# Data Structures

# BTree Analysis

CS 225

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Department of Computer Science

# Next Week Mattox Lectures!

I will return around time to introduce graphs.

# Learning Objectives

Review fundamentals of B Tree

Discuss the importance of M in a B Tree

Analyze the performance of the B Tree

# Engineering vs Theory Efficiency

| | Time x1 billion | Like |
|---|---|---|
| **L1 cache reference** | 0.5 seconds | Heartbeat 💓 |
| **Branch mispredict** | 5 seconds | Yawn 😯 |
| **L2 cache reference** | 7 seconds | Long yawn 😯😯😯 |
| **Mutex lock/unlock** | 25 seconds | Make coffee ☕ |
| **Main memory reference** | 100 seconds | Brush teeth |
| **Compress 1K bytes** | 50 minutes | TV show 📺 |
| **Send 2K bytes over 1 Gbps network** | 5.5 hours | (Brief) Night's sleep 🛌 |
| **SSD random read** | 1.7 days | Weekend |
| **Read 1 MB sequentially from memory** | 2.9 days | Long weekend |
| **Read 1 MB sequentially from SSD** | 11.6 days | 2 weeks for delivery 📦 |
| **Disk seek** | 16.5 weeks | Semester |
| **Read 1 MB sequentially from disk** | 7.8 months | Human gestation 🐣 |
| **Above two together** | 1 year | 🌍☀️ |
| **Send packet CA->Netherlands->CA** | 4.8 years | Ph.D. 🎓 |

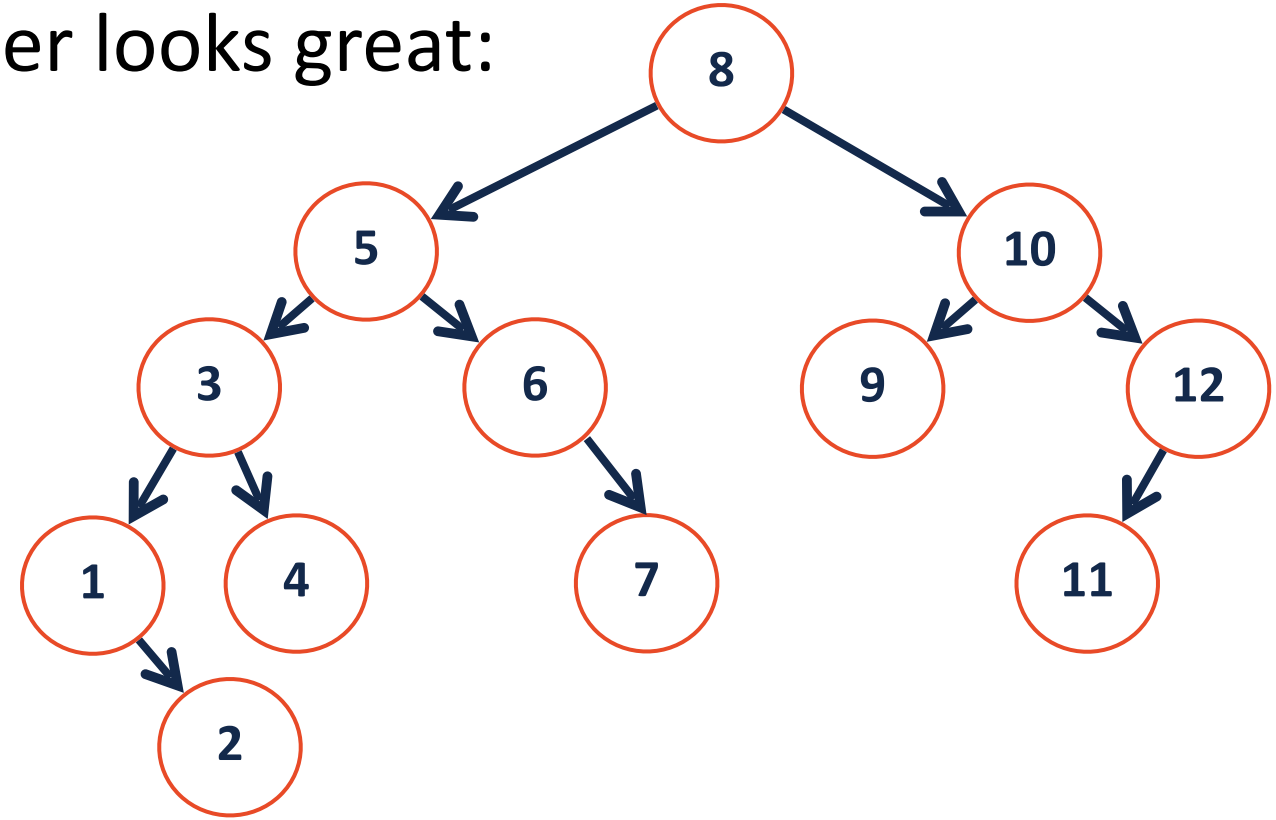
(Care of https://gist.github.com/hellerbarde/2843375)

# Engineering vs Theory Efficiency

In Big-O we have assumed uniform time for all operations, but this isn't always true.

