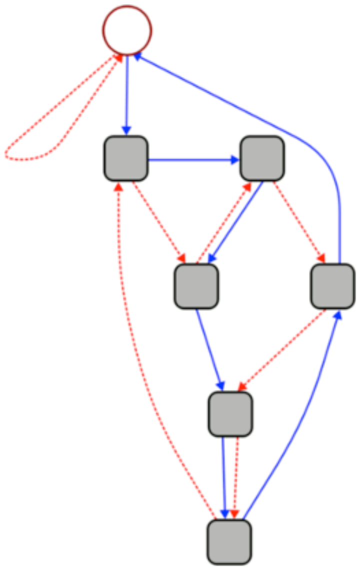
To study all of these structures:

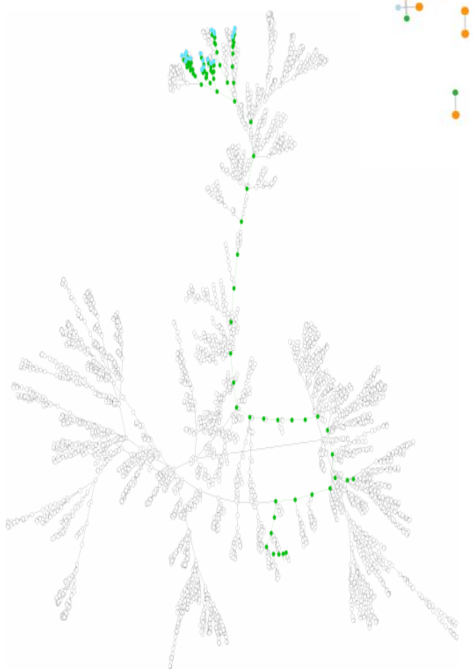
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

