CS 225

Data Structures

April 9 – Graphs Intro

Wade Fagen-Ulmschneider

Disjoint Sets Analysis

The **iterated log** function:

The number of times you can take a log of a number.

log*(n) =

0 , $n \le 1$

1 + log*(log(n)) , $n > 1$

What is **lg*(2^{65536})**?

Disjoint Sets Analysis

In an Disjoint Sets implemented with smart **unions** and path compression on **find**:

Any sequence of **m** **union** and **find** operations result in the worse case running time of O(____________),

where **n** is the number of items in the Disjoint Sets.

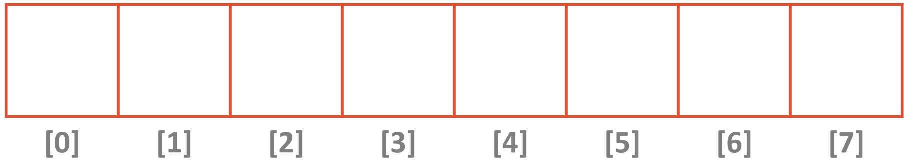
In Review: Data Structures

Array

- Sorted Array
- Unsorted Array
  - Stacks
  - Queues
  - Hashing
  - Heaps
    - Priority Queues
  - UpTrees
    - Disjoint Sets

List

- Doubly Linked List
- Skip List
- Trees
  - BTree
  - Binary Tree
    - Huffman Encoding
    - kd-Tree
    - AVL Tree



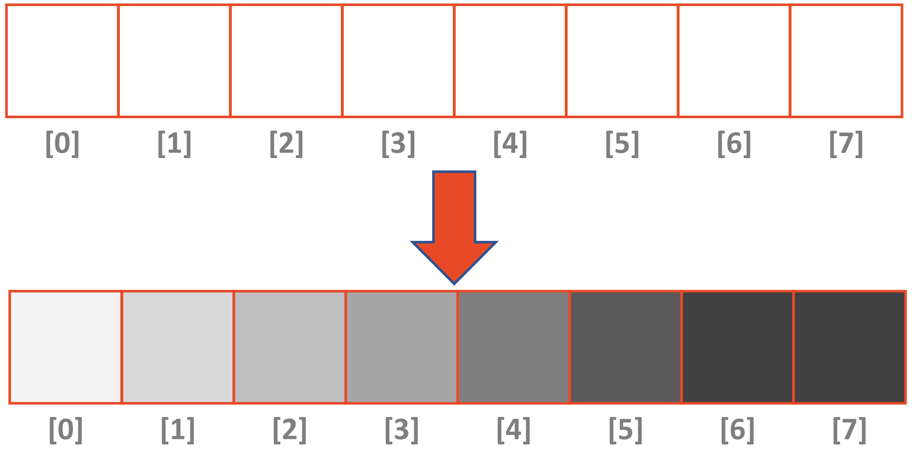
Array

- Constant time access to any element, given an index

  a[k] is accessed in O(1) time, no matter how large the array grows

- Cache-optimized

  Many modern systems cache or pre-fetch nearby memory values due the “Principle of Locality”. Therefore, arrays often perform faster than lists in identical operations.



Array

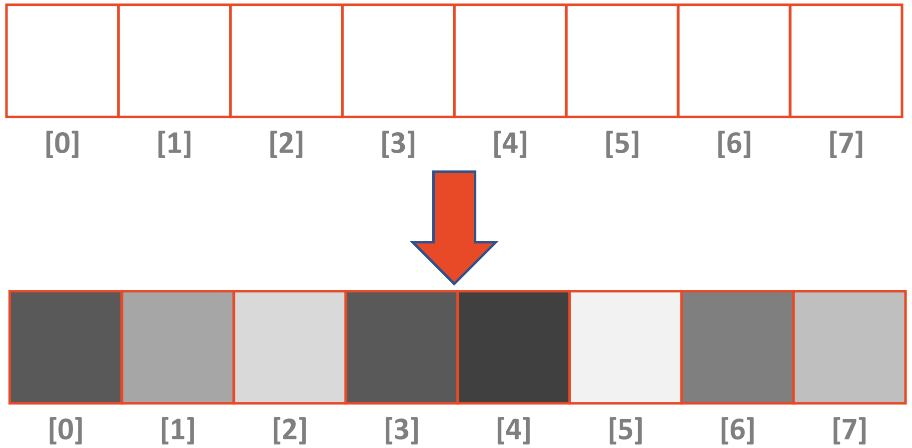
Sorted Array

- Efficient general search structure

  Searches on the sort property run in O(lg(n)) with Binary Search

- Inefficient insert/remove

  Elements must be inserted and removed at the location dictated by the sort property, resulting shifting the array in memory – an O(n) operation



Array

Unsorted Array

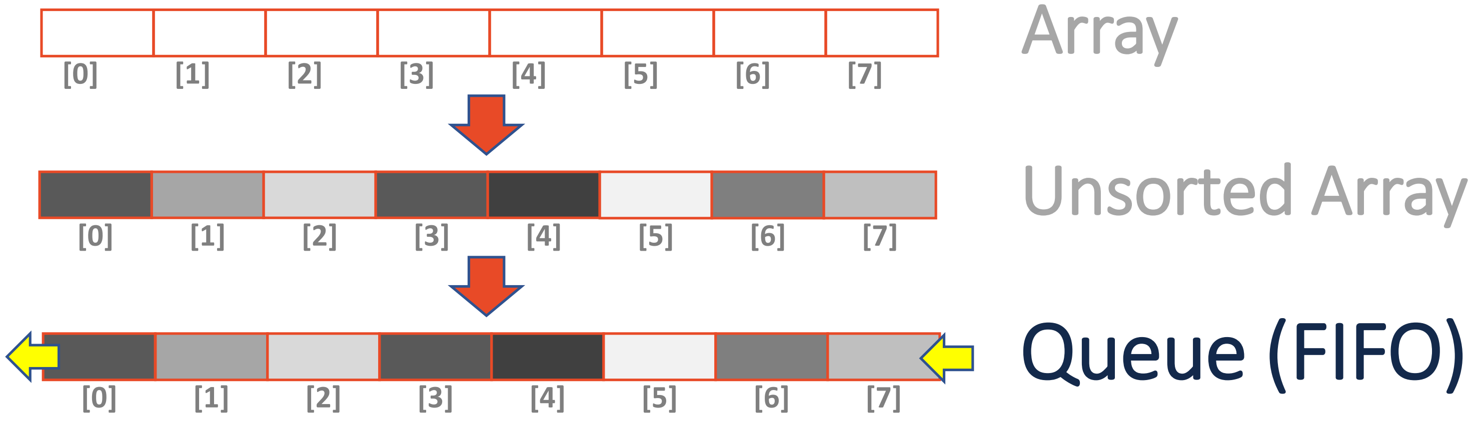
- Constant time add/remove at the beginning/end

  Amortized O(1) insert and remove from the front and of the array

  <u>Idea:</u> Double on resize

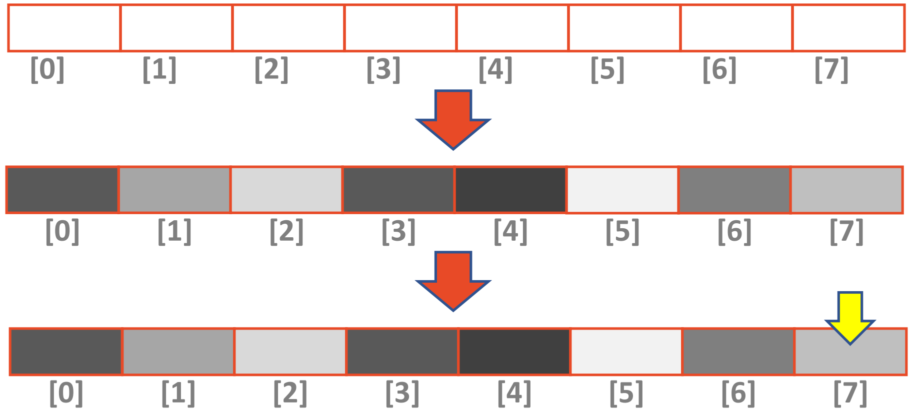
- Inefficient global search structure

  With no sort property, all searches must iterate the entire array; O(1) time



- First In First Out (FIFO) ordering of data
  Maintains an arrival ordering of tasks, jobs, or data

- All ADT operations are constant time operations
  enqueue() and dequeue() both run in O(1) time



Array

Unsorted Array

Stack (LIFO)

- Last In First Out (LIFO) ordering of data
  Maintains a “most recently added” list of data

- All ADT operations are constant time operations
  push() and pop() both run in O(1) time

