



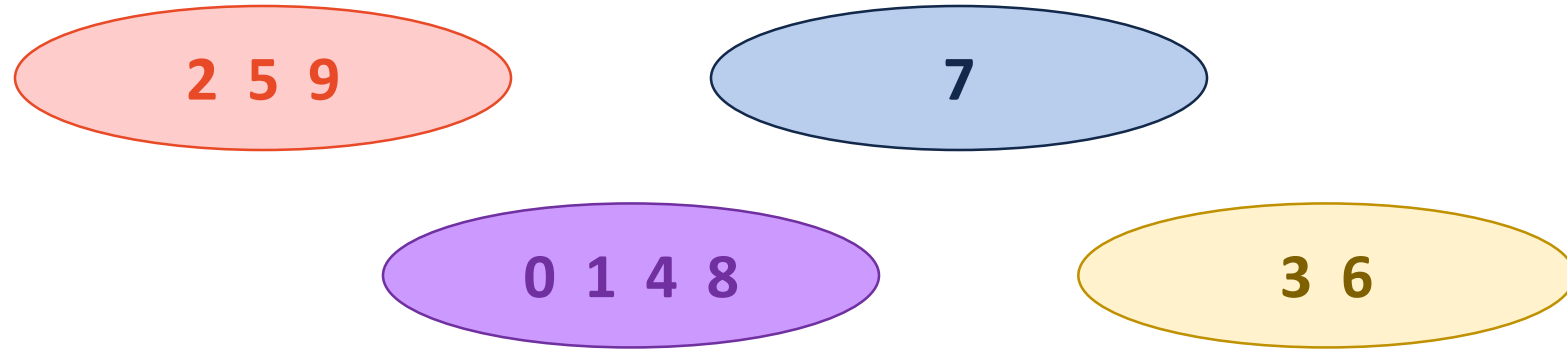
CS 225

Data Structures

April 4 – Disjoint Sets

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Disjoint Sets



Key Ideas:

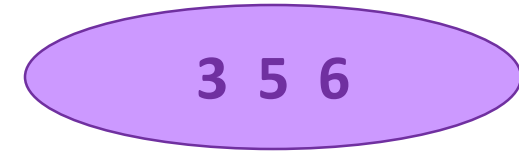
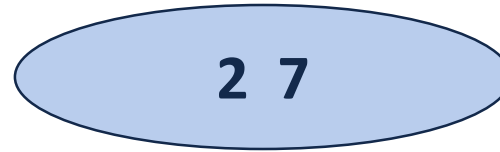
- Each element exists in exactly one set.
- Every set is an equitant representation.
 - Mathematically: $4 \in [0]_R \rightarrow 8 \in [0]_R$
 - Programmatically: `find(4) == find(8)`

Disjoint Sets ADT

- Maintain a collection $S = \{s_0, s_1, \dots, s_k\}$
- Each set has a representative member.
- API:

```
void makeSet(const T & t);  
void union(const T & k1, const T & k2);  
T & find(const T & k);
```

Implementation #1



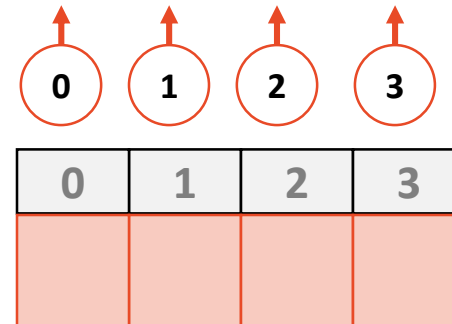
0	1	2	3	4	5	6	7

Find(k):

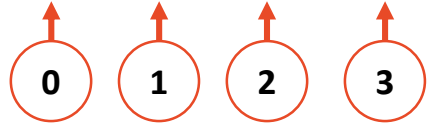
Union(k1, k2):

Implementation #2

- We will continue to use an array where the index is the key
- The value of the array is:
 - **-1**, if we have found the representative element
 - **The index of the parent**, if we haven't found the rep. element
- We will call these **UpTrees**:



UpTrees



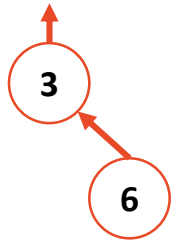
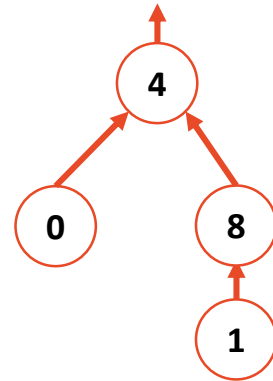
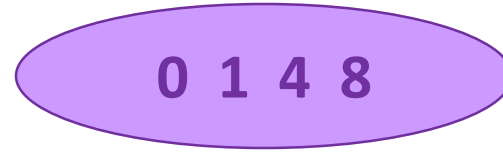
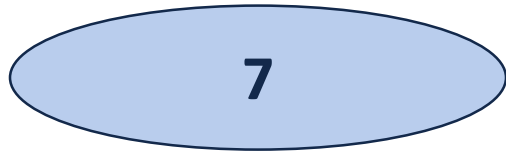
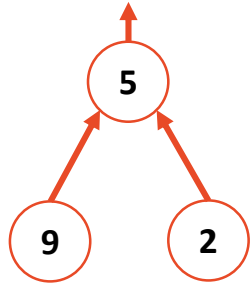
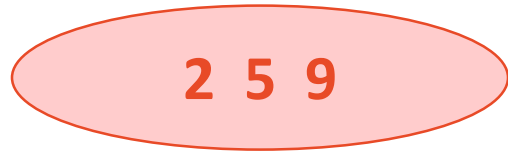
0	1	2	3
-1	-1	-1	-1

0	1	2	3

0	1	2	3

0	1	2	3

Disjoint Sets



0	1	2	3	4	5	6	7	8	9
4	8	5	6	-1	-1	-1	-1	4	5

Disjoint Sets Find

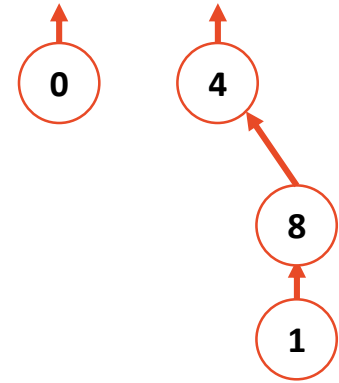
```
1 int DisjointSets::find() {  
2     if ( s[i] < 0 ) { return i; }  
3     else { return _find( s[i] ); }  
4 }
```

Running time?

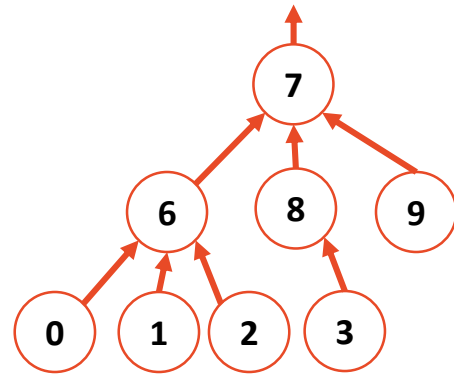
What is the ideal UpTree?

Disjoint Sets Union

```
1 void DisjointSets::union(int r1, int r2) {  
2  
3  
4 }
```