However, seeking data from the cloud may take 40ms+.

…an O(lg(n)) AVL tree no longer looks great:



# BTree Design Motivations

When large seek times become an issue, we address this by:

1) Keep the number of seeks low

2) When possible keep data stored locally

3) Make sure the data we look up is relevant!

# BTree Properties

A **BTrees** of order **m** is an m-ary tree and by definition:

- All keys within a node are ordered
- All nodes contain no more than **m-1** keys.
- All internal nodes have exactly **one more child than keys**
- All leaves in the tree are at the same level.

**Today: How does the above lead to these improvements?**

1) Keep the number of seeks low

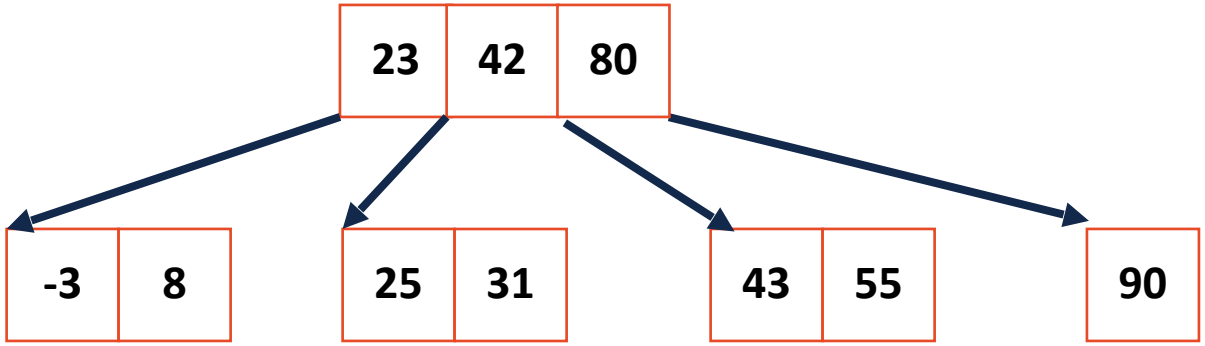
2) When possible keep data stored locally

3) Make sure the data we look up is relevant!

# Before we get there… lets review!

Join Code: 225

What values of M are allowed in the following tree?



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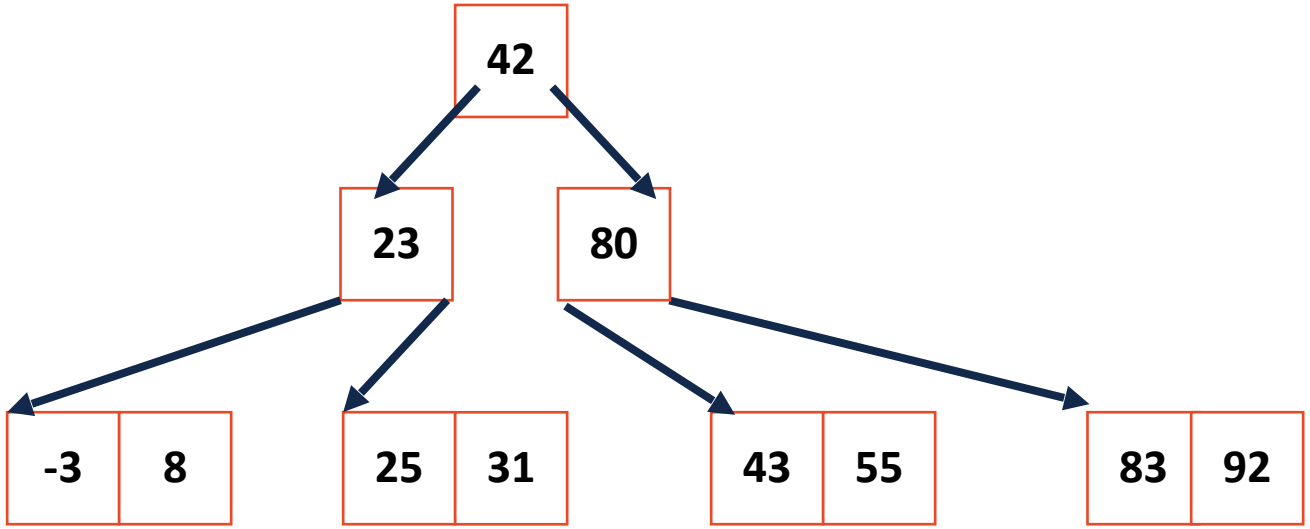