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Graphs

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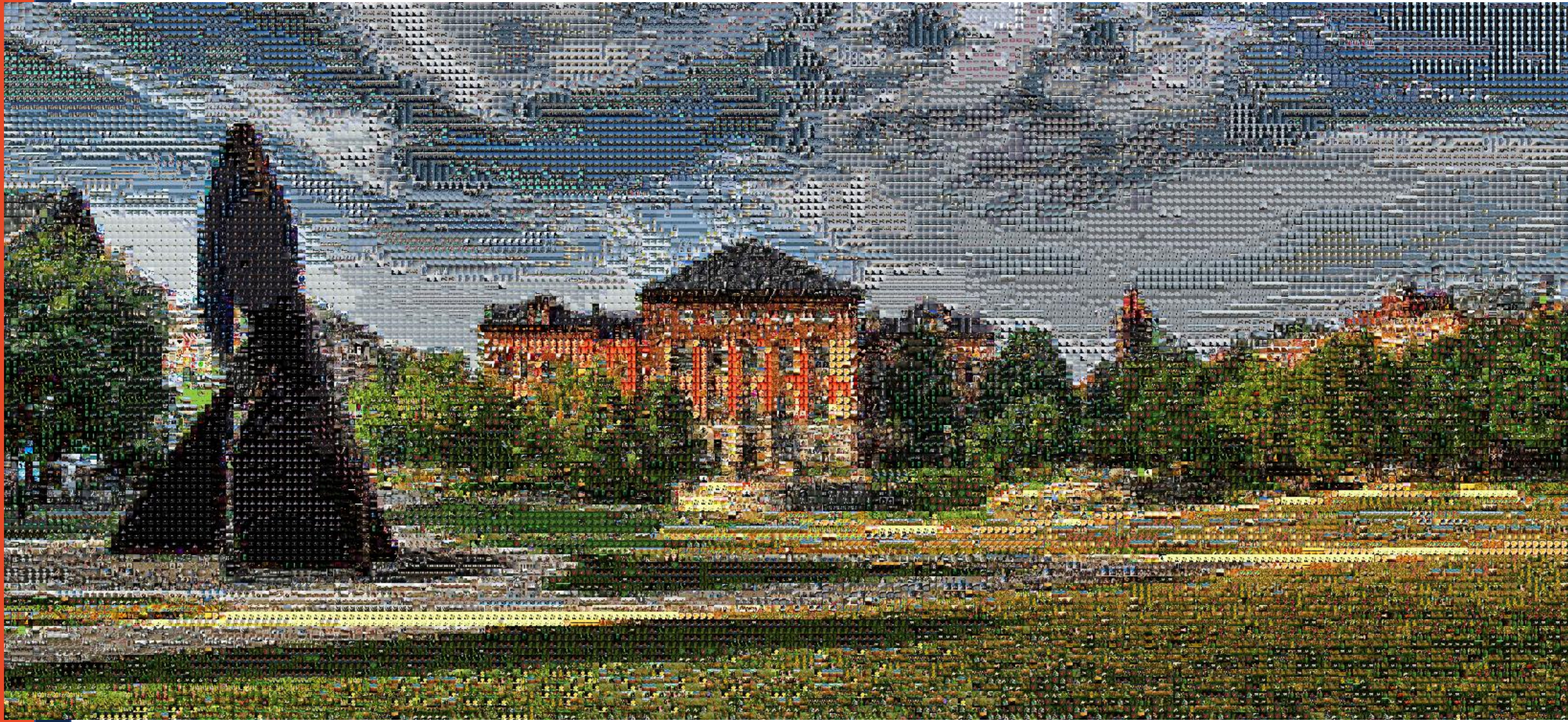
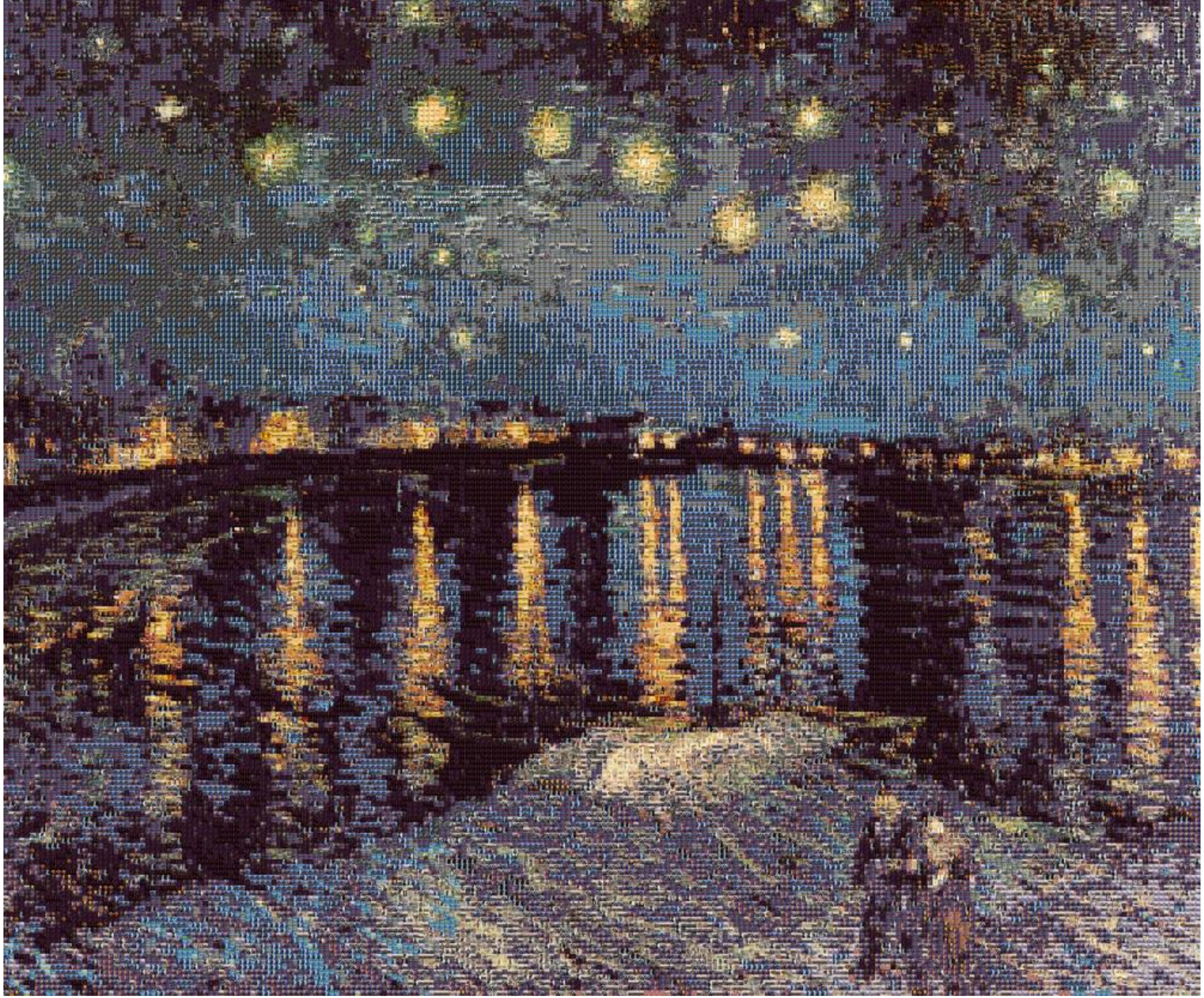