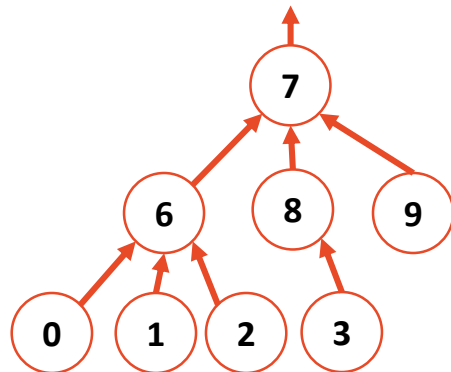


Disjoint Sets – Union



0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8	-1	10	7	-1	7	7	4	5

Disjoint Sets – Smart Union

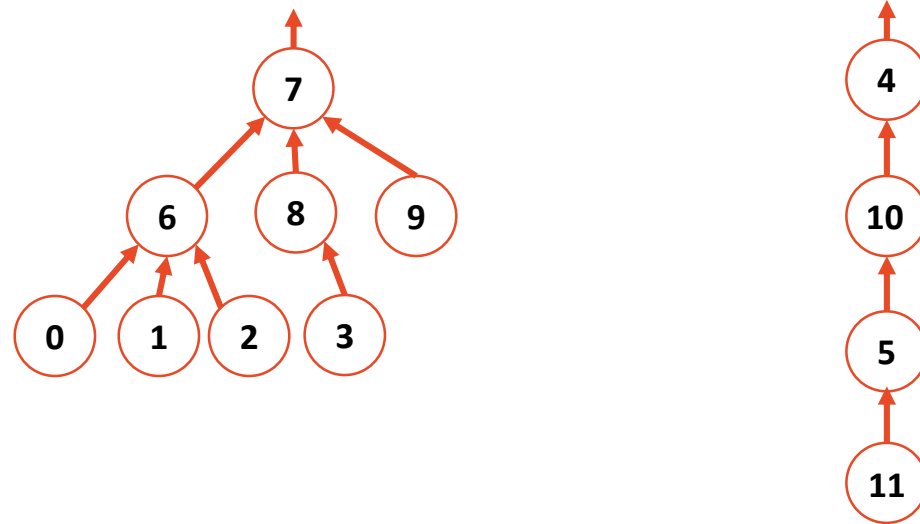


Union by height

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Keep the height of the tree as small as possible.

Disjoint Sets – Smart Union



Union by height

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Keep the height of the tree as small as possible.

Union by size

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8		10	7		7	7	4	5

Idea: Minimize the number of nodes that increase in height

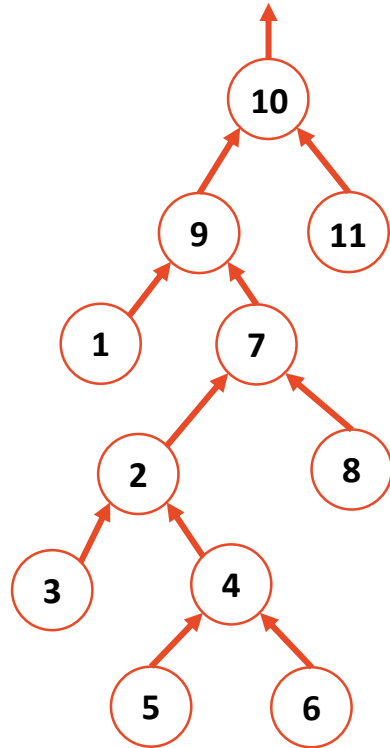
Both guarantee the height of the tree is: _____.

Disjoint Sets Find

```
1 int DisjointSets::find(int i) {  
2     if ( s[i] < 0 ) { return i; }  
3     else { return _find( s[i] ); }  
4 }
```

```
1 void DisjointSets::unionBySize(int root1, int root2) {  
2     int newSize = arr_[root1] + arr_[root2];  
3  
4     // If arr_[root1] is less than (more negative), it is the larger set;  
5     // we union the smaller set, root2, with root1.  
6     if ( arr_[root1] < arr_[root2] ) {  
7         arr_[root2] = root1;  
8         arr_[root1] = newSize;  
9     }  
10  
11     // Otherwise, do the opposite:  
12     else {  
13         arr_[root1] = root2;  
14         arr_[root2] = newSize;  
15     }  
16 }
```

Path Compression



Disjoint Sets Analysis

The **iterated log** function:

The number of times you can take a log of a number.

$\log^*(n) =$

0, $n \leq 1$

$1 + \log^*(\log(n))$, $n > 1$

What is $\lg^*(2^{65536})$?

Disjoint Sets Analysis

In an Disjoint Sets implemented with smart **unions** and path compression on **find**:

Any sequence of **m union** and **find** operations result in the worse case running time of $O(\text{_____})$,
where **n** is the number of items in the Disjoint Sets.