# BTree of Order M (In Practice)

**Brainstorm:** In the real world, what value of **m** would be useful?

Join Code: 225

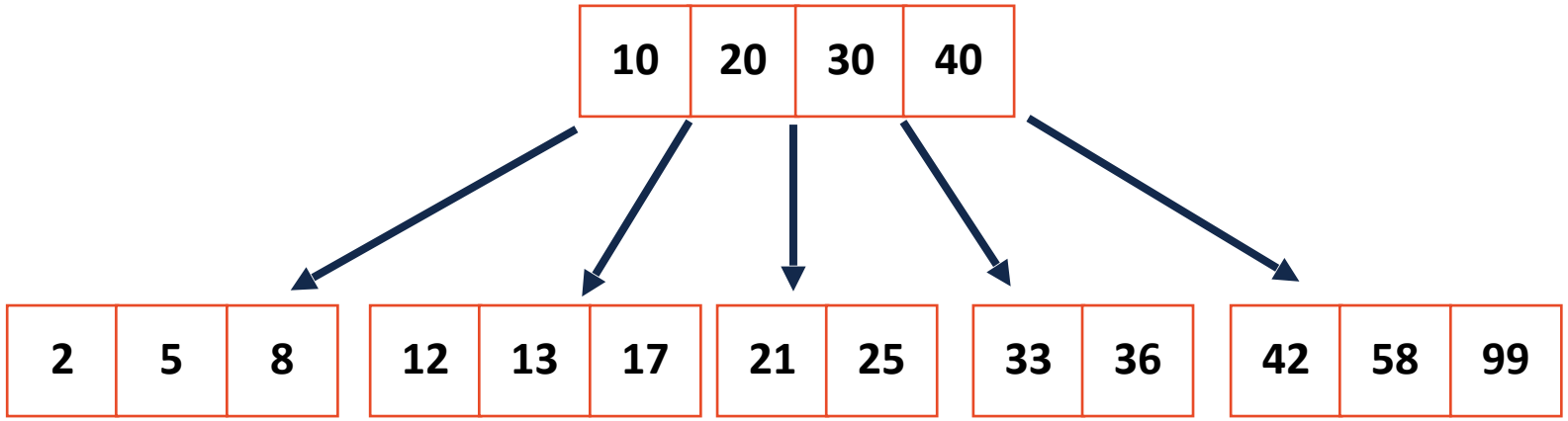
# BTree of Order M

**Questions about M?**

# BTree Find

**How many keys are looked at to complete this search?**

Join Code: 225




`Find(4)`

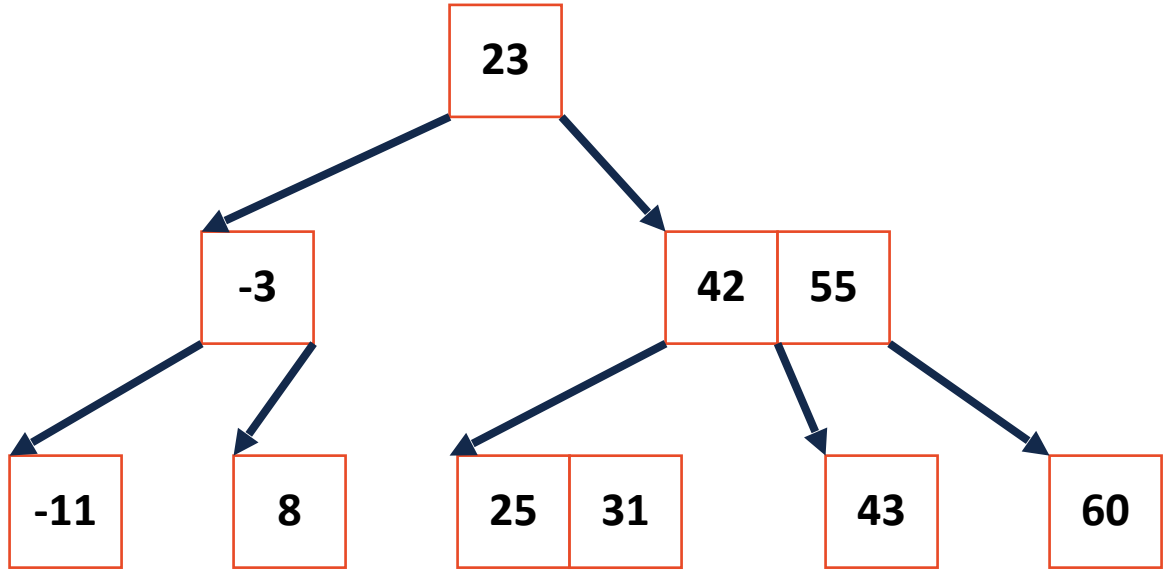
# BTree Find

`Find(31)`

Base Cases:

If root is empty, return

If leaf, do array find() and return




Recursive Step:

Array find() for match or first greater value

Recurse on appropriate child

**Tip:** Index of first greater value is index of child we want to visit!

# BTree Insertion

M = 5

Given appropriate leaf, insert is sorted array insert

Insert(1)

Insert(2)

Insert(3)

Insert(4)

Insert(5)

Insert(6)

Insert(7)

Insert(8)