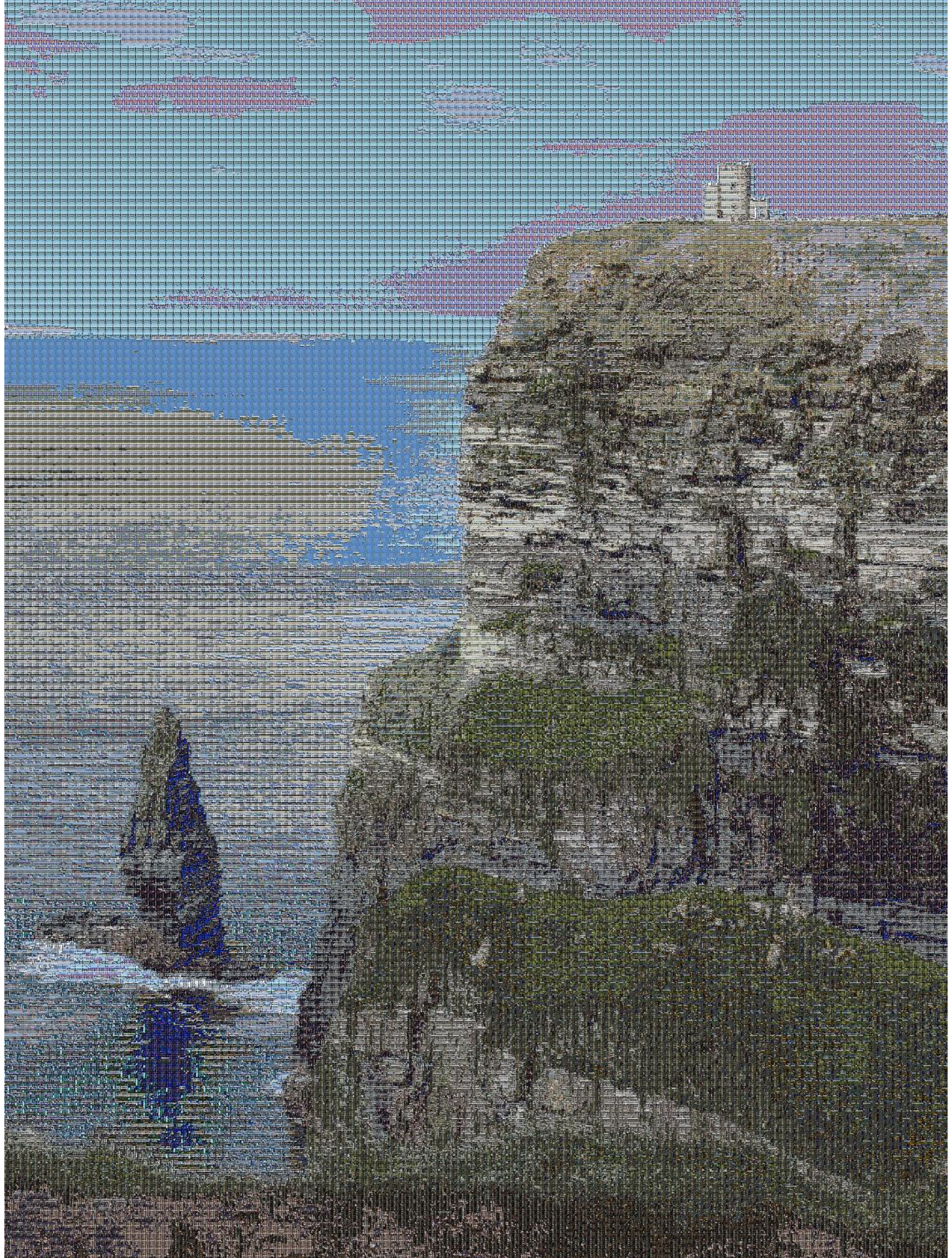
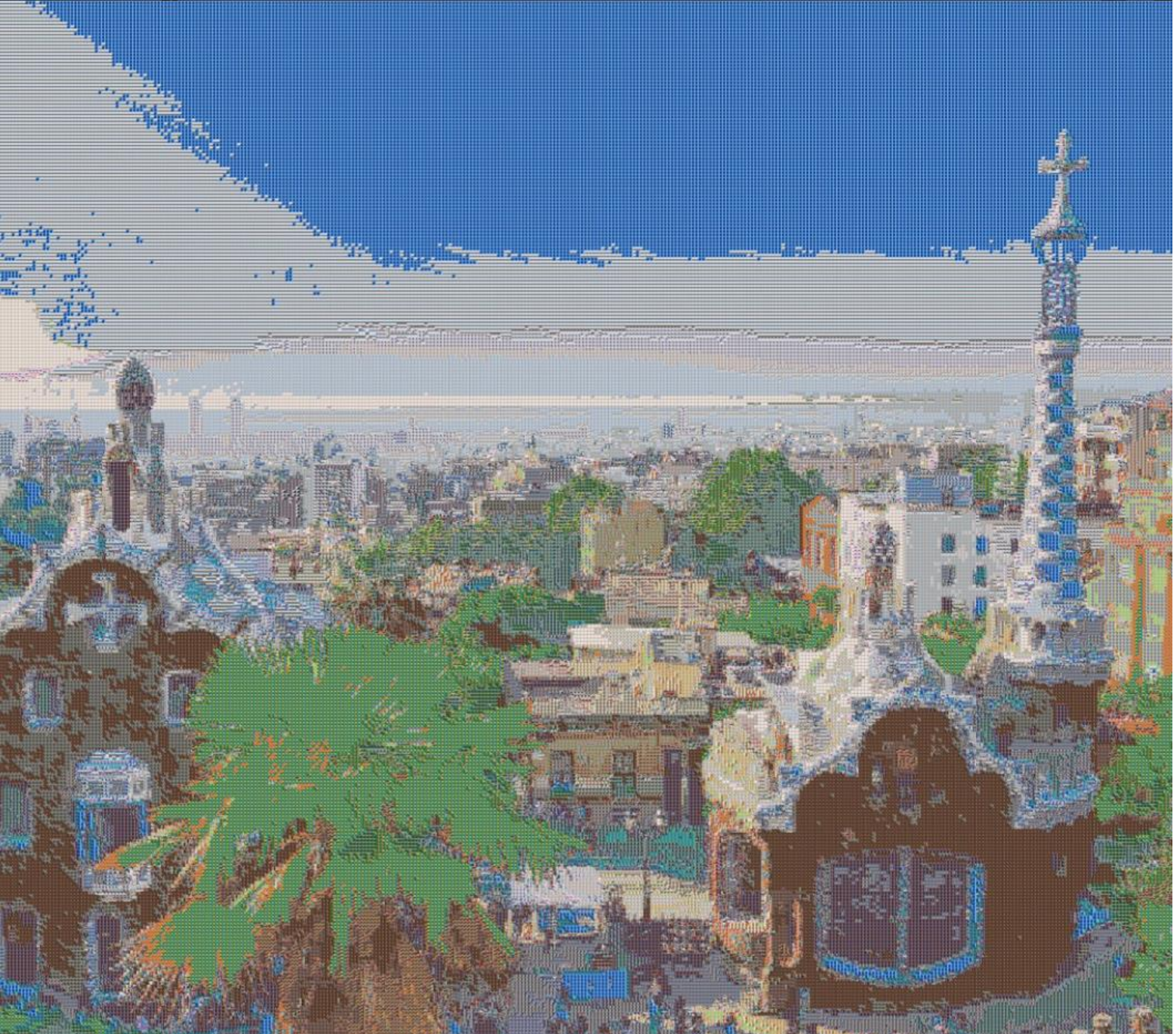
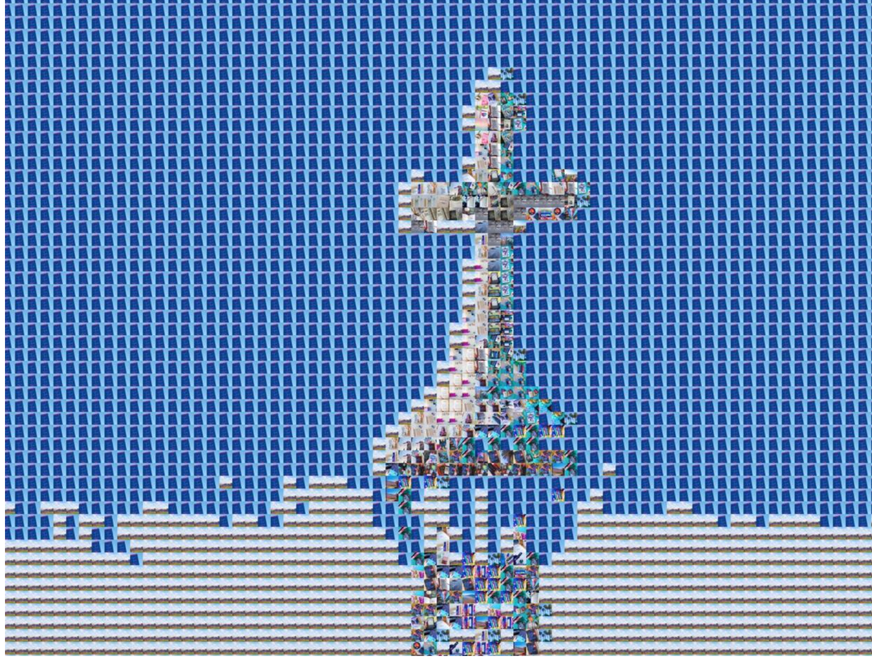
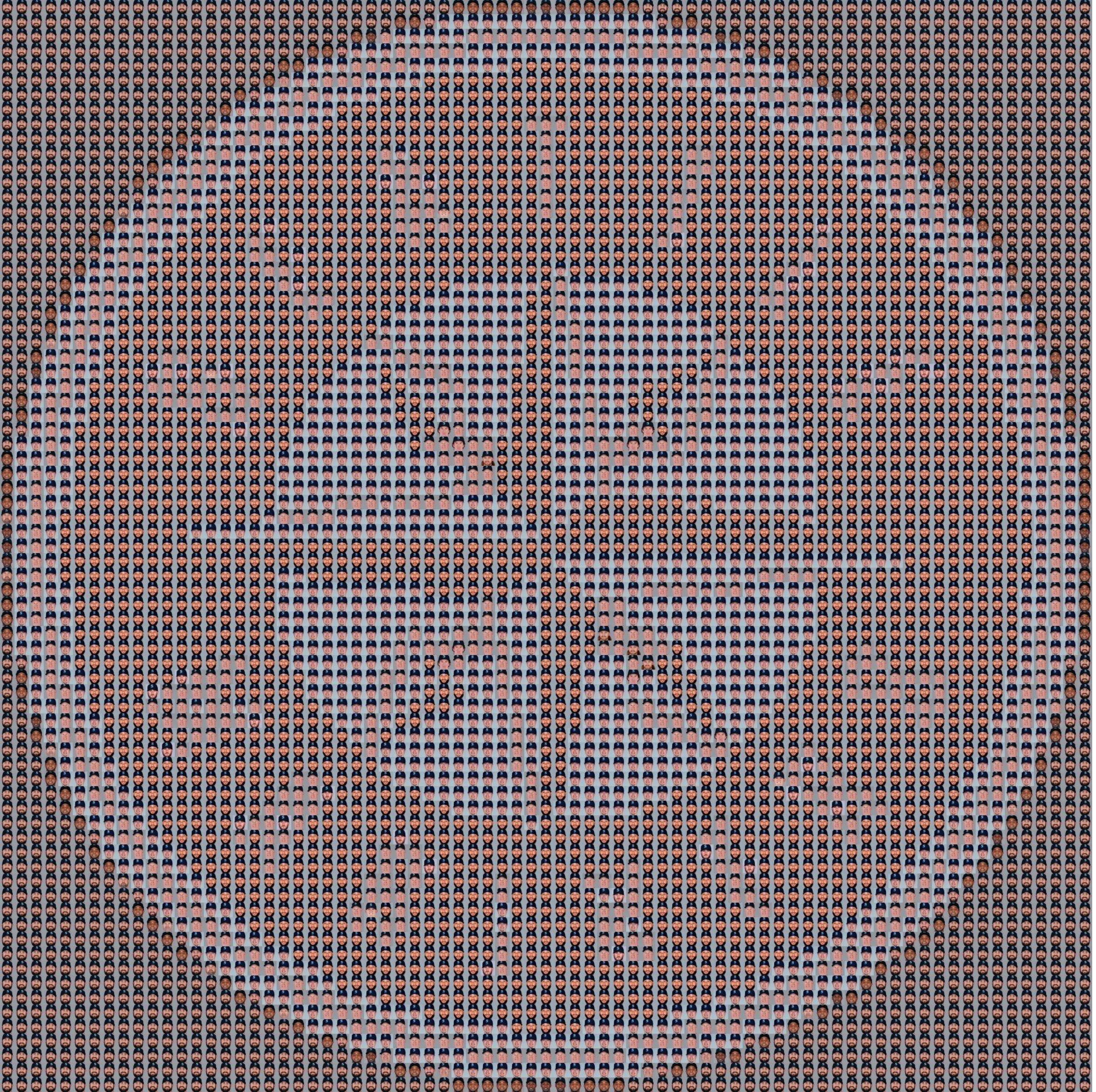
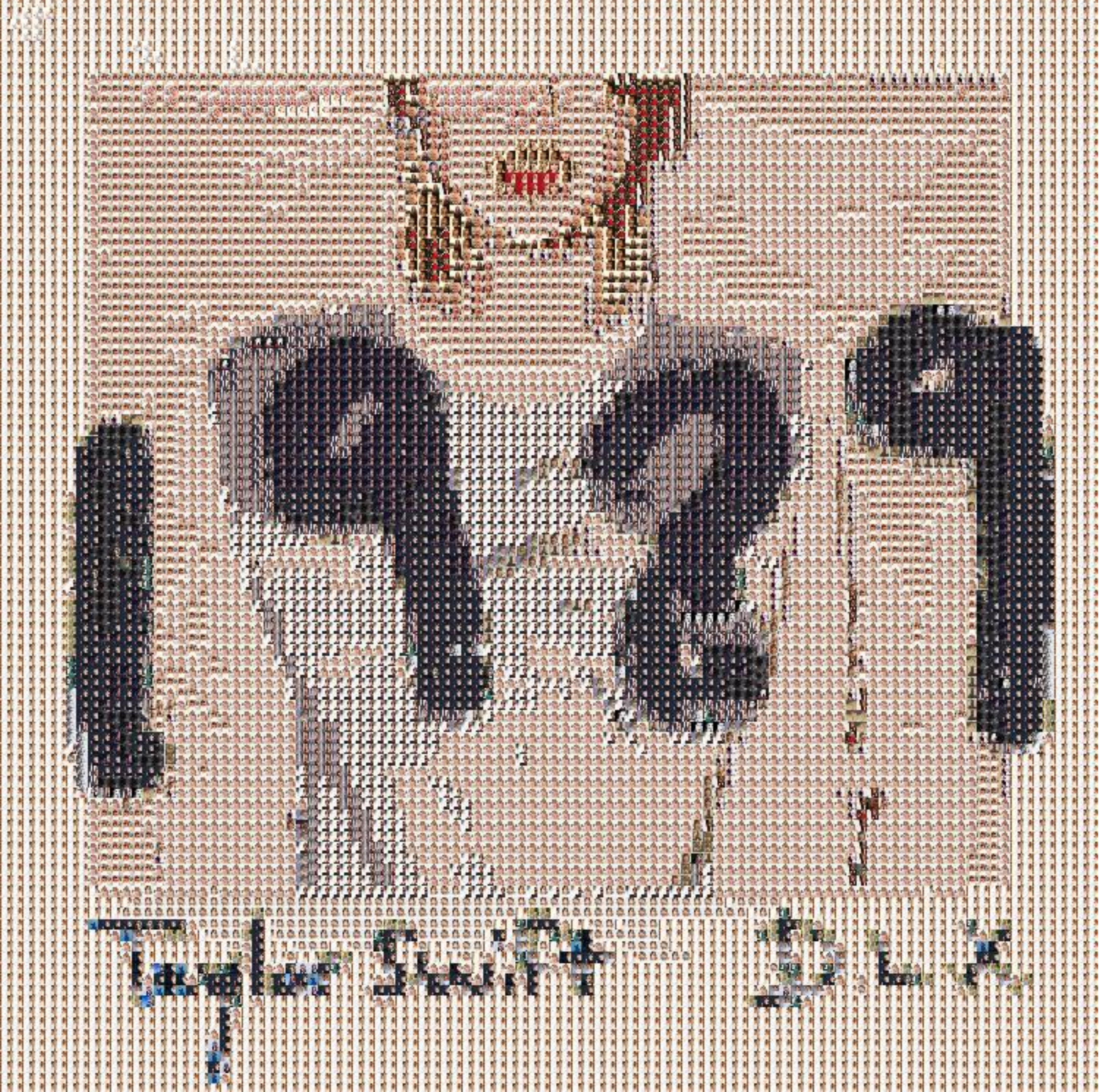






