

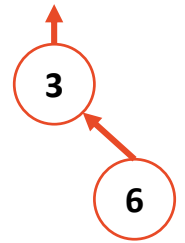
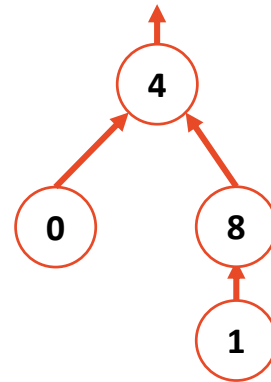
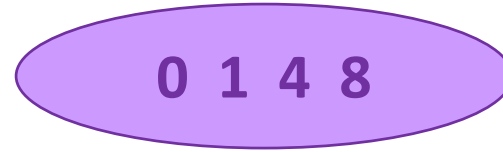
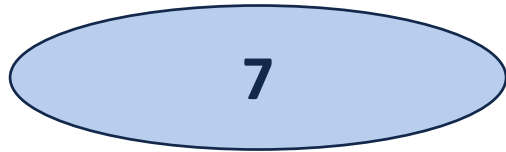
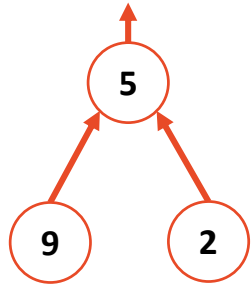
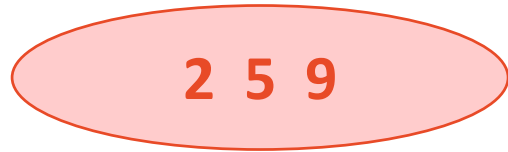
CS 225

Data Structures

April 6 – Disjoint Sets Implementation

Wade Fagen-Ulmschneider

Disjoint Sets



0	1	2	3	4	5	6	7	8	9
4	8	5	-1	-1	-1	3	-1	4	5

Disjoint Sets Find

```
1 int DisjointSets::find() {  
2     if ( s[i] < 0 ) { return i; }  
3     else { return _find( s[i] ); }  
4 }
```

Running time?

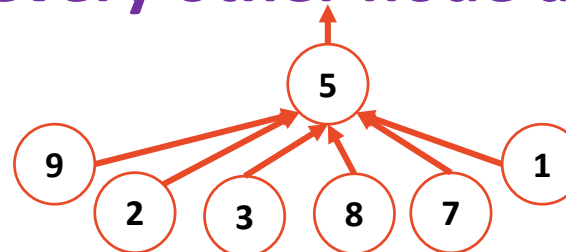
Structure: A structure similar to a linked list

Running time: $O(h) == O(n)$

What is the ideal UpTree?

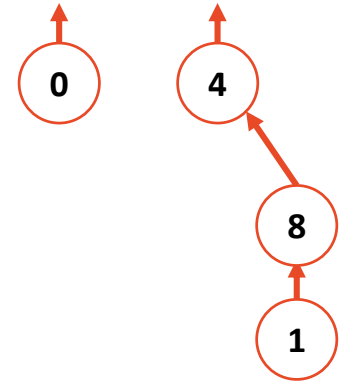
Structure: One root node with every other node as it's child

Running Time: $O(1)$

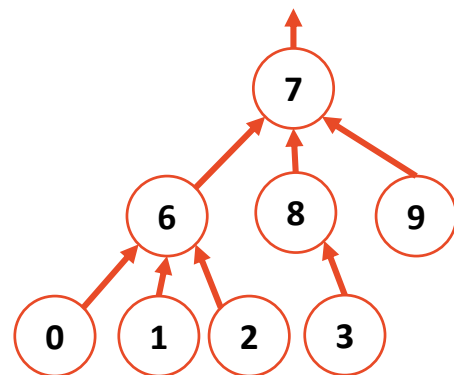


Disjoint Sets Union

```
1 void DisjointSets::union(int r1, int r2) {  
2  
3  
4 }
```

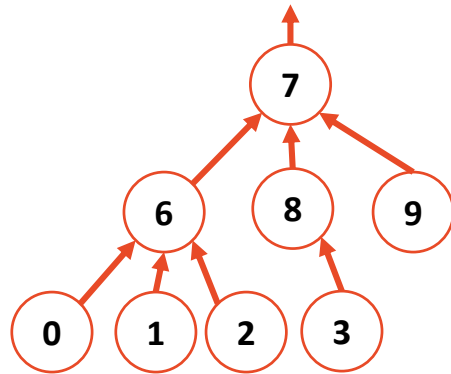


Disjoint Sets – Union



0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8	-1	10	7	-1	7	7	4	5

Disjoint Sets – Smart Union

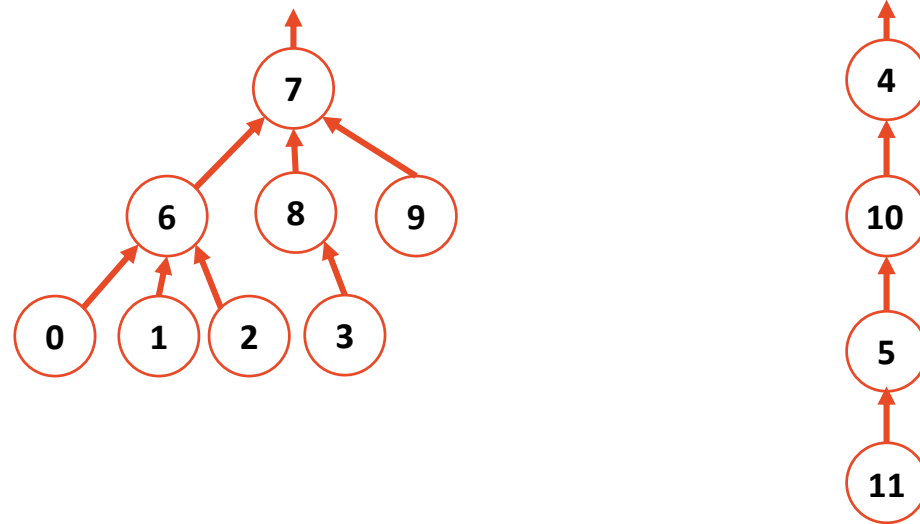


Union by height

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Keep the height of the tree as small as possible.

Disjoint Sets – Smart Union



Union by height

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Keep the height of the tree as small as possible.