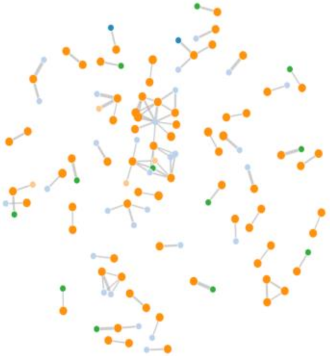










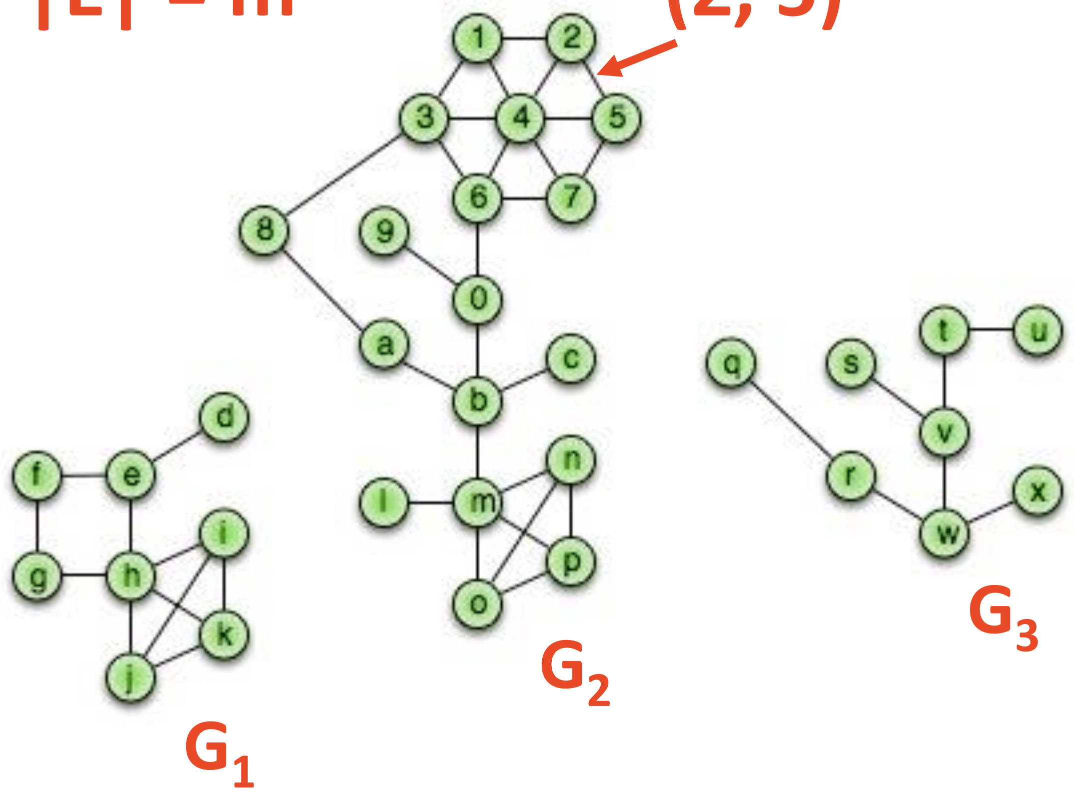


Graph Vocabulary

$G = (V, E)$

$|V| = n$

$|E| = m$



Incident Edges:
$I(v) = \{ (x, v) \text{ in } E \}$

Degree(v): $|I|$

Adjacent Vertices:
$A(v) = \{ x : (x, v) \text{ in } E \}$

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

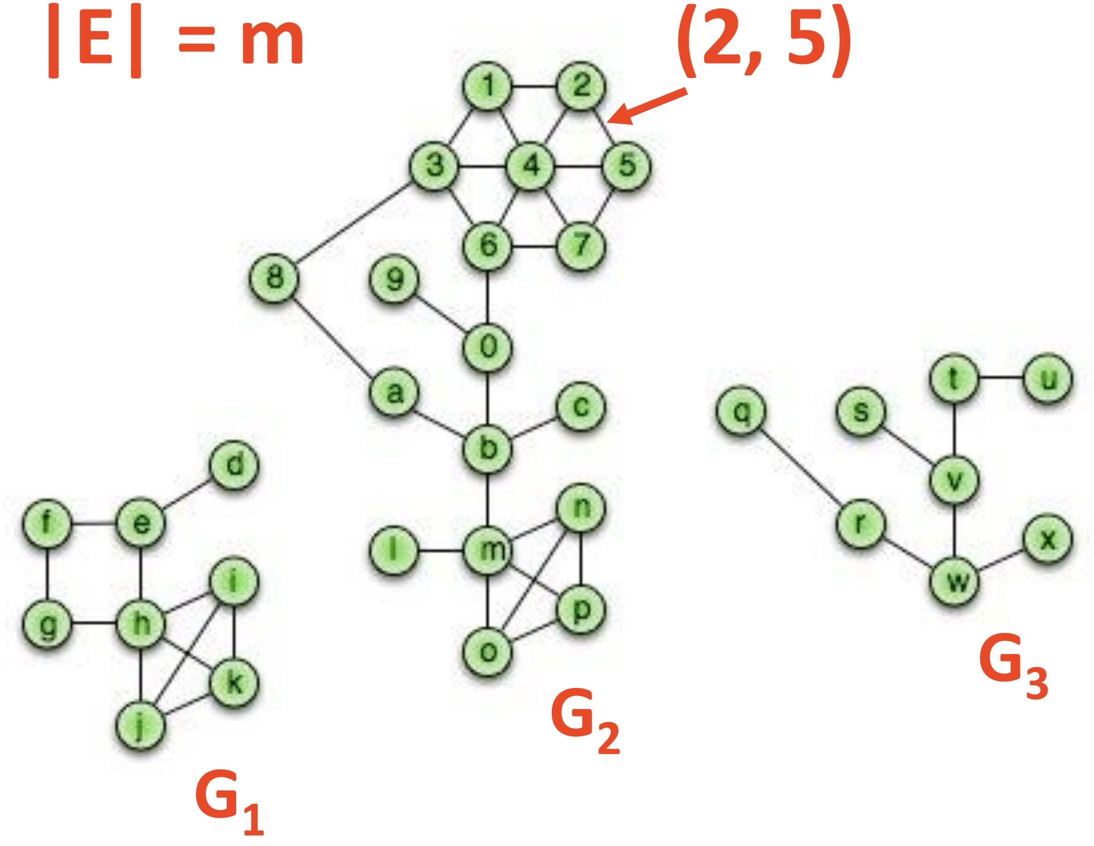
Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

G = (V, E)

|V| = n

|E| = m



Subgraph(G):

G' = (V', E'):

- V' ∈ V, E' ∈ E, and
- (u, v) ∈ E → u ∈ V', v ∈ V'

Complete subgraph(G)

Connected subgraph(G)

Connected component(G)

Acyclic subgraph(G)

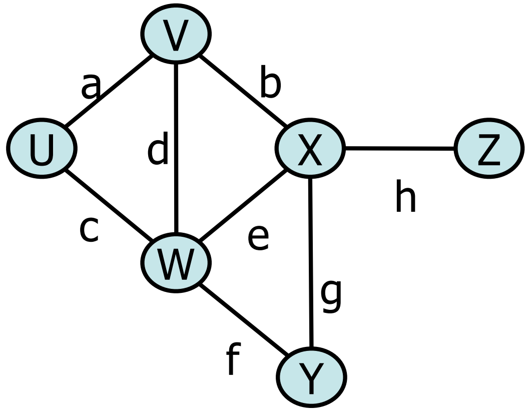
Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges? **Minimum edges:**