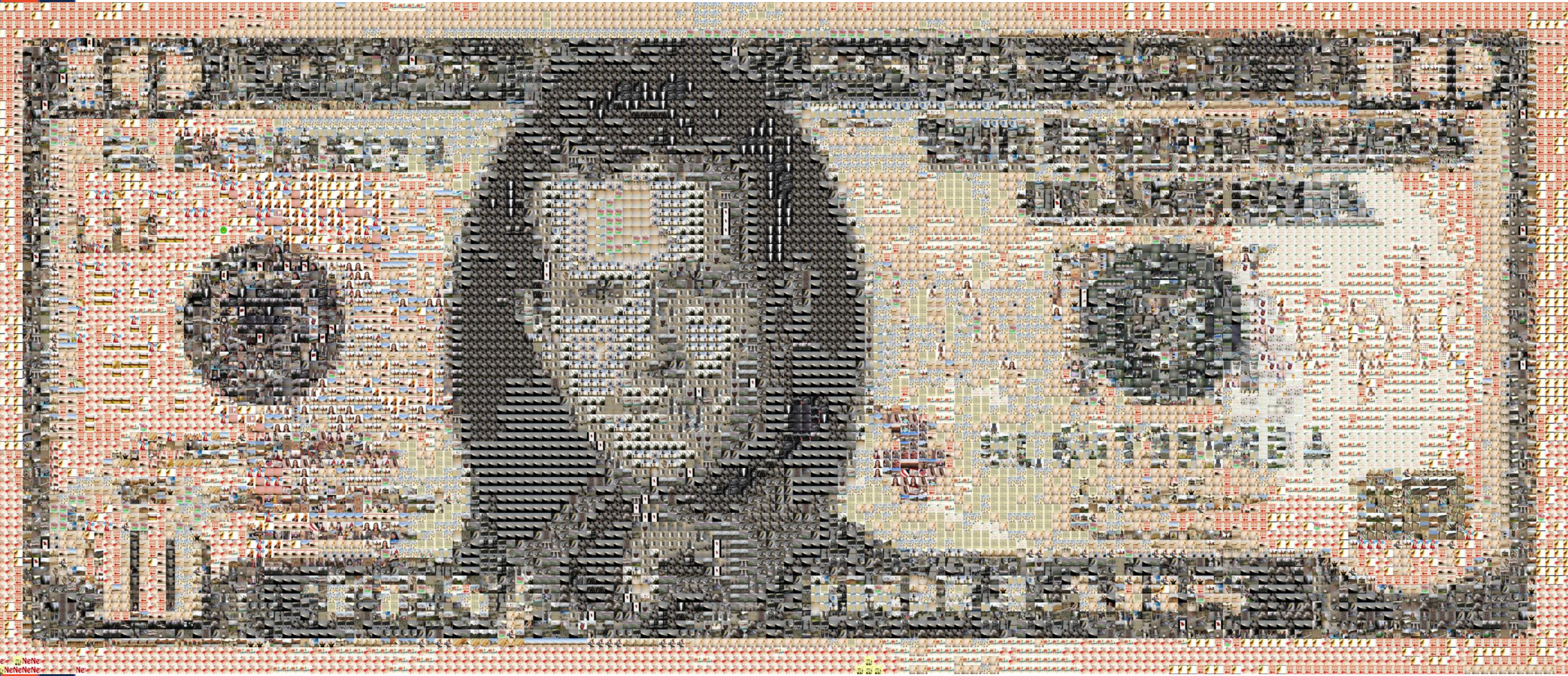



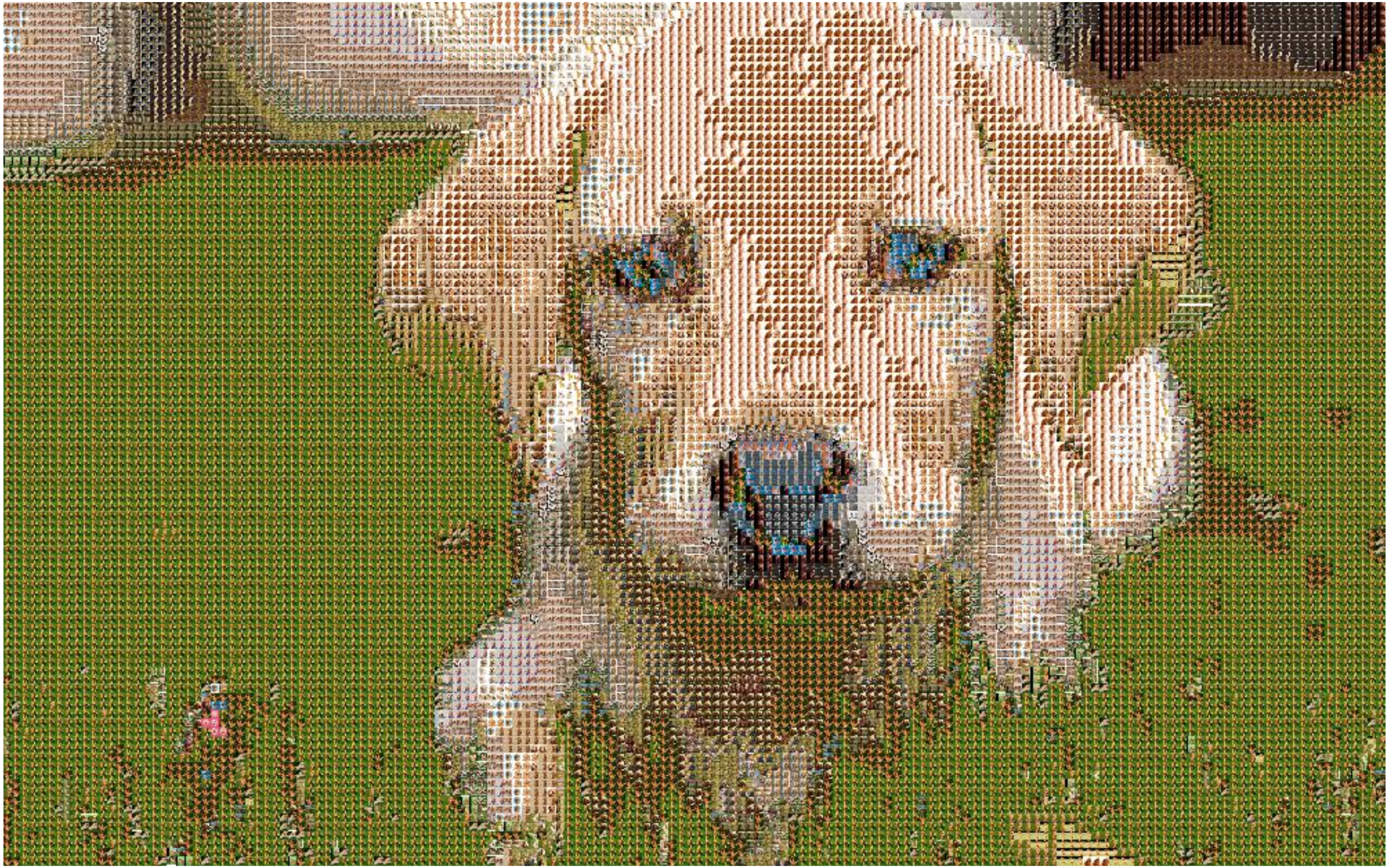



1989
Taylor Swift


ring
ring!
WHO IS IT?
it's a...

"When you're asked about kd-trees in an interview and Wade comes to mind:"



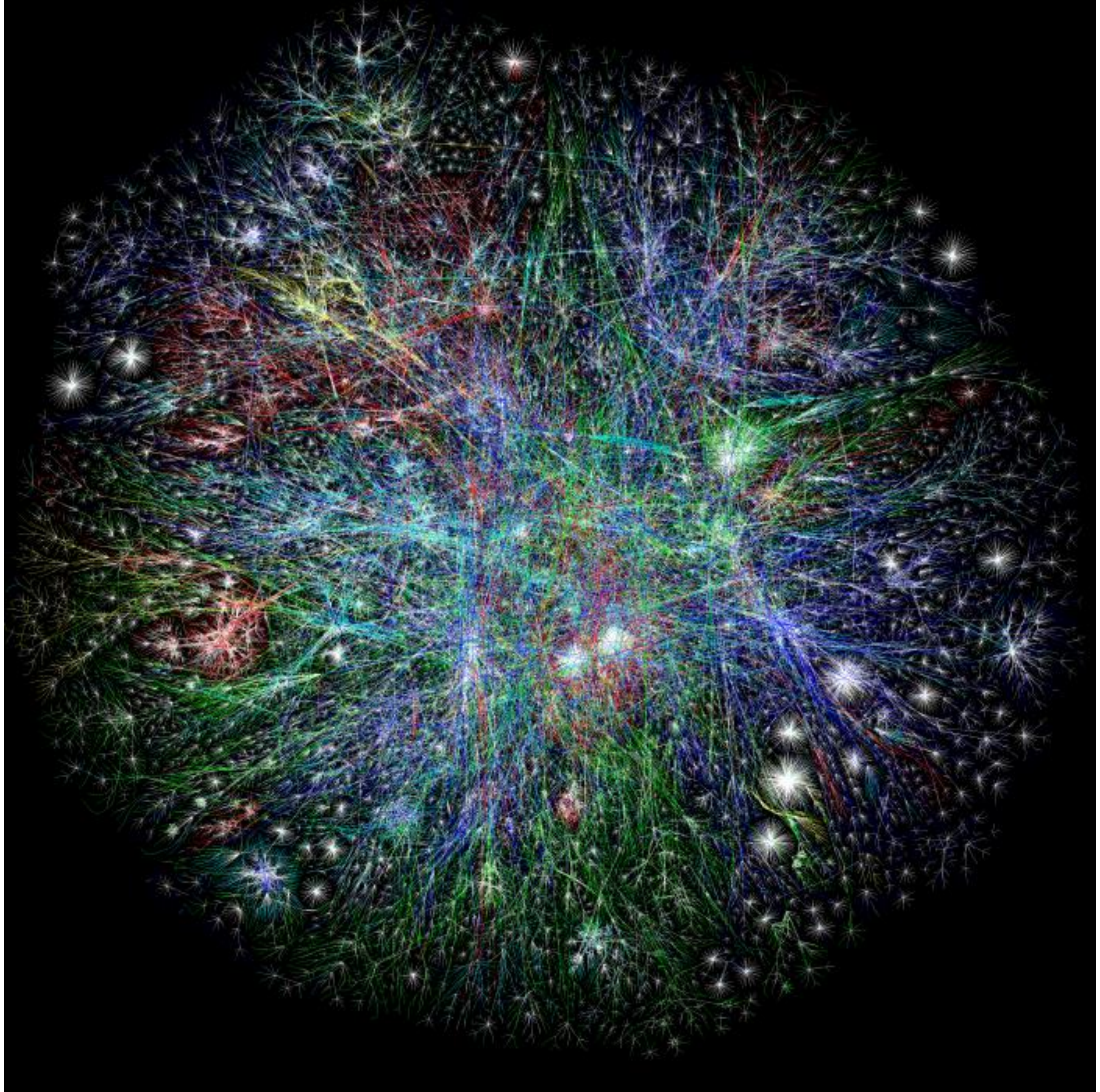


Aw, Snap!

Something went wrong while displaying this webpage.