Union by size

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Minimize the number of nodes that increase in height

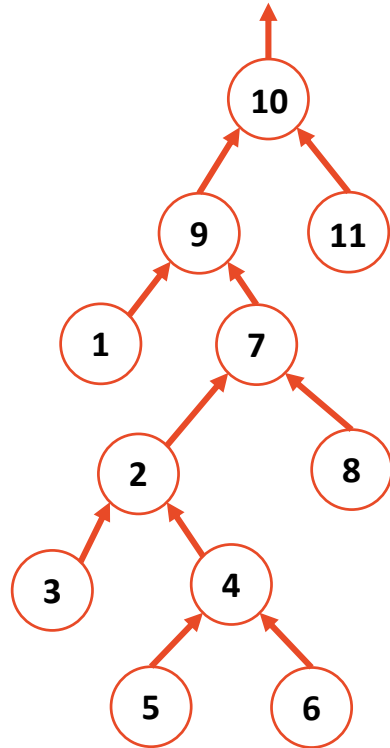
Both guarantee the height of the tree is: _____.

Disjoint Sets Find

```
1 int DisjointSets::find(int i) {  
2     if ( s[i] < 0 ) { return i; }  
3     else { return _find( s[i] ); }  
4 }
```

```
1 void DisjointSets::unionBySize(int root1, int root2) {  
2     int newSize = arr_[root1] + arr_[root2];  
3  
4     // If arr_[root1] is less than (more negative), it is the larger set;  
5     // we union the smaller set, root2, with root1.  
6     if ( arr_[root1] < arr_[root2] ) {  
7         arr_[root2] = root1;  
8         arr_[root1] = newSize;  
9     }  
10  
11     // Otherwise, do the opposite:  
12     else {  
13         arr_[root1] = root2;  
14         arr_[root2] = newSize;  
15     }  
16 }
```


Path Compression



Disjoint Sets Analysis

The **iterated log** function:

The number of times you can take a log of a number.

$\log^*(n) =$

0, $n \leq 1$

$1 + \log^*(\log(n))$, $n > 1$

What is $\lg^*(2^{65536})$?

Disjoint Sets Analysis

In an Disjoint Sets implemented with smart **unions** and path compression on **find**:

Any sequence of **m union** and **find** operations result in the worse case running time of $O(\text{_____})$,
where **n** is the number of items in the Disjoint Sets.

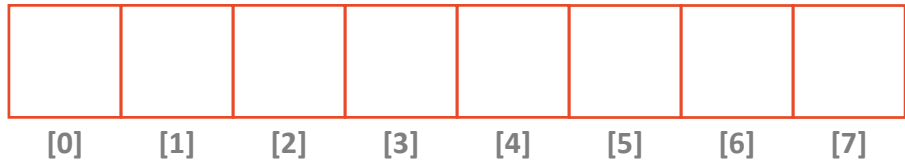
In Review: Data Structures

Array

- Sorted Array
- Unsorted Array
- Stacks
- Queues
- Hashing
- Heaps
 - Priority Queues
- UpTrees
 - Disjoint Sets

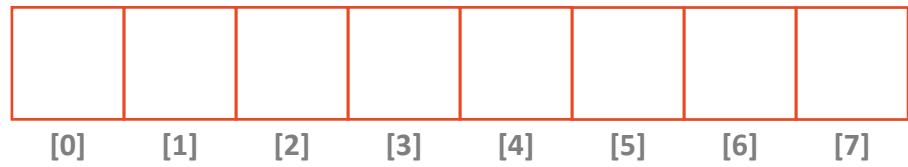
List

- Doubly Linked List
- Skip List
- Trees
 - BTree
 - Binary Tree
 - Huffman Encoding
 - kd-Tree
 - AVL Tree

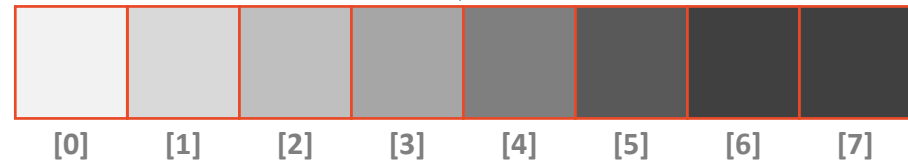


Array

- **Constant time access to any element, given an index $a[k]$ is accessed in $O(1)$ time, no matter how large the array grows**
- **Cache-optimized**
Many modern systems cache or pre-fetch nearby memory values due the “Principle of Locality”. Therefore, arrays often perform faster than lists in identical operations.

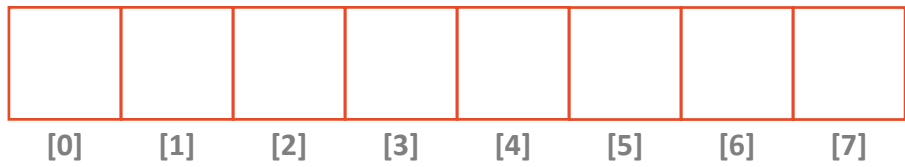


Array

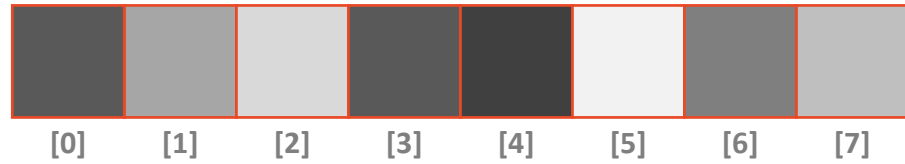


Sorted Array

- Efficient general search structure
Searches on the sort property run in $O(\lg(n))$ with Binary Search
- Inefficient insert/remove
Elements must be inserted and removed at the location dictated by the sort property, resulting shifting the array in memory – an $O(n)$ operation

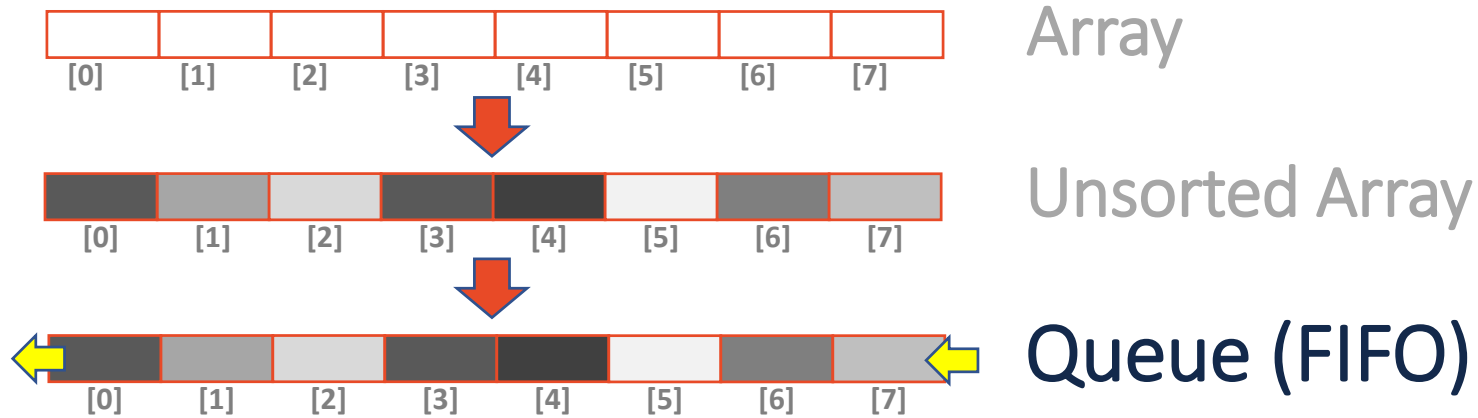


Array



Unsorted Array

- Constant time add/remove at the beginning/end
Amortized $O(1)$ insert and remove from the front and of the array
Idea: Double on resize
- Inefficient global search structure
With no sort property, all searches must iterate the entire array; $O(1)$ time