Not Connected:

Connected*:



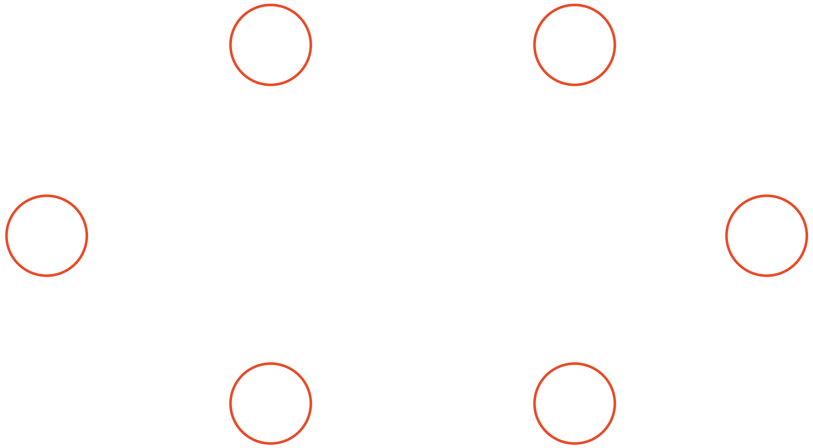
Maximum edges:

Simple:

Not simple:

$$\sum_{v \in V} \deg(v) =$$

Connected Graphs



Proving the size of a minimally connected graph

Theorem:
Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Thm: Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Proof: Consider an arbitrary, minimally connected graph **G=(V, E)**.

Lemma 1: Every connected subgraph of **G** is minimally connected.
(Easy proof by contradiction left for you.)

Inductive Hypothesis: For any **j < |V|**, any minimally connected graph of **j** vertices has **j-1** edges.

Suppose |V| = 1:

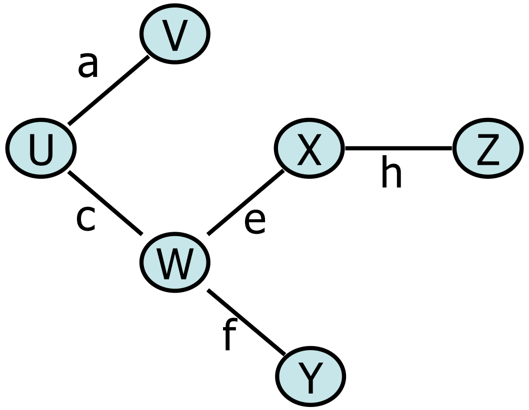
Definition: A minimally connected graph of 1 vertex has 0 edges.

Theorem: |V|-1 edges → 1-1 = 0.

Suppose |V| > 1:

Choose any vertex **u** and let **d** denote the degree of **u**.

Remove the incident edges of **u**, partitioning the graph into ___ components: $\mathbf{C_0 = (V_0, E_0)}$, …, $\mathbf{C_d = (V_d, E_d)}$.



By Lemma 1, every component $\mathbf{C_k}$ is a minimally connected subgraph of **G**.

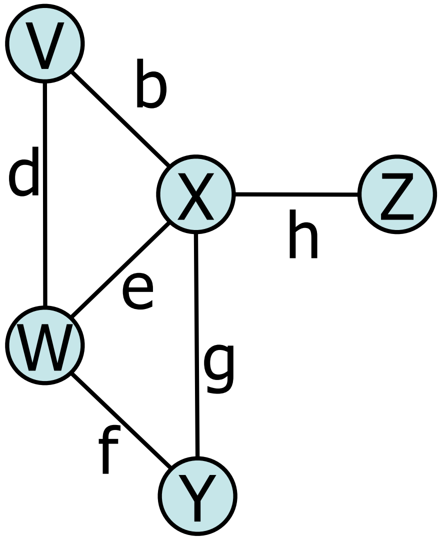
By our ________: ____________________.