# BTree Insertion

M = 5

Given appropriate leaf, insert is sorted array insert

```
Insert(1)
Insert(2)
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Insert(4)
Insert(5)
Insert(6)
Insert(7)
Insert(8)
```
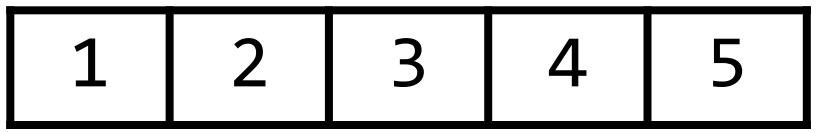




For LAB: Why is this size 5 and not size 4?

# BTree Insertion

**M = 5**

When we hit **M** items, split into three nodes!

Insert(1)

Insert(2)

Insert(3)

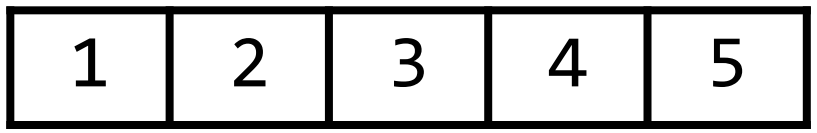
Insert(4)

Insert(5)

**Insert(6)**

**Insert(7)**

**Insert(8)**



# BTree Insertion

M = 5

In larger trees, use BTree Find() to get appropriate leaf

Insert(1)

Insert(2)

Insert(3)

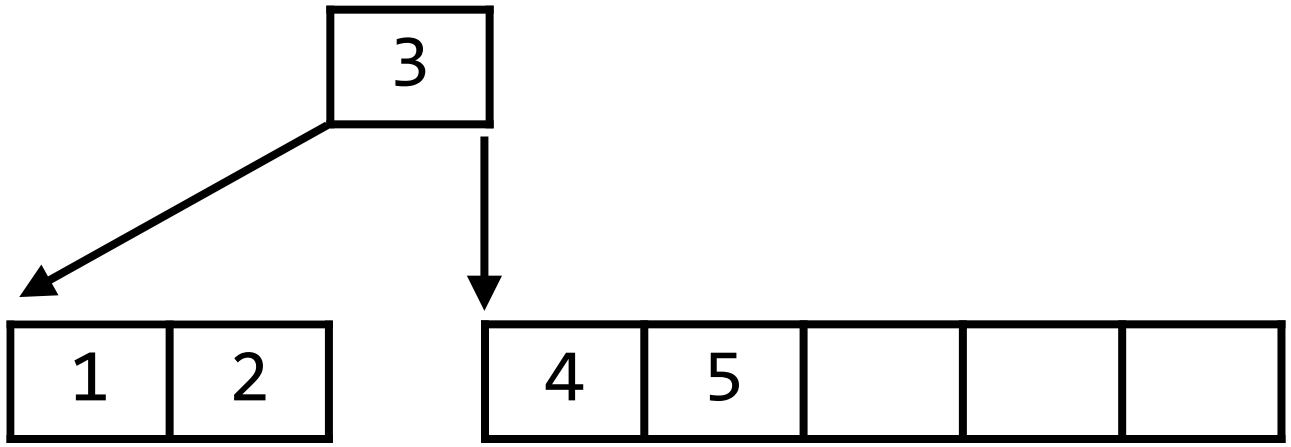
Insert(4)

Insert(5)

Insert(6)

Insert(7)

Insert(8)



# BTree Insertion

M = 5

If parent node already exists, split adds new key.