- **First In First Out (FIFO) ordering of data**
Maintains an arrival ordering of tasks, jobs, or data
- **All ADT operations are constant time operations**
enqueue() and dequeue() both run in $O(1)$ time



Array



Unsorted Array



Stack (LIFO)

- Last In First Out (LIFO) ordering of data
Maintains a “most recently added” list of data
- All ADT operations are constant time operations
`push()` and `pop()` both run in $O(1)$ time

In Review: Data Structures

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In Review: Data Structures

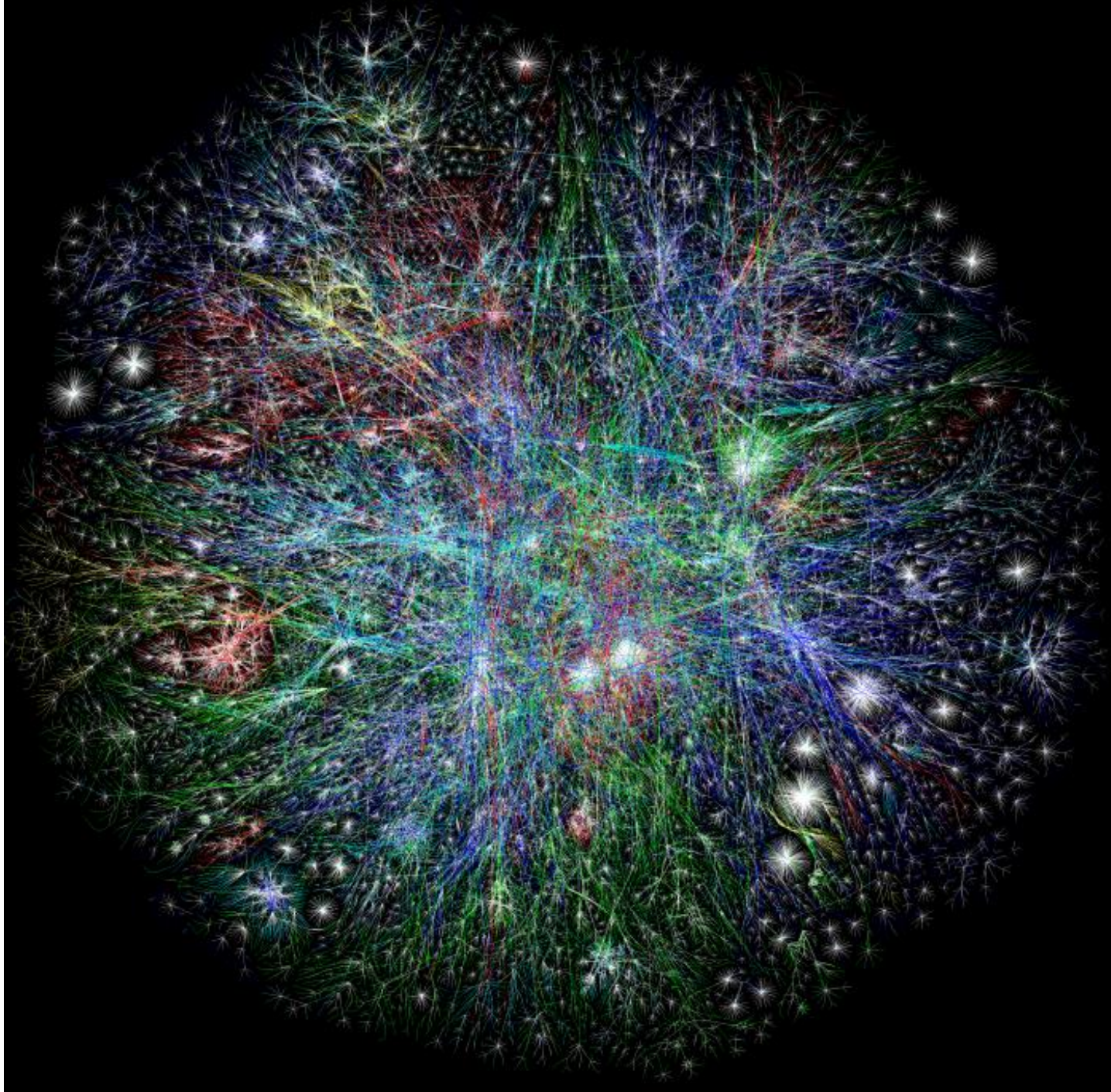
Array

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Graphs

List

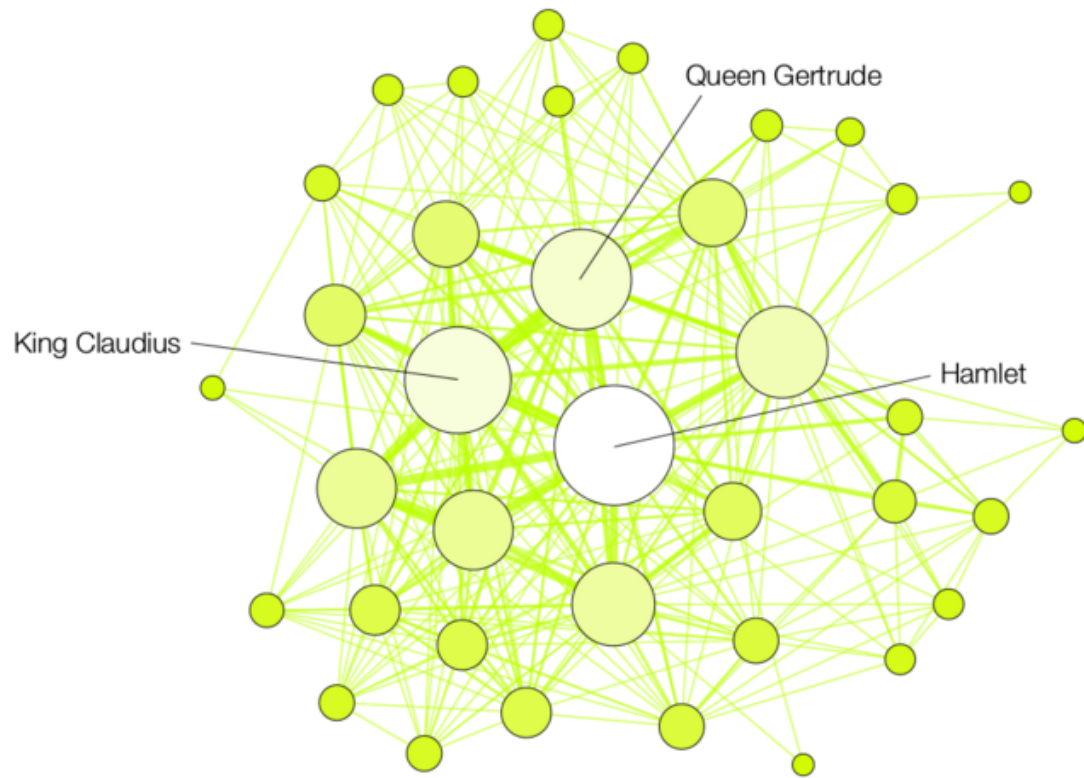
- Doubly Linked List
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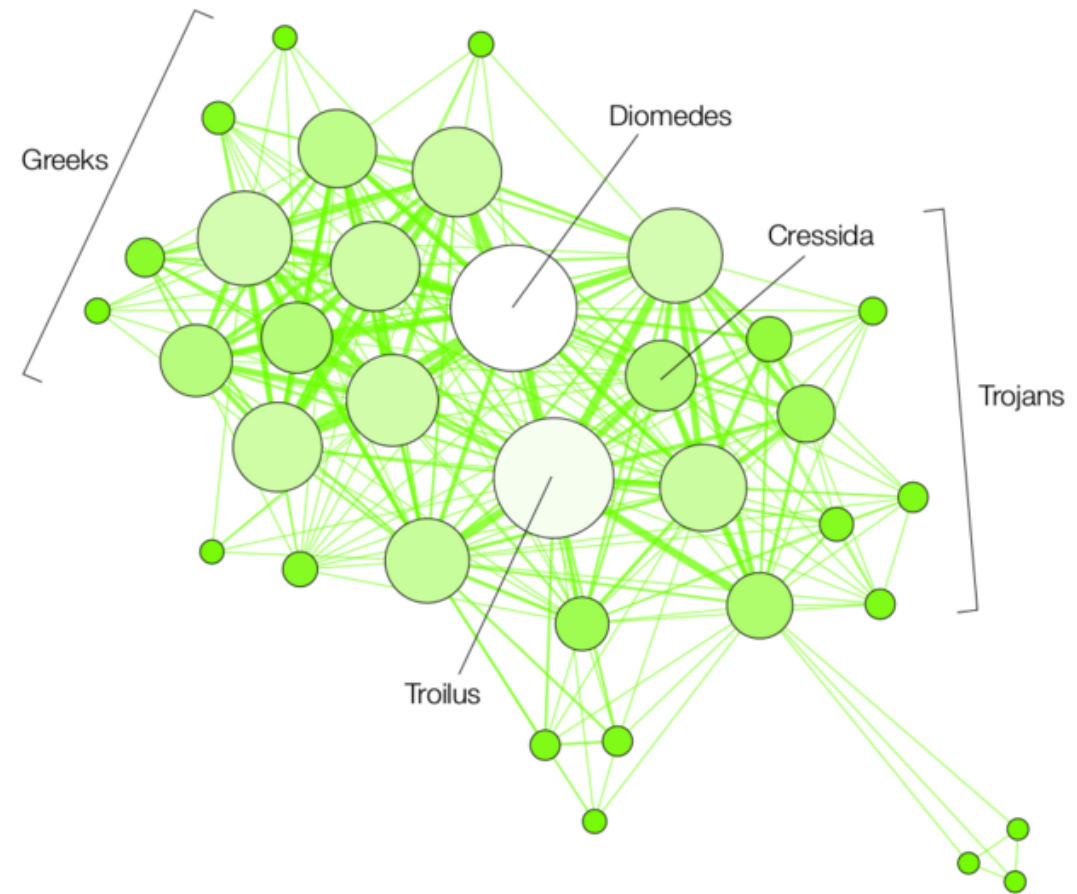
The Internet 2003

The OPTE Project (2003)

Map of the entire internet; nodes are routers; edges are connections.