Learn more

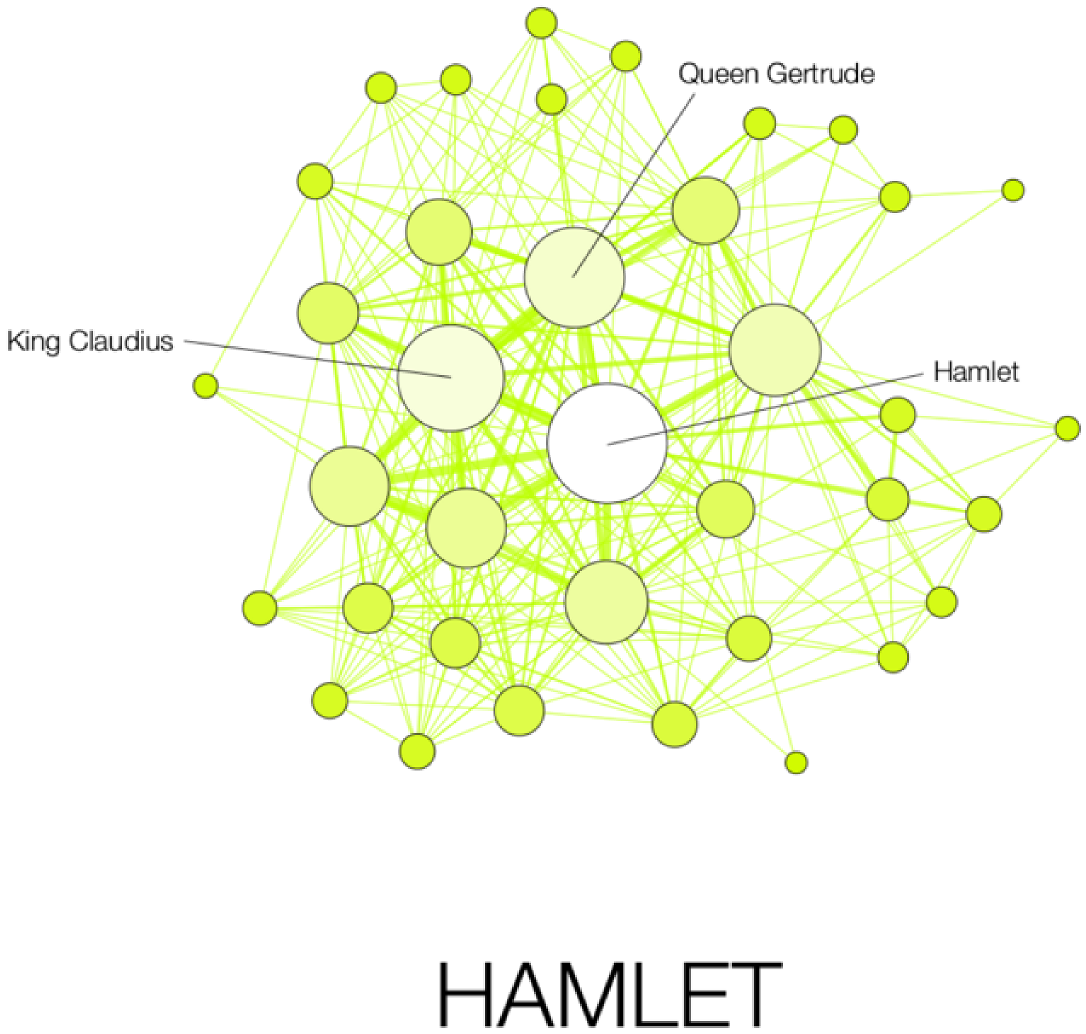
Reload

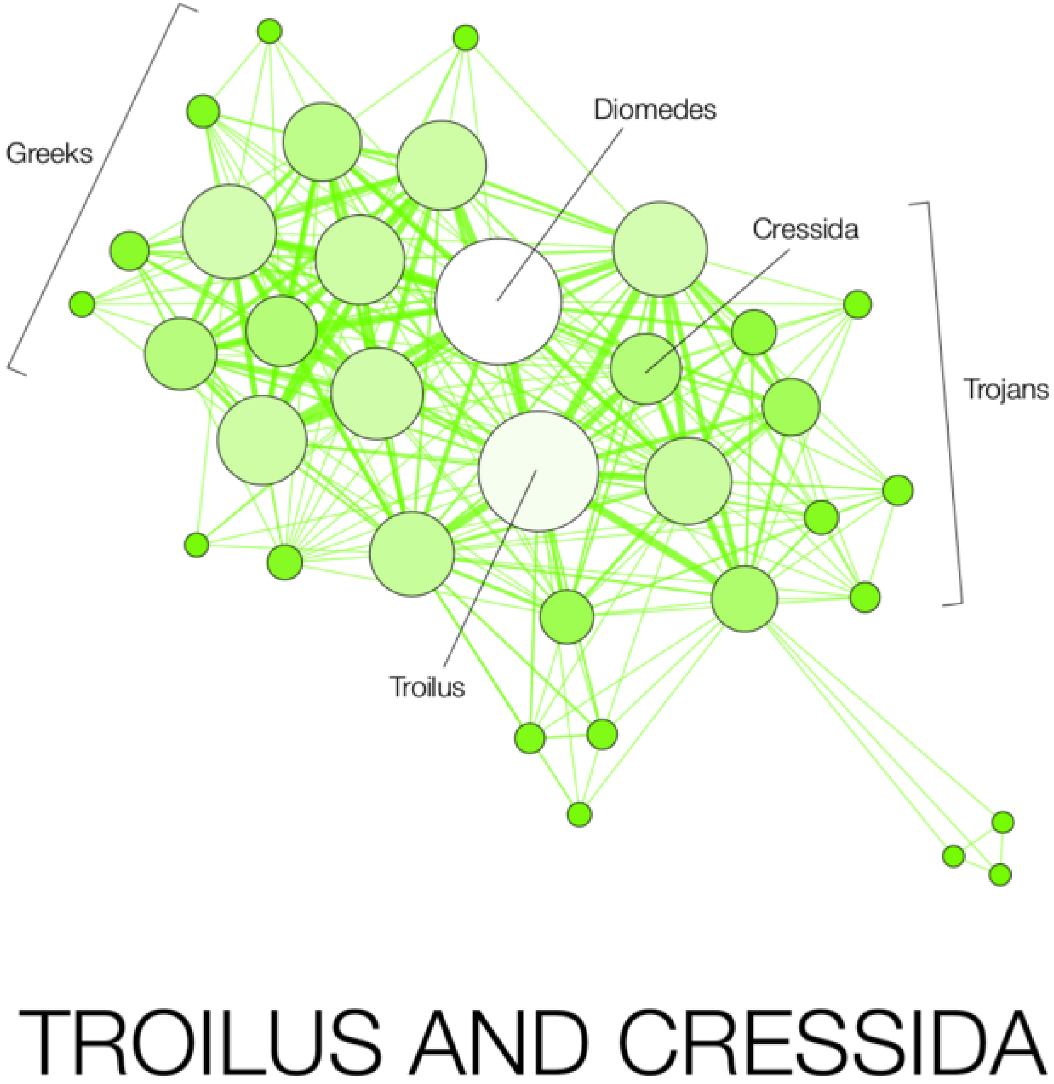


The Internet, 2003

The OPTE Project (2003)

Map of the entire internet; nodes are routers; edges are connections.

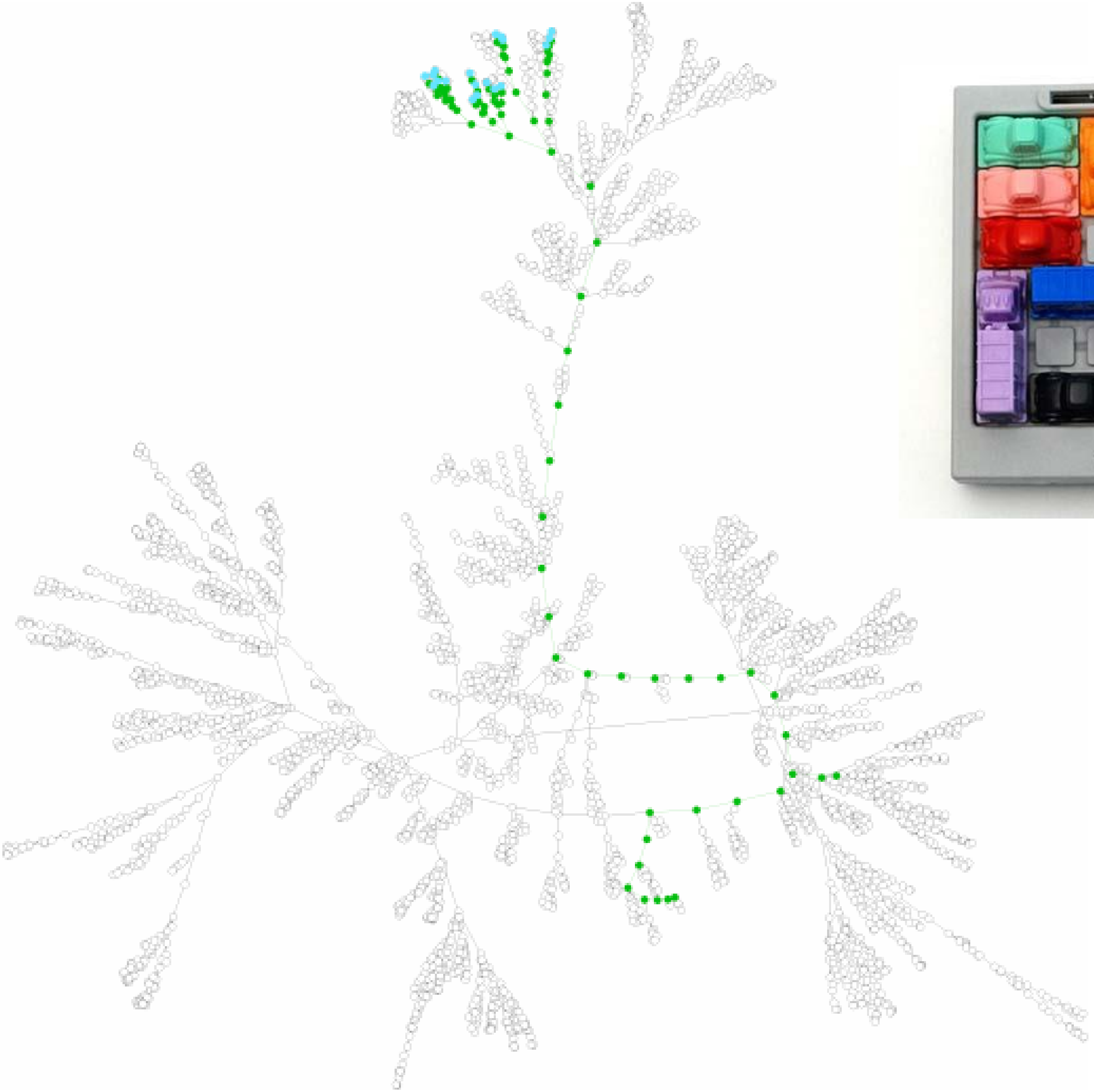




Who's the real main character in Shakespearean tragedies?

