Final thoughts on arrays (& lists)

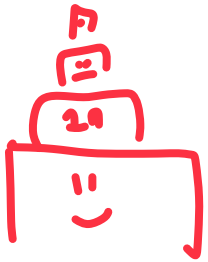
Last content on exam 1

Data Structures + Stacks and Queues

CS 225

Brad Solomon

September 9, 2026



UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Department of Computer Science

workshop + info meeting

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6PM TIL 8PM

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FROM **6-8PM** ON **SEP 14**

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Learning Objectives

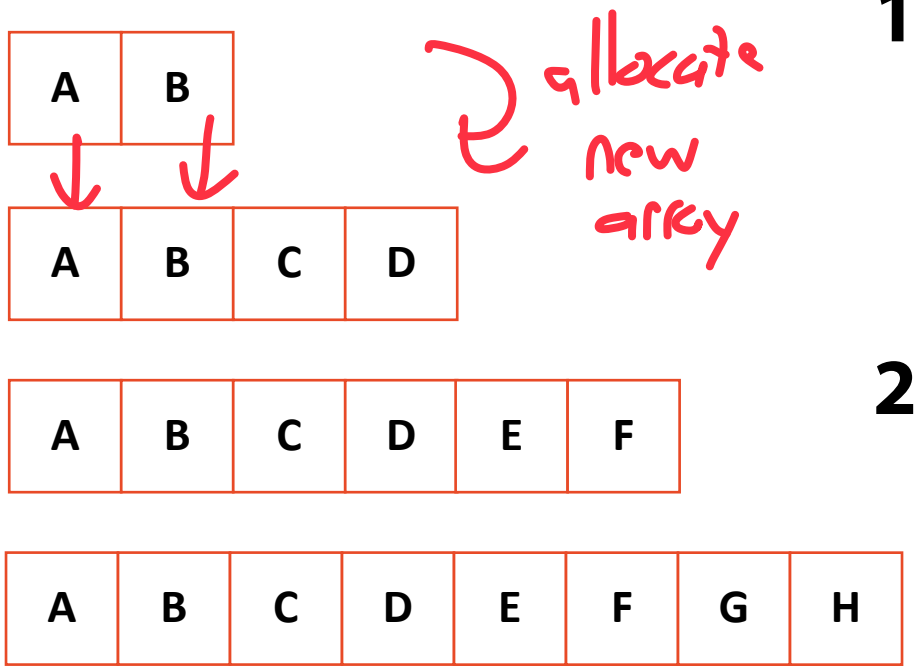
Finalize resize implementation for an array list

Consider extensions to lists (data structure tradeoffs)

Introduce the stack and the queue data structure

Introduce and explore iterators

Resize Strategy: +2 elements every time



1) How many copy calls per reallocation?

For reallocation i, 2i copy calls are made

2) Total reallocations for N objects?