HAMLET

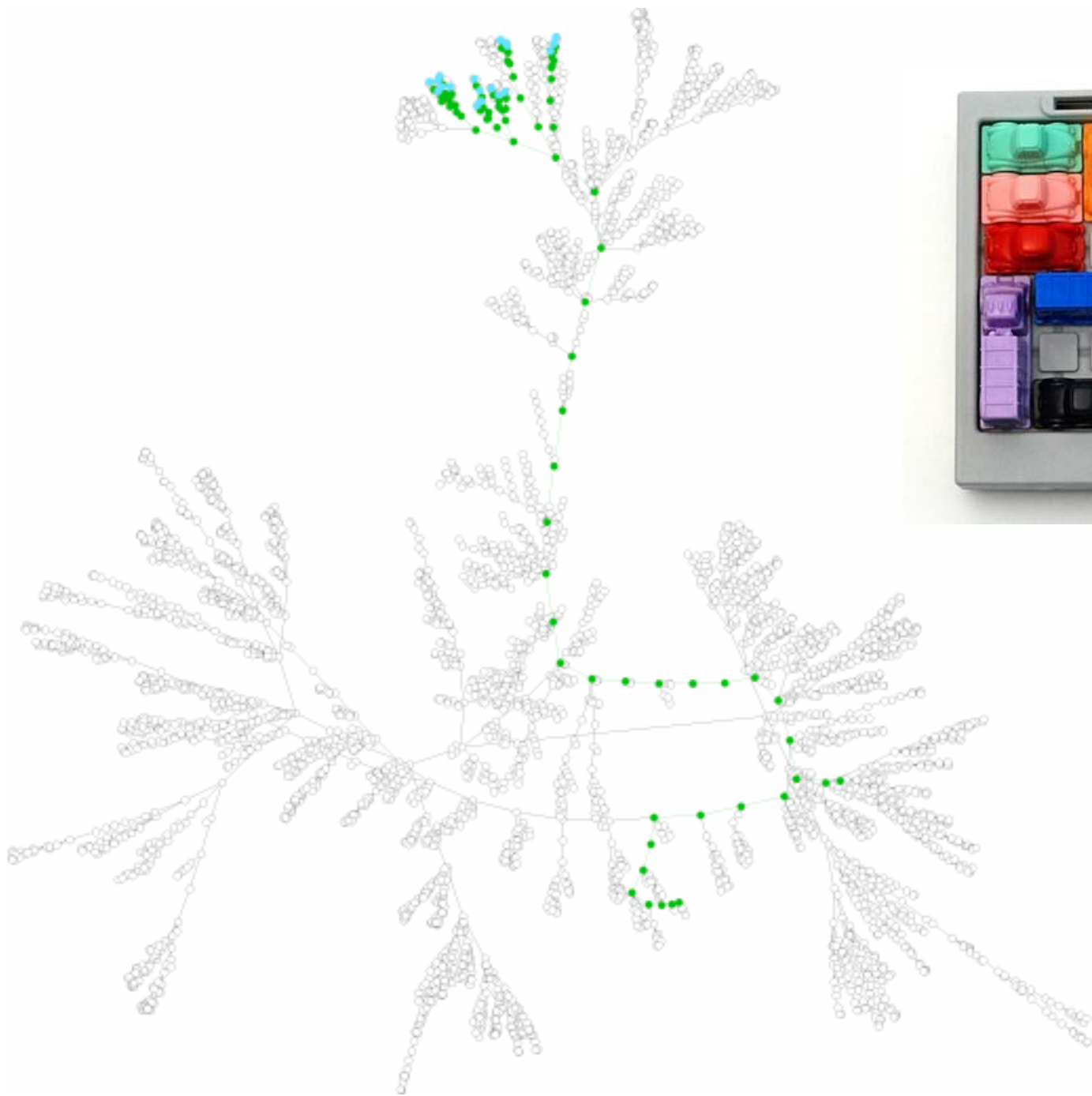


TROILOUS AND CRESSIDA

Who's the real main character in Shakespearean tragedies?

Martin Grandjean (2016)

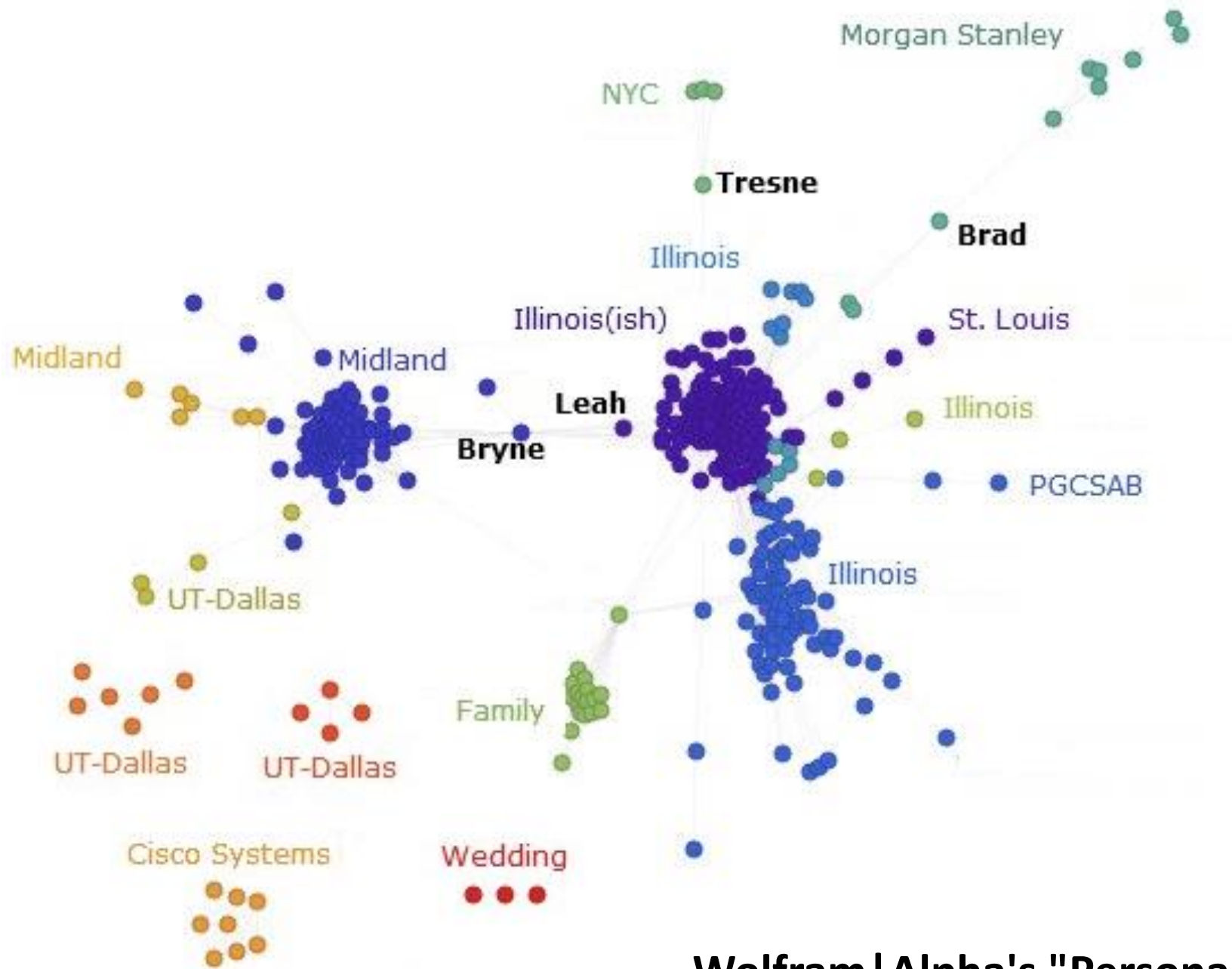
<https://www.pbs.org/newshour/arts/whos-the-real-main-character-in-shakespearean-tragedies-heres-what-the-data-say>