Insert(1)

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Insert(3)

Insert(4)

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Insert(7)

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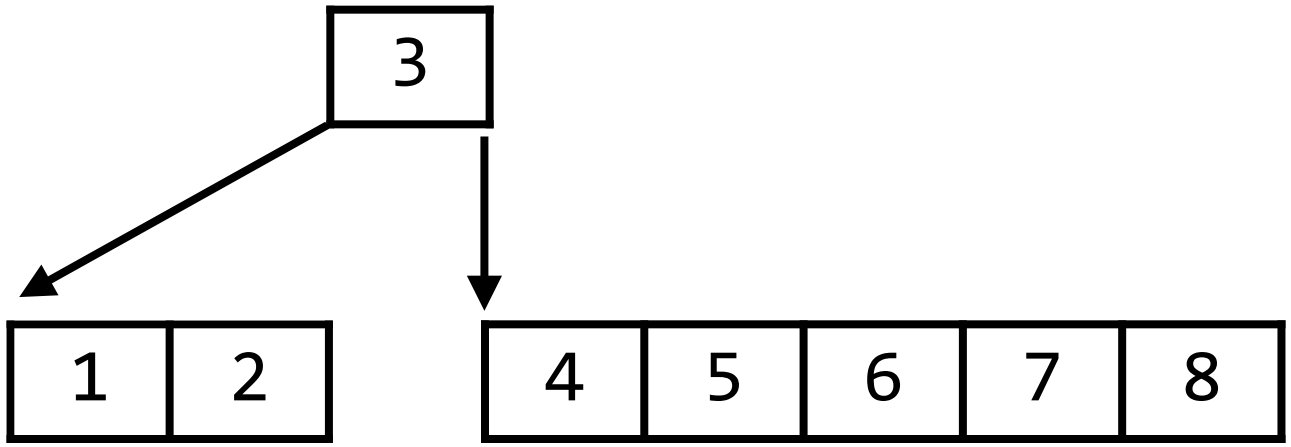
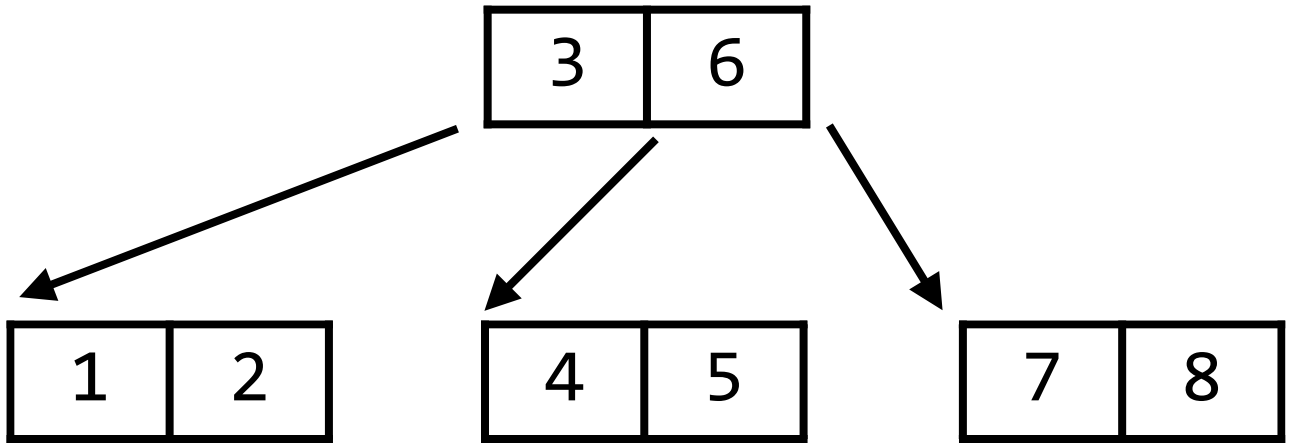


# BTree Insertion

M = 5

If parent node already exists, split instead adds new key.