Martin Grandjean (2016)

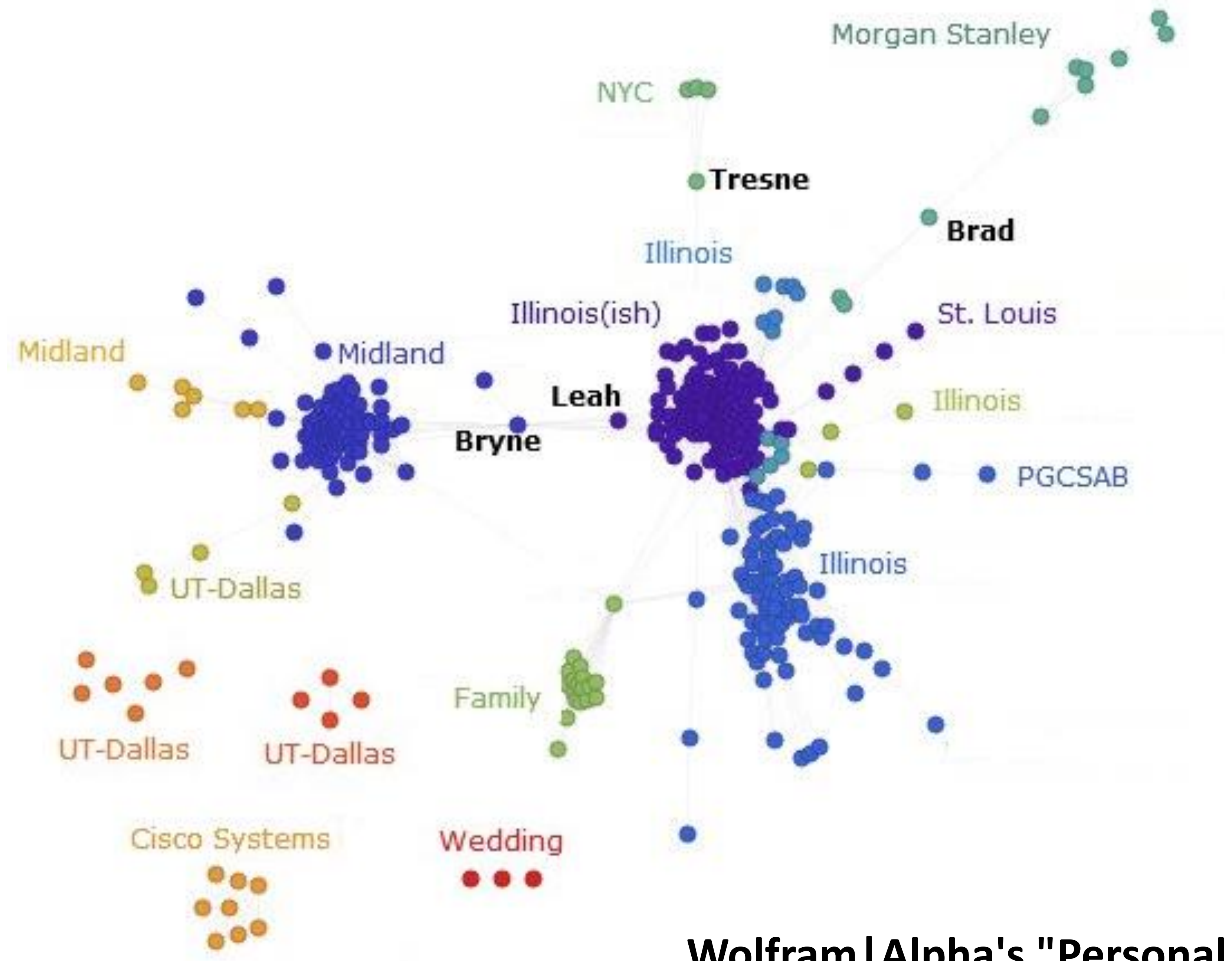
https://www.pbs.org/newshour/arts/whos-the-real-main-character-in-shakespearen-tragedies-heres-what-the-data-say



“Rush Hour” Solution

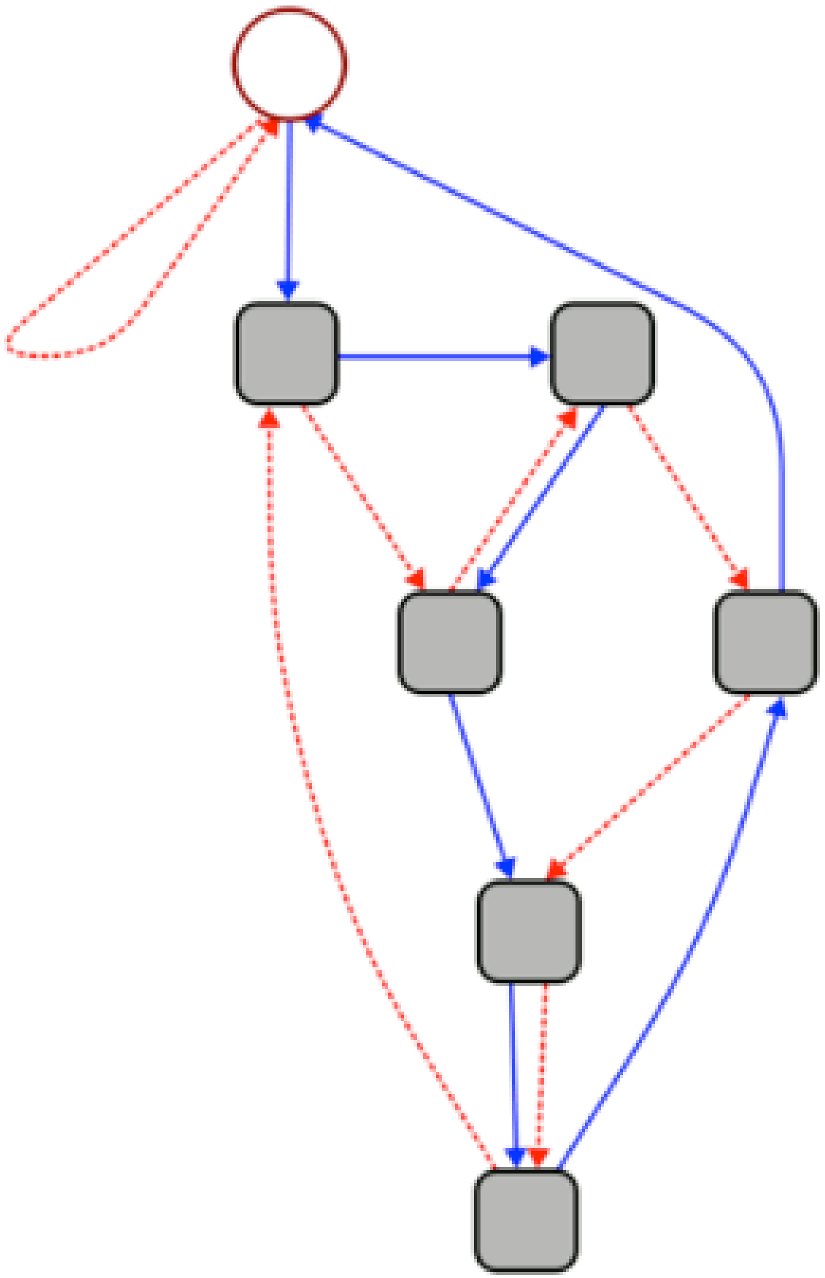
Unknown Source

Presented by Cinda Heeren, 2016



Wolfram|Alpha's "Personal Analytics" for Facebook

Generated: April 2013 using Wade Fagen-Ulmschneider's Profile Data



This graph can be used to quickly calculate whether a given number is divisible by 7.

1. Start at the circle node at the top.
2. For each digit **d** in the given number, follow **d** blue (solid) edges in succession. As you move from one digit to the next, follow **1** red (dashed) edge.
3. If you end up back at the circle node, your number is divisible by 7.

3703

“Rule of 7”

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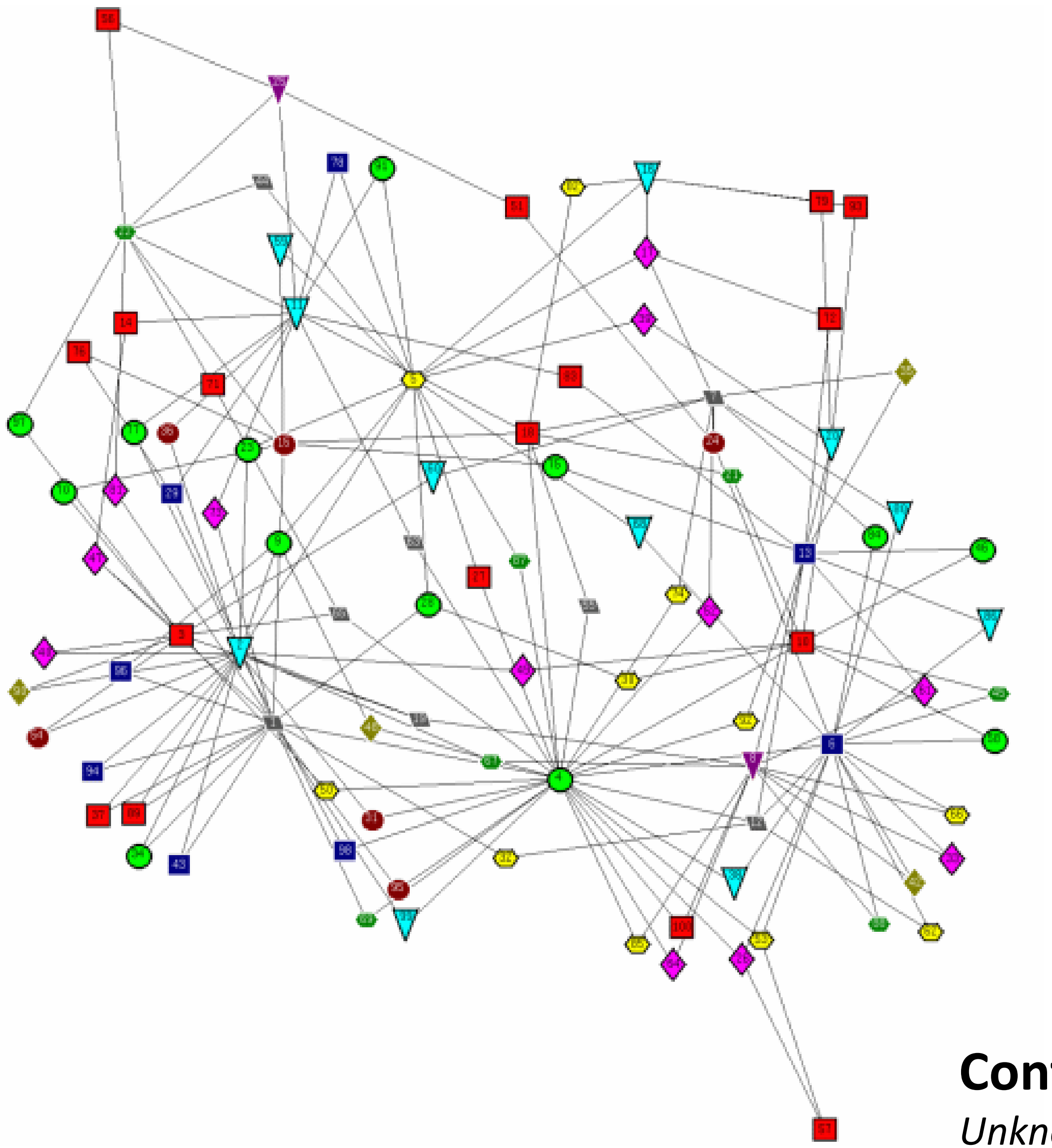
Presented by Cinda Heeren, 2016