“Rush Hour” Solution

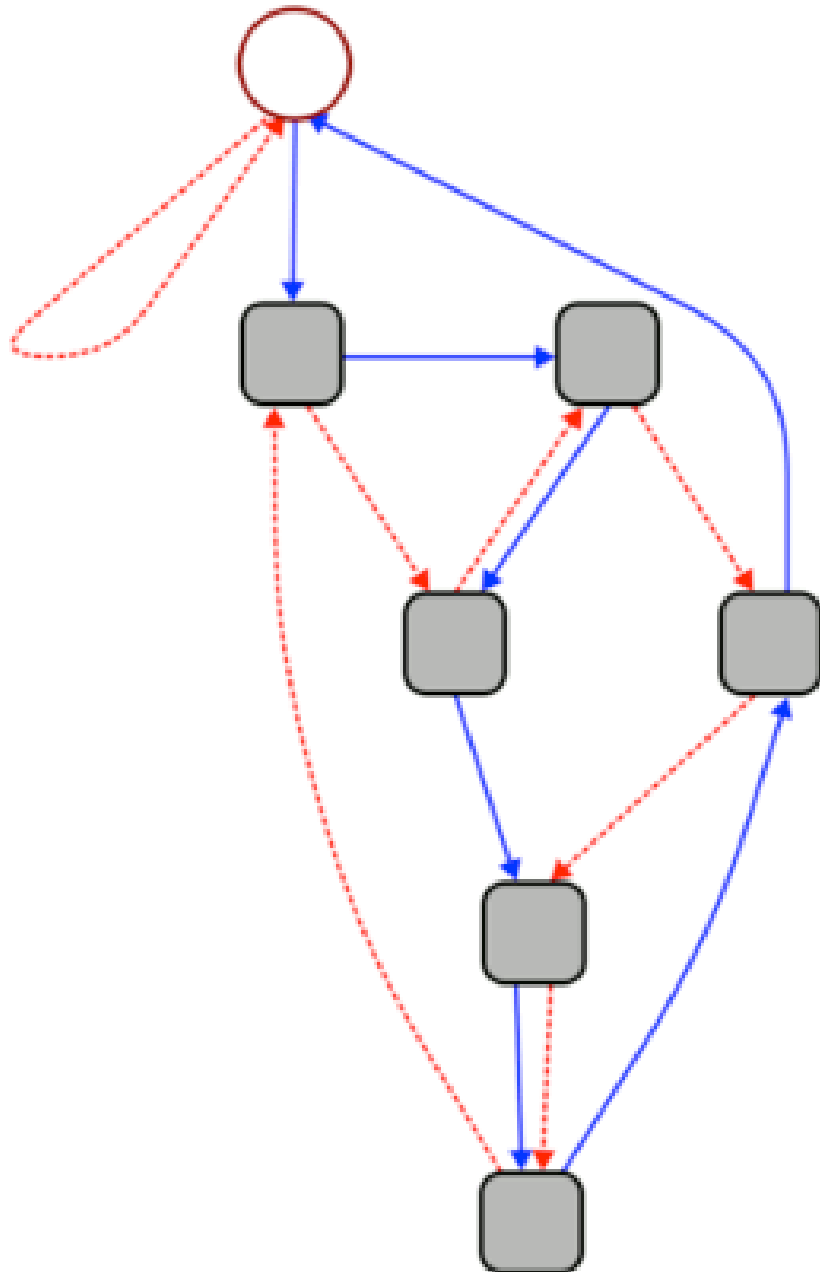
Unknown Source

Presented by Cinda Heeren, 2016



Wolfram|Alpha's "Personal Analytics" for Facebook

Generated: April 2013 using Wade Fagen-Ulmschneider's Profile Data



This graph can be used to quickly calculate whether a given number is divisible by 7.

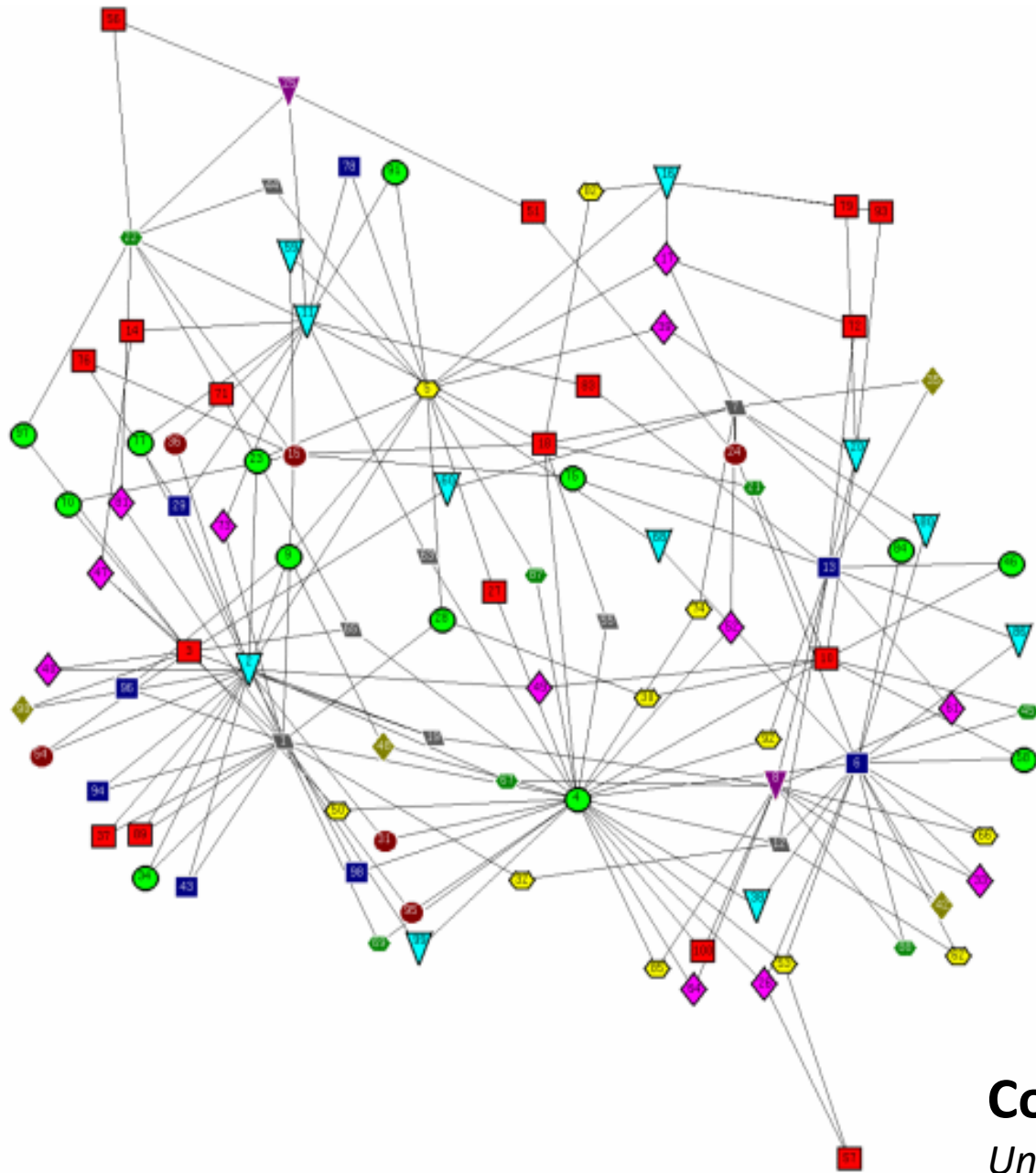
1. Start at the circle node at the top.
2. For each digit **d** in the given number, follow **d** blue (solid) edges in succession. As you move from one digit to the next, follow **1** red (dashed) edge.
3. If you end up back at the circle node, your number is divisible by 7.