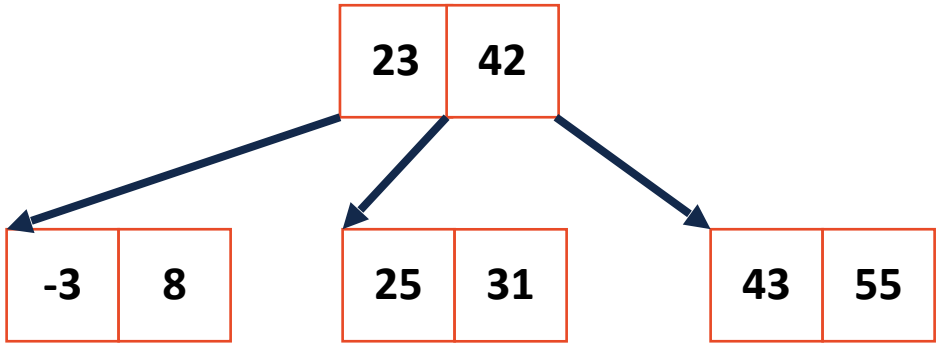
```
Insert(1)
Insert(2)
Insert(3)
Insert(4)
Insert(5)
Insert(6)
Insert(7)
Insert(8)
```



# BTree Recursive Insert

`Insert(56), M = 3`

Insert always starts at a leaf but can propagate up repeatedly.



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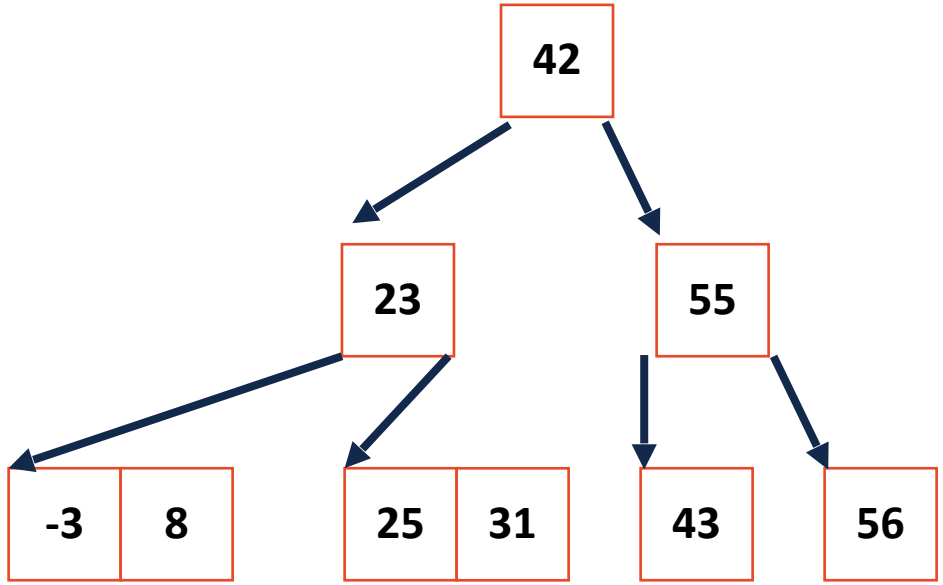
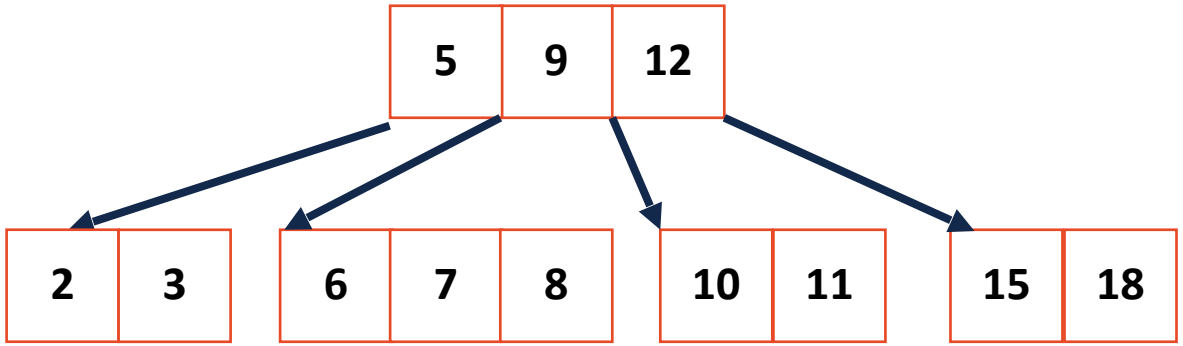


# BTree Remove

BTree removal is complicated! **It won't be part of the lab.**

If we have time at the end of the day today we will discuss it



# BTree Implementation

**Any questions about BTree implementation details?**

# BTree Properties

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# BTree Analysis

Like the BST, BTree height determines the runtime of our operations!