Conflict-Free Final Exam Scheduling Graph

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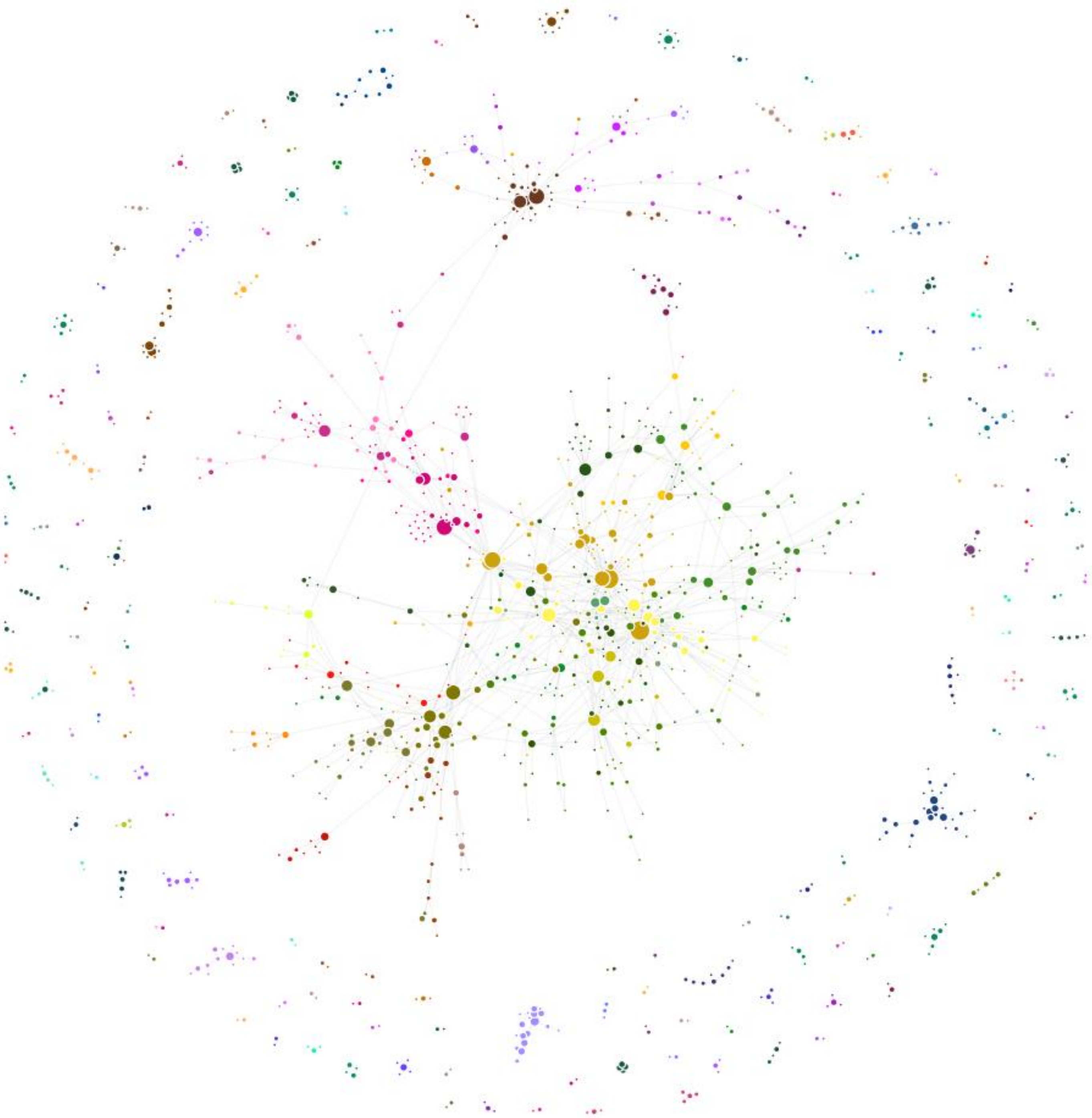
Presented by Cinda Heeren, 2016

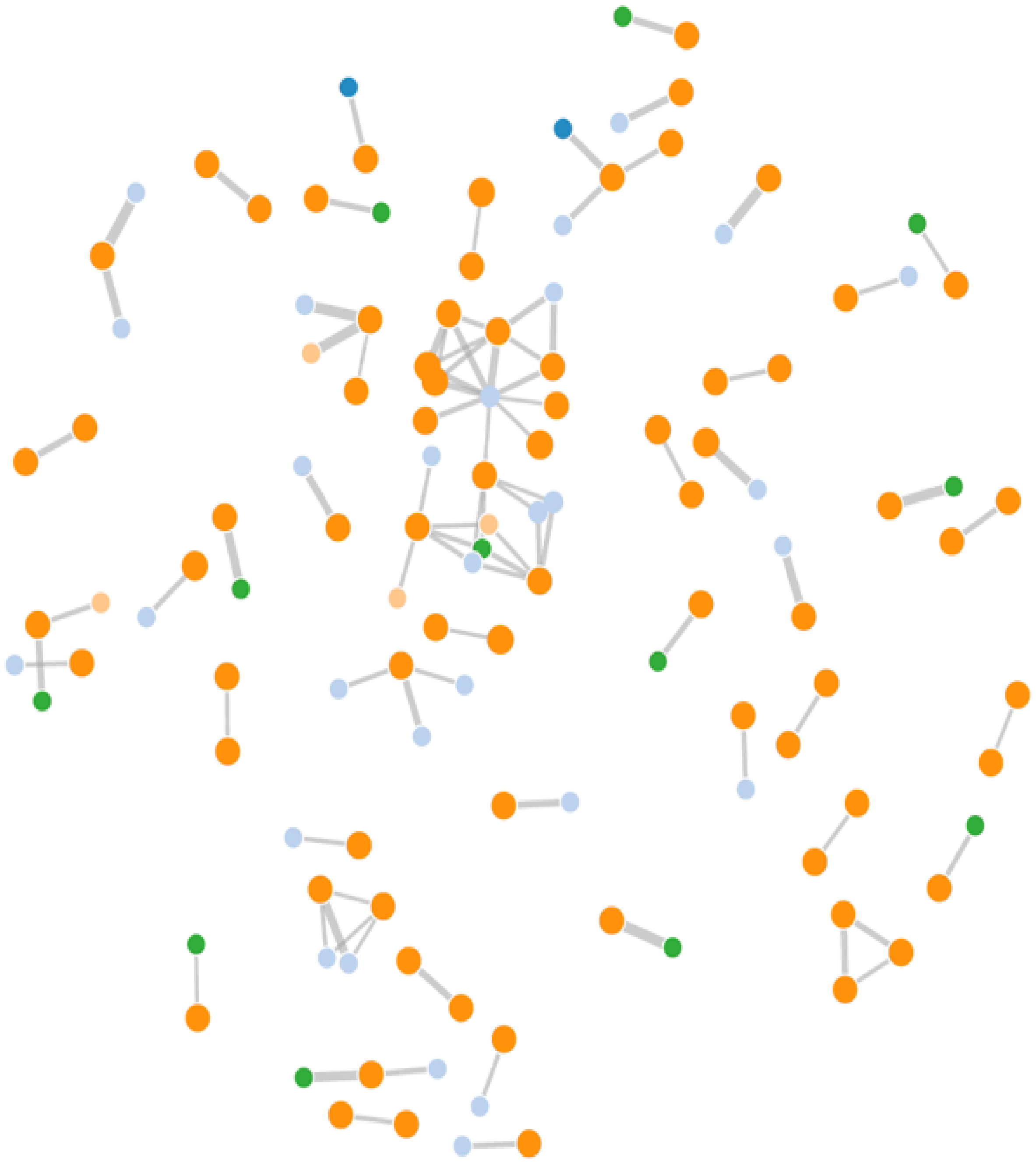


Class Hierarchy At University of Illinois Urbana-Champaign

A. Mori, W. Fagen-Ulmschneider, C. Heeren

Graph of every course at UIUC; nodes are courses, edges are prerequisites

http://waf.cs.illinois.edu/discovery/class_hierarchy_at_illinois/



MP Collaborations in CS 225

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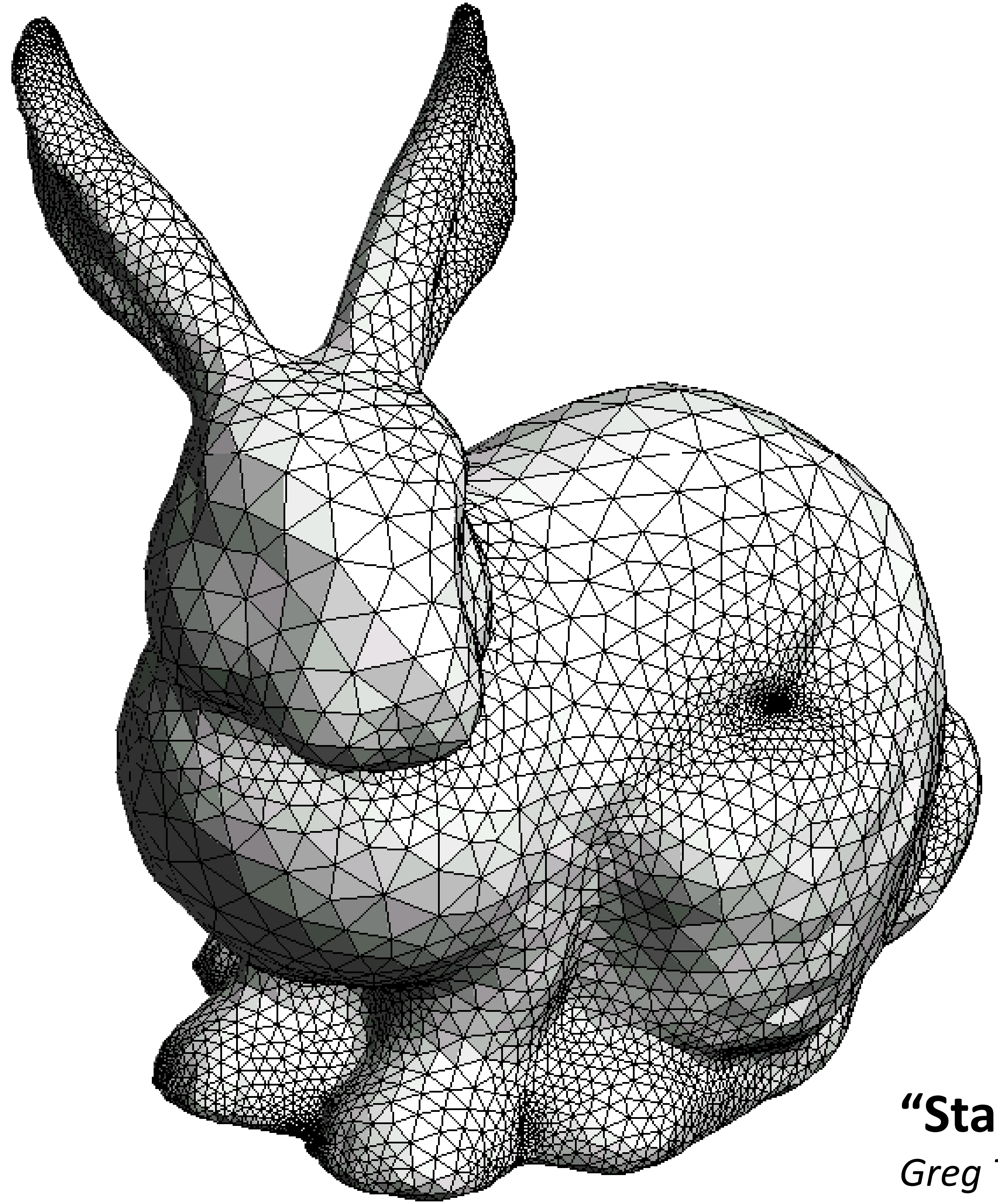
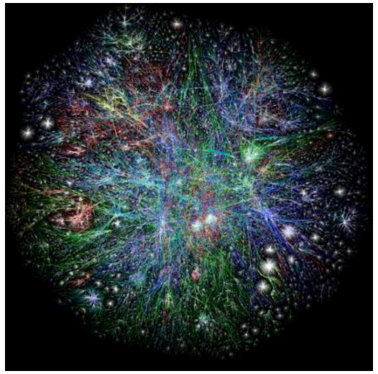
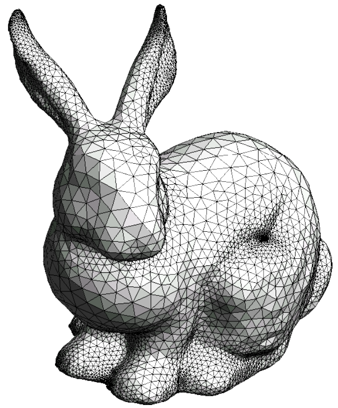