3703

“Rule of 7”

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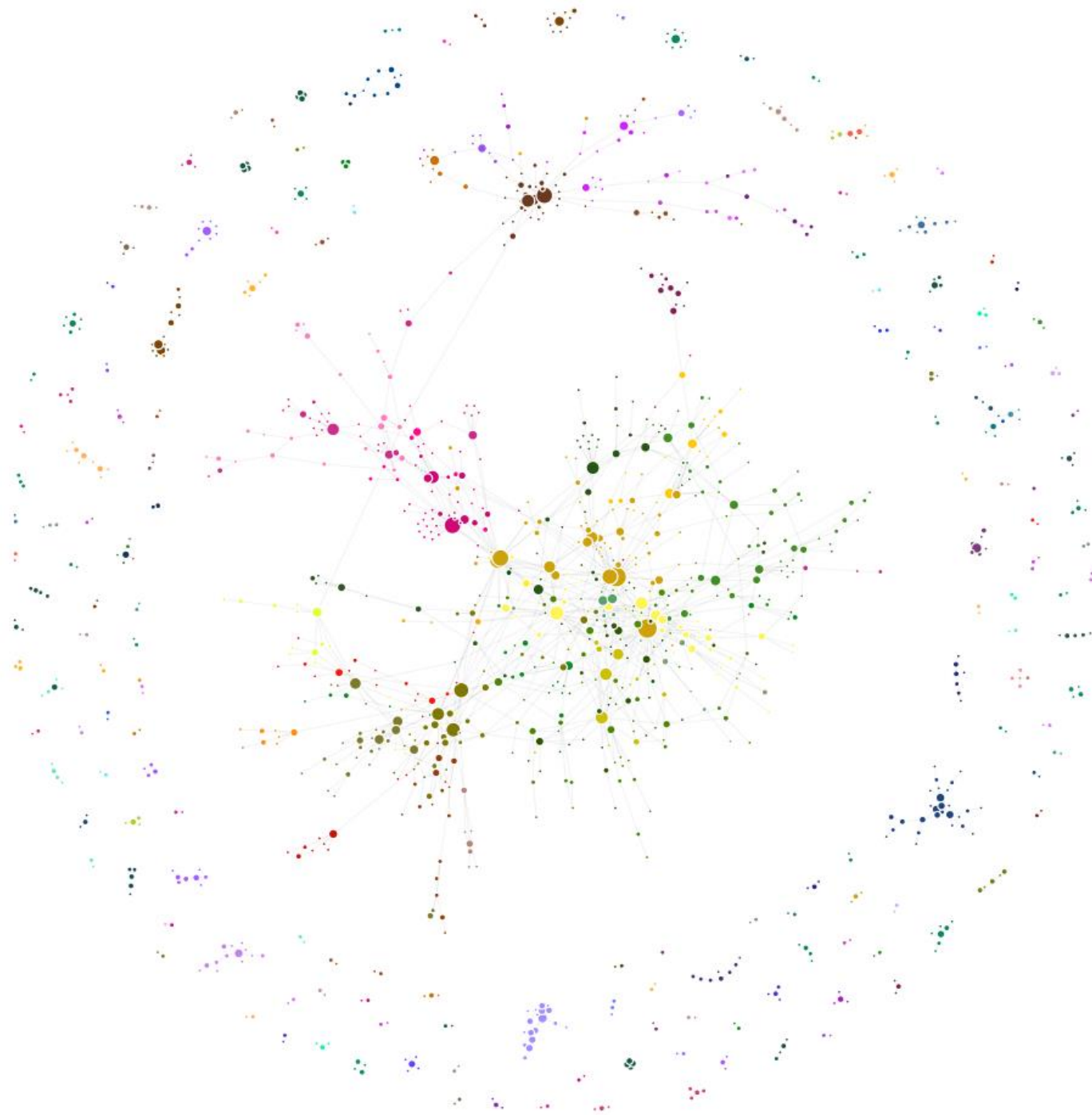
Presented by Cinda Heeren, 2016



Conflict-Free Final Exam Scheduling Graph

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Presented by Cinda Heeren, 2016