Finally, we count edges:

Graph ADT

Data:

- **Vertices**
- **Edges**
- **Some data structure maintaining the structure between vertices and edges.**



Functions:

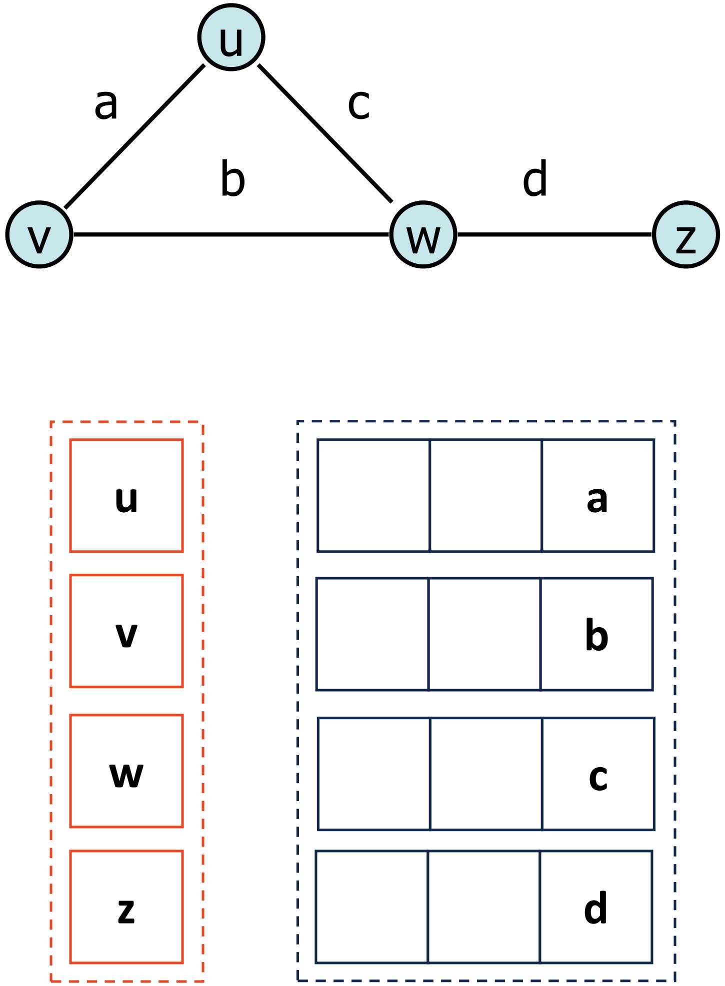
- **insertVertex(K key);**
- **insertEdge(Vertex v1, Vertex v2, K key);**

- **removeVertex(Vertex v);**
- **removeEdge(Vertex v1, Vertex v2);**

- **incidentEdges(Vertex v);**
- **areAdjacent(Vertex v1, Vertex v2);**

- **origin(Edge e);**
- **destination(Edge e);**

Graph Implementation: Edge List



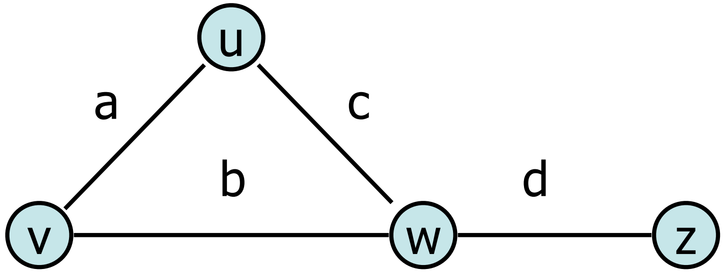
insertVertex(K key);

removeVertex(Vertex v);

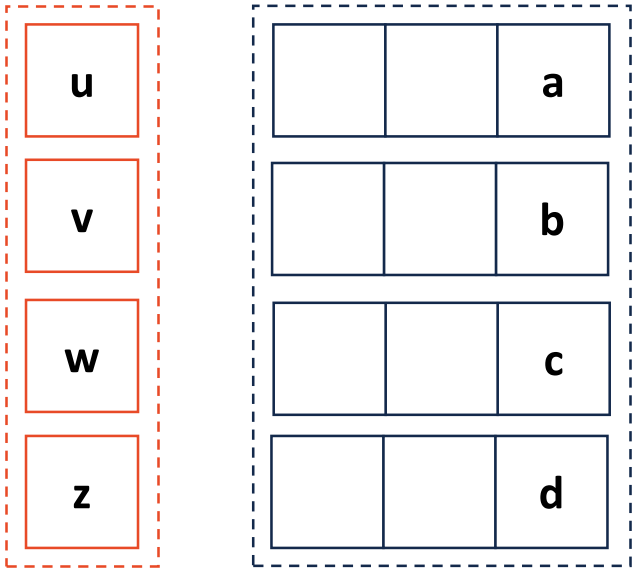
areAdjacent(Vertex v1, Vertex v2);

incidentEdges(Vertex v);

Graph Implementation: Adjacency Matrix



insertVertex(K key);
removeVertex(Vertex v);
areAdjacent(Vertex v1, Vertex v2);
incidentEdges(Vertex v);



	u	v	w	z
u				
v				
w				
z				