“Stanford Bunny”

Greg Turk and Mark Levoy (1994)

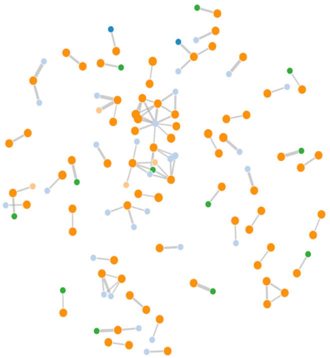
Graphs

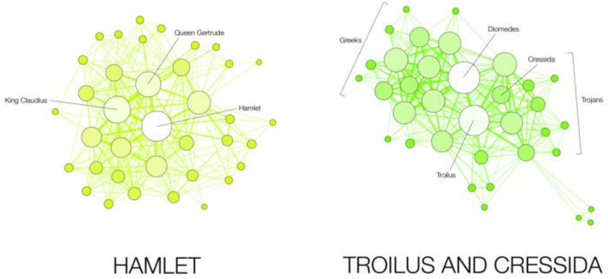


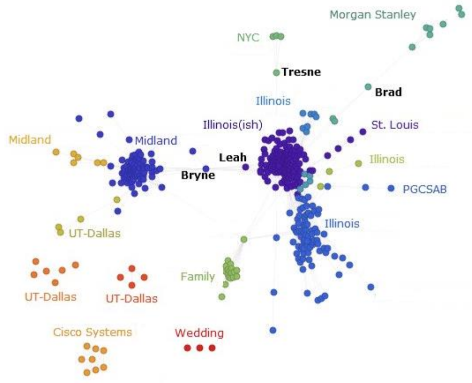


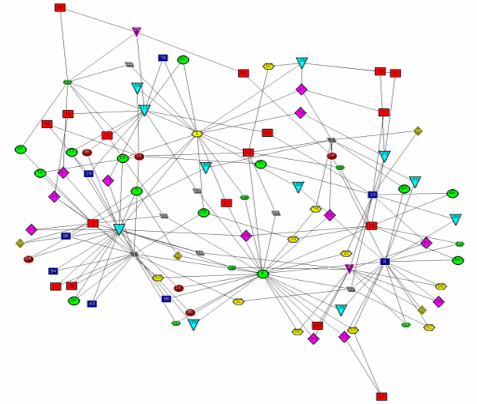
To study all of these structures:

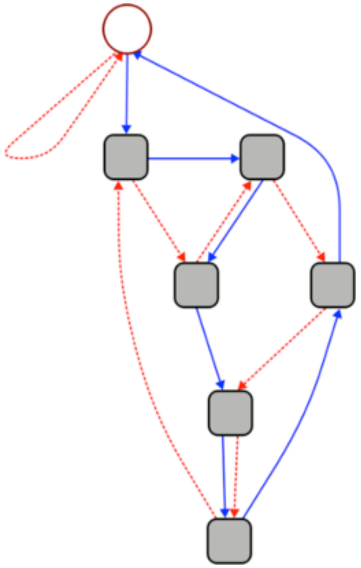
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

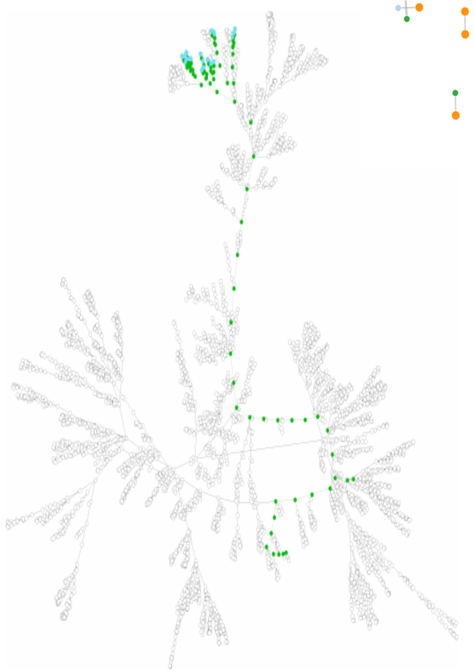










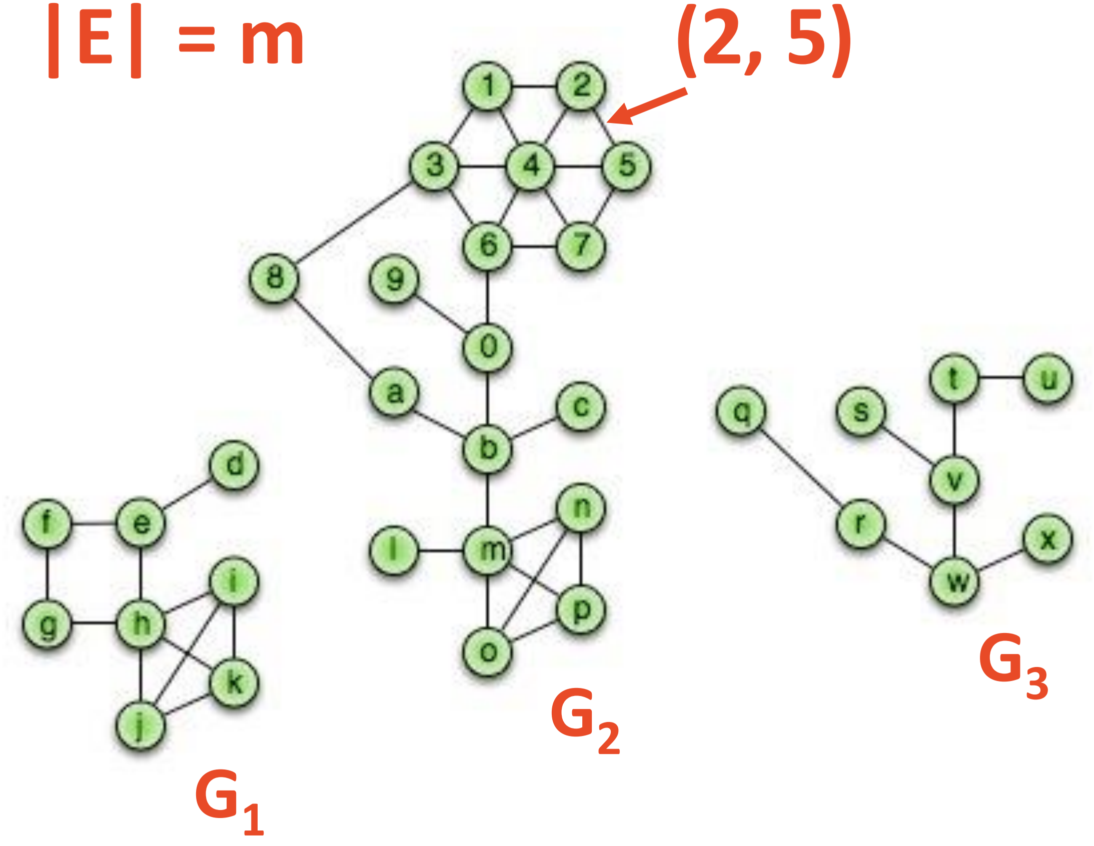


Graph Vocabulary

$G = (V, E)$

$|V| = n$

$|E| = m$



Incident Edges:
$I(v) = \{ (x, v) \text{ in } E \}$

Degree(v): $|I|$

Adjacent Vertices:
$A(v) = \{ x : (x, v) \text{ in } E \}$

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

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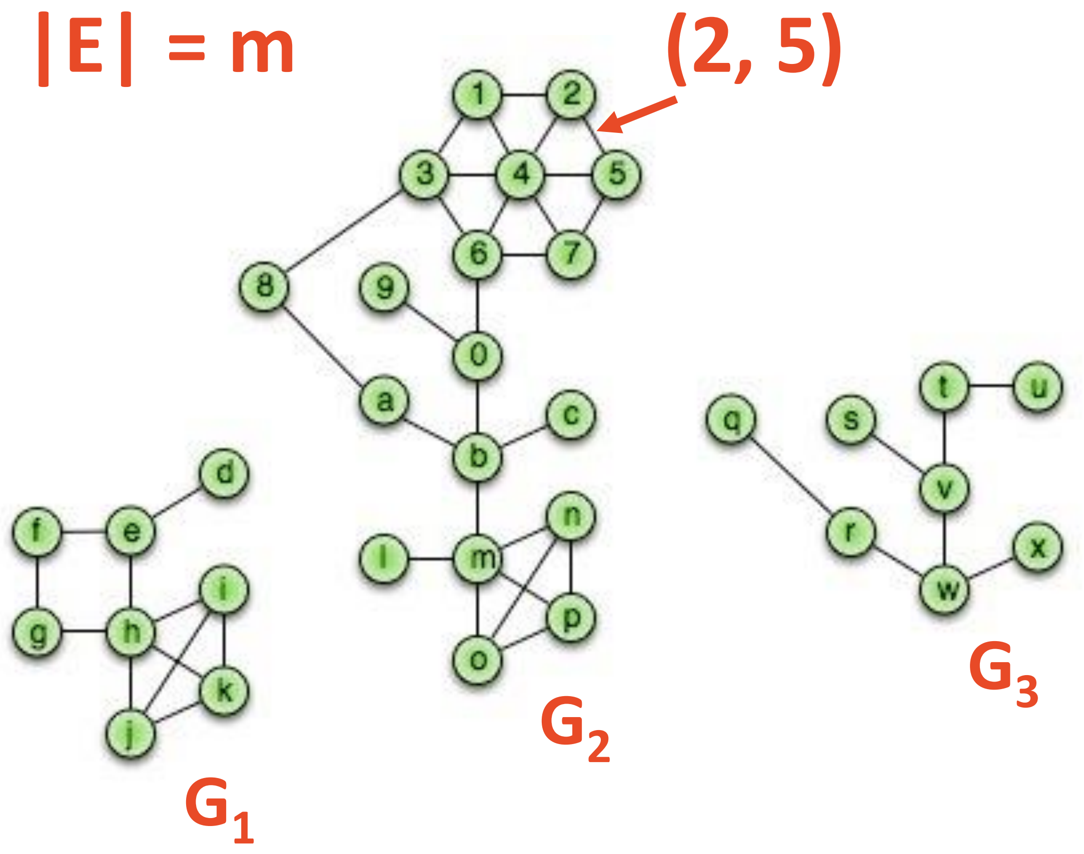


Subgraph(G):

$G' = (V', E')$:

$V' \in V$, $E' \in E$, and

$(u, v) \in E \rightarrow u \in V', v \in V'$

Complete subgraph(G)

Connected subgraph(G)

Connected component(G)

Acyclic subgraph(G)

Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges? **Minimum edges:**

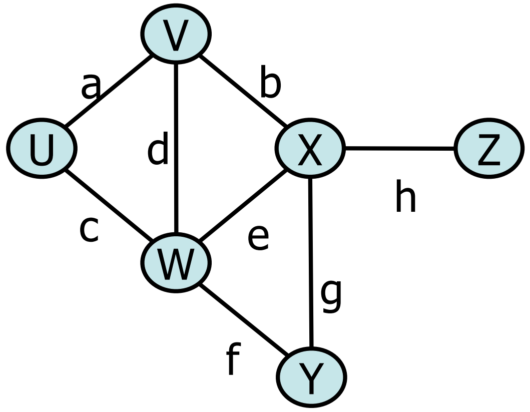
Not Connected:

Connected*:

Maximum edges:

Simple:

Not simple:



$$\sum_{v \in V} \deg(v) =$$