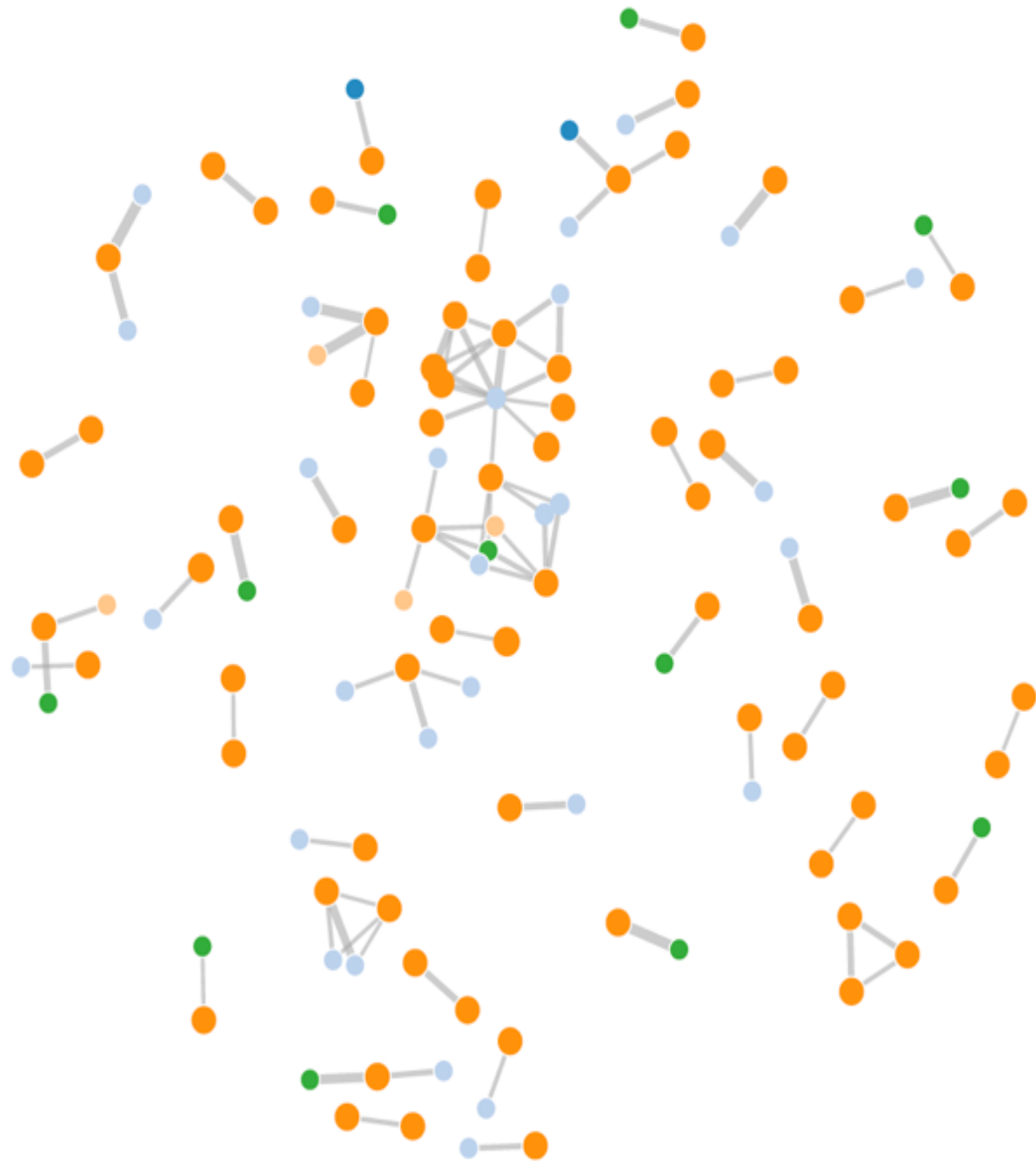


Class Hierarchy At University of Illinois Urbana-Champaign

A. Mori, W. Fagen-Ulmschneider, C. Heeren

Graph of every course at UIUC; nodes are courses, edges are prerequisites

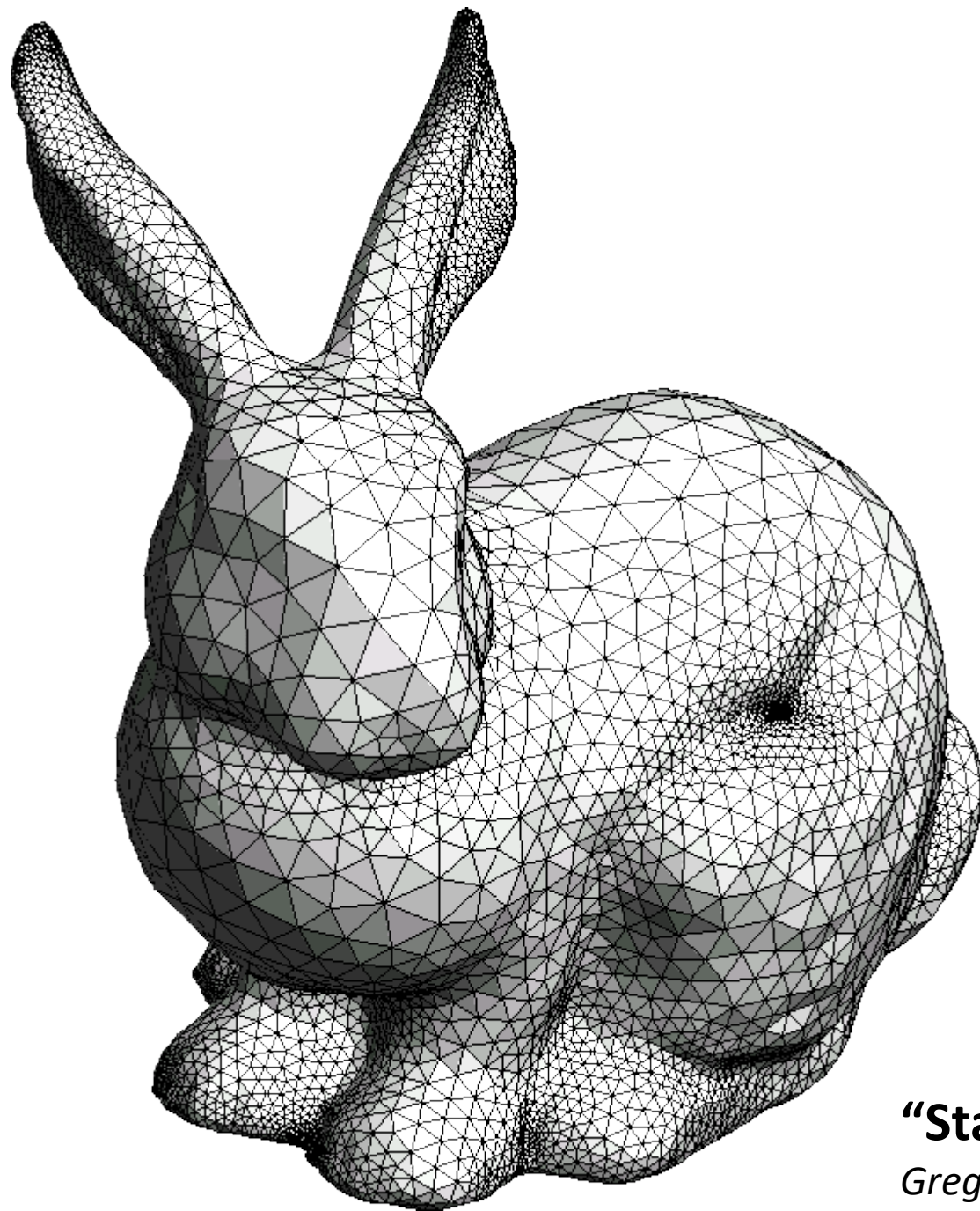
http://waf.cs.illinois.edu/discovery/class_hierarchy_at_illinois/



MP Collaborations in CS 225

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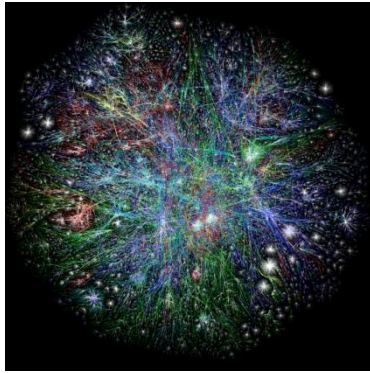
Presented by Cinda Heeren, 2016



“Stanford Bunny”

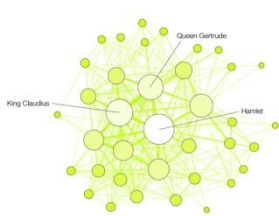
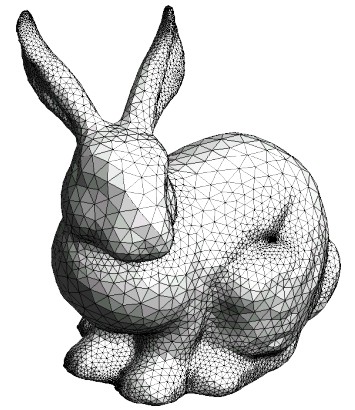
Greg Turk and Mark Levoy (1994)

Graphs

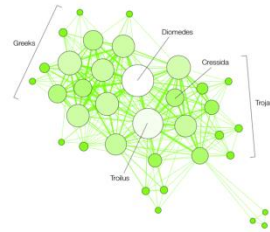


To study all of these structures:

1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms



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TROILUS AND CRESSIDA

