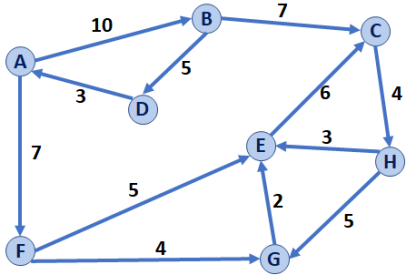
CS 225

#39: Dijkstra's Algorithm (for reals!)

April 25, 2018 · *Wade Fagen-Ulmschneider*

Shortest Path Home:

Dijkstra's Algorithm (Single Source Shortest Path)



Dijkstra's Algorithm Overview:

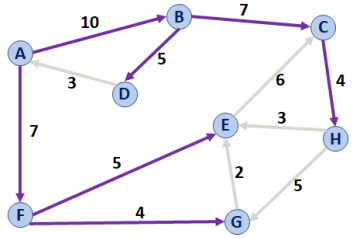
- The overall logic is the same as Prim's Algorithm
- We will modify the code in only two places – both involving the update to the distance metric.
- The result is a directed acyclic graph or DAG

Pseudocode for Dijkstra's SSSP Algorithm

```
1  DijkstraSSSP(G, s):
2    Input: G, Graph;
3           s, vertex in G, starting vertex of algorithm
4    Output: T, DAG w/ shortest paths (and distances) to s
5
6    foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9    d[s] = 0
10
11   PriorityQueue Q   // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T           // "labeled set"
14
15   repeat n times:
16     Vertex m = Q.removeMin()
17     T.add(m)
18     foreach (Vertex v : neighbors of m not in T):
19       if _____________________________ < d[v]:
20         d[v] = _____________________________
21         p[v] = m
22
23   return T
```

Backtracking in Dijkstra

Dijkstra's Algorithm gives us the shortest path from a single source to every connected vertex:



The data structure maintained by Dijkstra's Algorithm will have the following state after running Dijkstra's Algorithm:

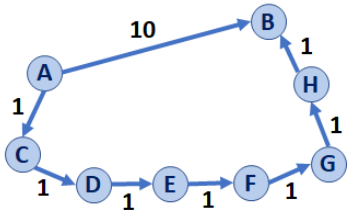
	A	B	C	D	E	F	G	H
p	NULL	A	B	B	F	A	F	C
d	0	10	17	15	12	7	11	21

Q: What is the shortest path from A to H?

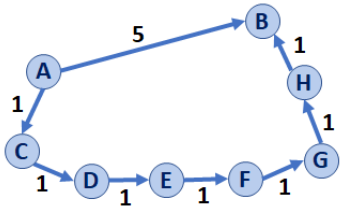
Q: What is the shortest path from A to E?

Examples: How is a single heavy-weight path vs. many light-weight paths handled?

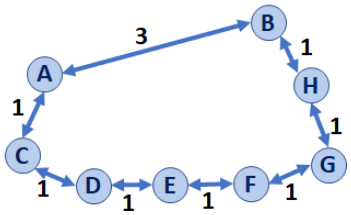
Ex 1:



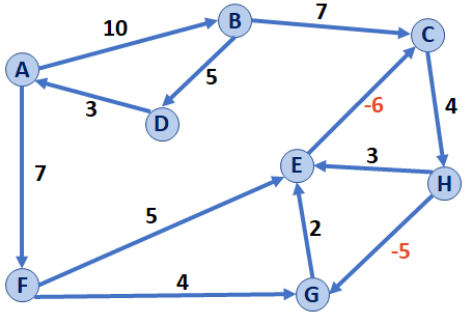
Ex 2:



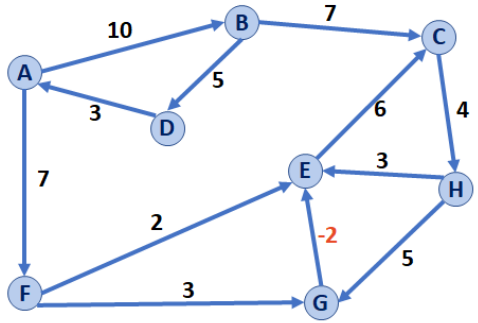
What about undirected graphs?



Dijkstra: What if we have a negative-weight cycle?



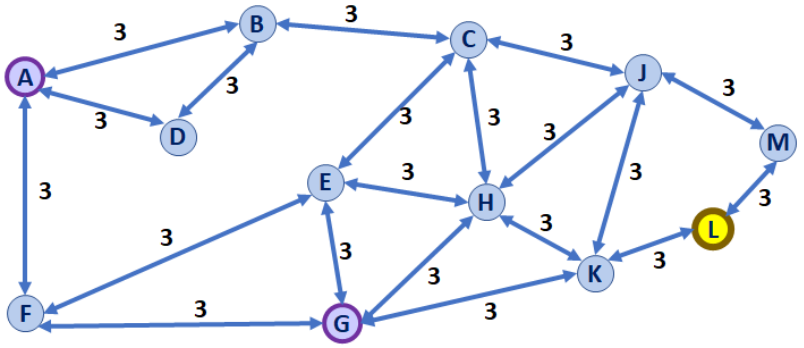
Dijkstra: What if we have a minimum-weight edge, without having a negative-weight cycle?



Dijkstra makes an assumption:

Dijkstra Algorithm: What is the running time?

Challenge: Landmark Path Problem



CS 225 – Things To Be Doing:
1. Final Exam runs Thursday, May 3 – Thursday, May 10 **2.** MP7 is released; MP7 deadline Monday, May 30 **3.** Final lab, **lab_ml**, released today; due Sunday, May 29 **4.** This week is the last week of POTDs *(last POTD is Friday!)*