Let k be the number of reallocs, $k = \frac{N}{2}$

Total number of copy calls: $\sum_{i=1}^{k} 2i = k(k+1) = k^2 + k$

… For N objects: $\frac{N^2 + 2N}{4}$

Resize Strategy: +2 elements every time

Total copies for N inserts: $\frac{N^2 + 2N}{4}$

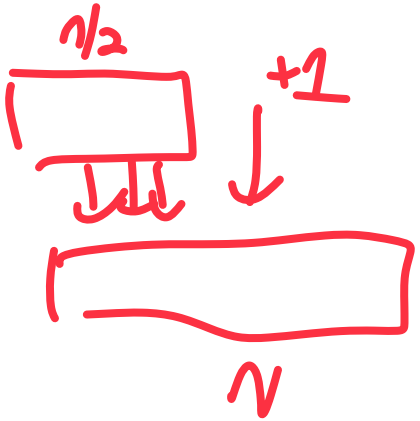
Amortized: ~N ops per insert

Precise total work over N calls

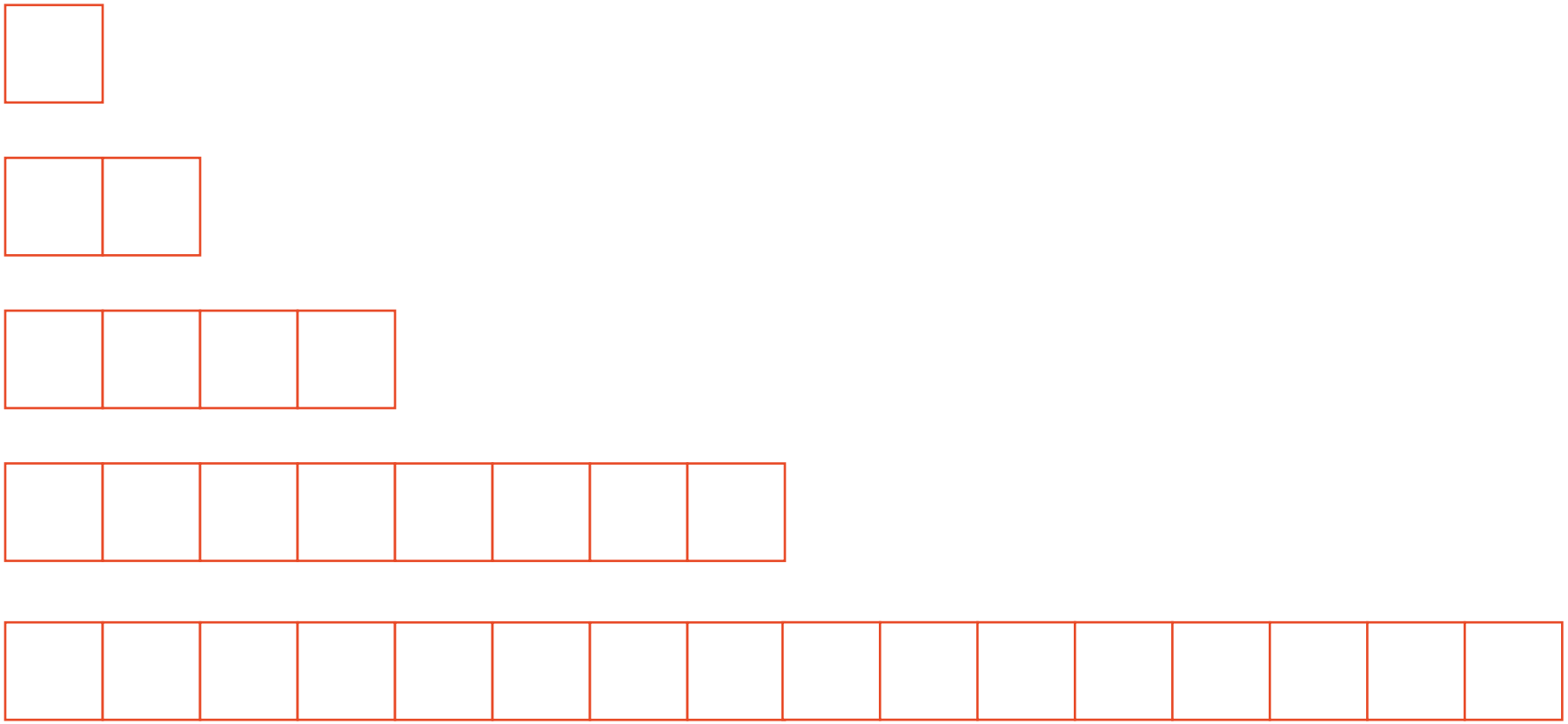
Big O: O(N)