**Claim:** The BTree structure limits our height to $O(log_m(n))$

**Proof:** We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

# BTree Analysis

**Strategy:**

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.

**Key Facts:**

Root nodes can be a leaf or have **[2, m]** children.

All non-root, internal nodes have **[ceil(m/2), m]** children.

# BTree Analysis

Min number of **nodes** for a BTree of order m **at each level:**

Root:

Level 1:

Level 2:

Level 3:

Level h:

# BTree Analysis

Min number of **nodes** for a BTree of order m **at each level:** $t = \lceil \frac{m}{2} \rceil$

Root: $1$

Level 1: $2$ (Each of these will have t children)

Level 2: $2t$ (Each of these will have t children)

Level 3: $2t^2$ (Each of these will have t children)

Level h: $2t^{h-1}$

# BTree Analysis

$$t = \lceil \frac{m}{2} \rceil$$

The **min total number of nodes** is the sum of all the levels:

$$1 + 2\sum_{k=0}^{h-1} t^k =$$

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}$$

# BTree Analysis

$$t = \lceil \frac{m}{2} \rceil$$

The **min total number of nodes** is the sum of all the levels:

$$1 + 2\sum_{k=0}^{h-1} t^k = 1 + 2\frac{t^h - 1}{t - 1}$$

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}$$

# BTree Analysis

$$t = \lceil \frac{m}{2} \rceil$$

The **min total number of nodes**: $1 + 2\frac{t^h - 1}{t - 1}$

The **min total number of keys**:

# BTree Analysis

$t = \lceil \frac{m}{2} \rceil$

The **min total number of nodes**: $1 + 2\frac{t^h - 1}{t - 1}$

The **min total number of keys**:

Root has how many keys? **1**

Internal nodes? $\lceil \frac{m}{2} \rceil - 1 = t - 1$

Leaf nodes? $\lceil \frac{m}{2} \rceil - 1 = t - 1$

So we can multiply the fraction by $t - 1$

$= 1 + 2\frac{t^h - 1}{t - 1} * (t - 1)$

$= 2t^h - 1$

# BTree Analysis

$$t = \lceil \frac{m}{2} \rceil$$

The **smallest total number of keys** is: $2t^h - 1$

So an inequality about **n**, the total number of keys:

Solving for **h**, since **h** is the max number of seek operations:

# BTree Analysis

$t = \lceil \frac{m}{2} \rceil$

The **smallest total number of keys** is: $2t^h - 1$

So an inequality about **n**, the total number of keys:

$$n \geq 2t^h - 1$$

$$n + 1 \geq 2t^h$$

$$log_m\,(n+1) \geq log_m\,\left(2\lceil\frac{m}{2}\rceil^h\right) = log_m\,(m^h) = h$$

Solving for **h**, since **h** is the max number of seek operations:

$$h = O(log_m\,n)$$

# BTree Analysis

This is very powerful!

As long as I am *at least* minimally sized, we are O(log n)!

# BTree Analysis

Given **m=101**, a tree of height **h=4** has:

Minimum Keys:

Maximum Keys:

# BTree

The BTree is still used heavily today!

Improvements such as B+Tree and B*Tree exist far outside class scope

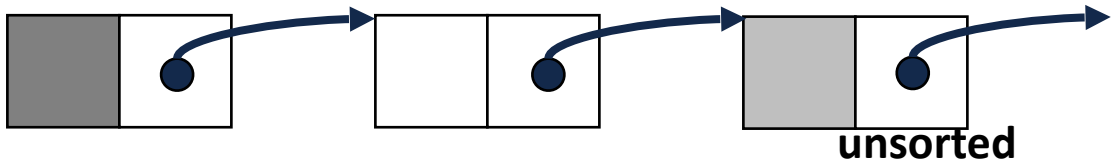
# Thinking conceptually: Sorting a queue

How might we build a ‘queue’ in which our front element is the min?

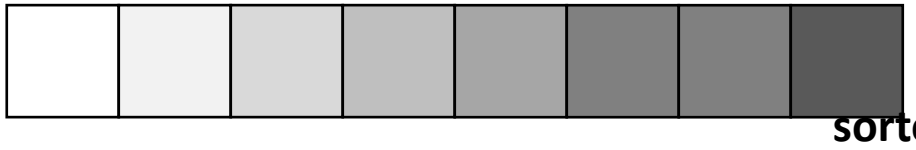
# Priority Queue Implementation

| insert | removeMin |
|---|---|
| O( n ) | O(n) |
| O(1) | O(n) |
| O( n ) | O(1) |
| O( n ) | O(1) |

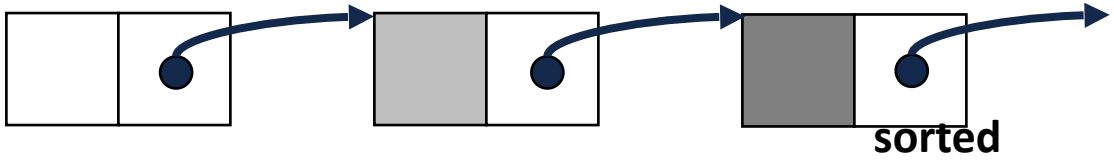
unsorted



unsorted



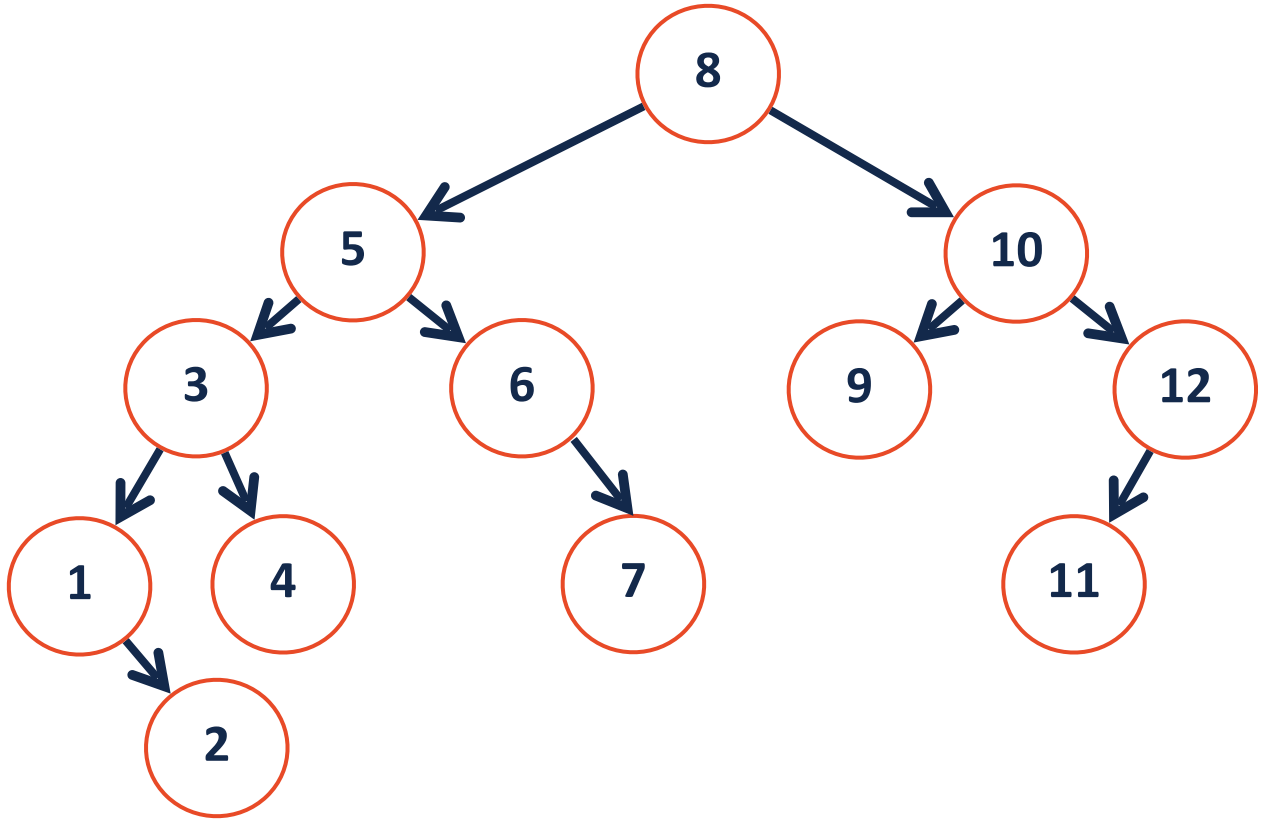
sorted



sorted

# Priority Queue Implementation

| insert | removeMin |
| --- | --- |
| | |



# Thinking conceptually: A tree without pointers

What class of (non-trivial) trees can we describe without pointers?