Upperbound on worst case

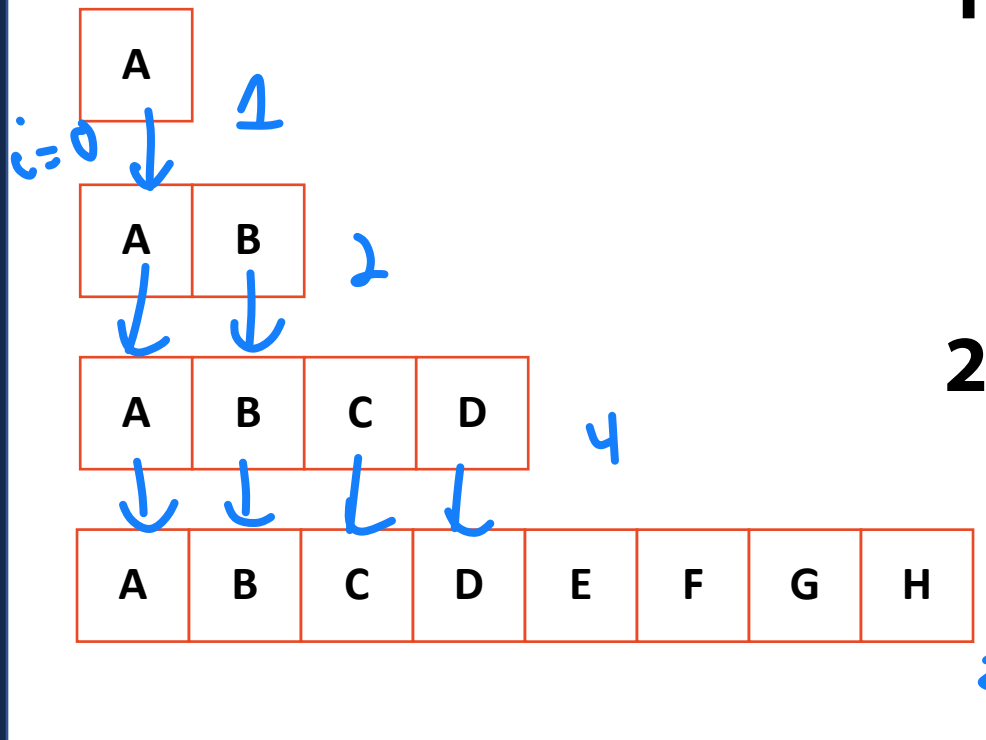
↳ I have to reallocate



Resize Strategy: x2 elements every time



Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

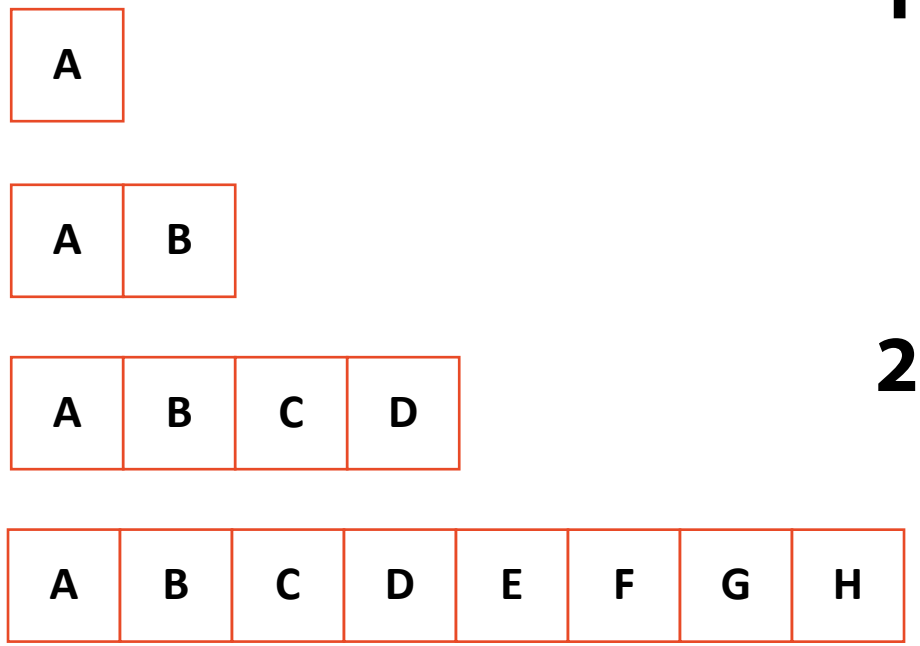
for realloc i , 2^i

2) Total reallocations for N objects?

$N \leq 2^k$ (k = # times realloc)

↳ $k = \log_2 N$

Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

For reallocation i, 2^i copy calls are made

vs $2 \cdot i$

2) Total reallocations for N objects?

k = final realloc needed = $\lceil log_2 N \rceil$

Total number of copy calls: $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

plug in k

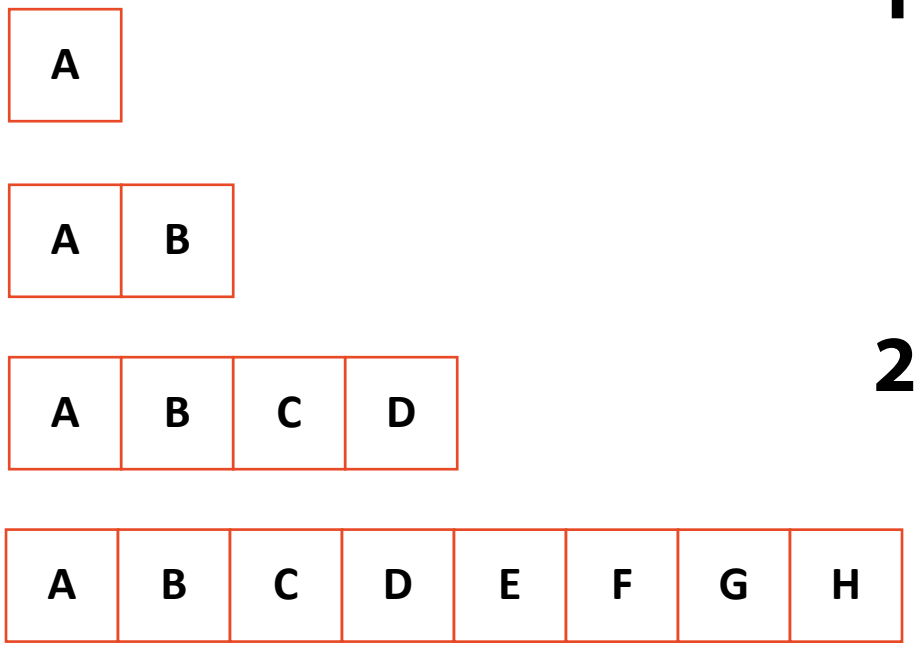
vs $k = N/2$ *

$2^{\log_2 N + 1} - 1 = 2 \cdot 2^{\log_2 N} - 1 \simeq 2N - 1$

Tradeoff in action!

x2 less frequent reallocations each reallocation is larger

Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

For reallocation i, 2^i copy calls are made

2) Total reallocations for N objects?

k = final realloc needed = $\lceil log_2 N \rceil$

Total number of copy calls: $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

… For N objects: $2N - 1$

Resize Strategy: x2 elements every time

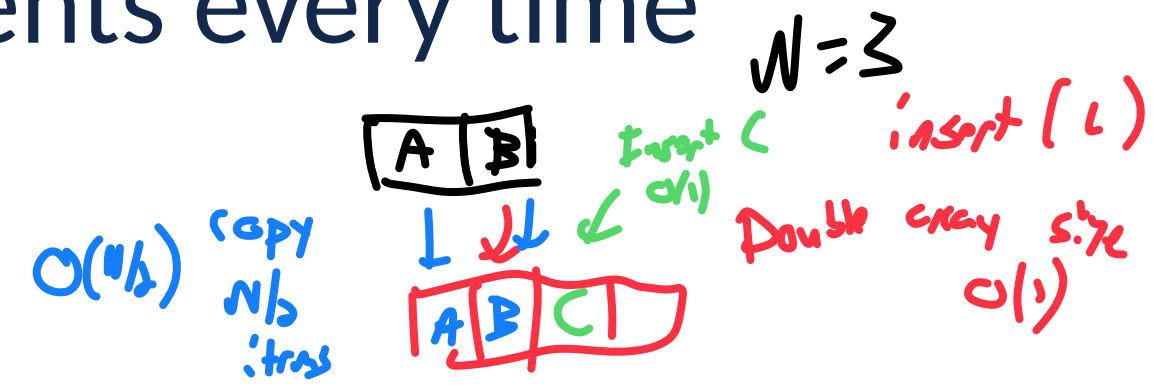
Total copies for n inserts: $2N - 1$

Amortized:

Big O:

Resize Strategy: x2 elements every time

Total copies for n inserts: $2N - 1$



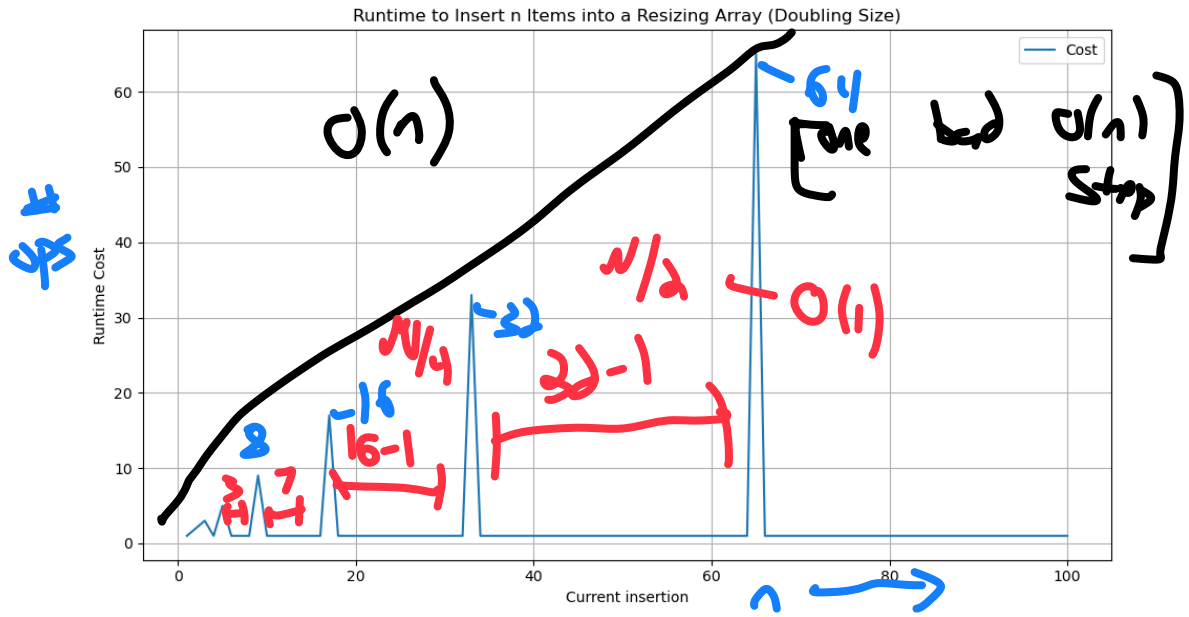
Amortized: $O(1)^*$

Big O: $O(n)$

Precise total work over N calls

$2N-1 / N$ = # work per insert is ~2

Upperbound on worst case



Distinguishing amortized vs Big O

1) Every insert operation has a Big O of O(N)

2) By totaling up the total work over N inserts, we see a different story:

- The 'bad' insert steps happen roughly once every n inserts

Ex: n = 2, 4, 8, 16, 32, …

- The amount of work in the bad insert also grows linearly

- All other inserts are 'good' inserts each taking O(1) time

infrequent

When viewed in the lens of 'total work over N inserts' we get O(1) *

List Implementation

	Singly Linked List	Array
Look up **arbitrary** location	O(n)	O(1) ☺
Insert after **given** element w/ ref to pointer	O(1) ☺	O(n)
Remove after **given** element w/ ref to pointer	O(1) ☺	O(n)
Insert at **arbitrary** location	O(n) find O(1) insert O(n)	O(1) find O(n) insert O(n)
Remove at **arbitrary** location	O(n) find O(1) del O(n)	O(1) find O(n) remove O(n)
Search for an input **value**	O(n)	O(n)

Special cases:

insert/Remove front O(1)

insert/Remove Back O(1)*

* array not full

List Implementation

	Singly Linked List	Array
Look up **arbitrary** location	O(n)	O(1)
Insert after **given** element	O(1)	O(n)
Remove after **given** element	O(1)	O(n)
Insert at **arbitrary** location	O(n)	O(n)
Remove at **arbitrary** location	O(n)	O(n)
Search for an input **value**	O(n)	O(n)

Special Cases: insertFront insertBack (not full)

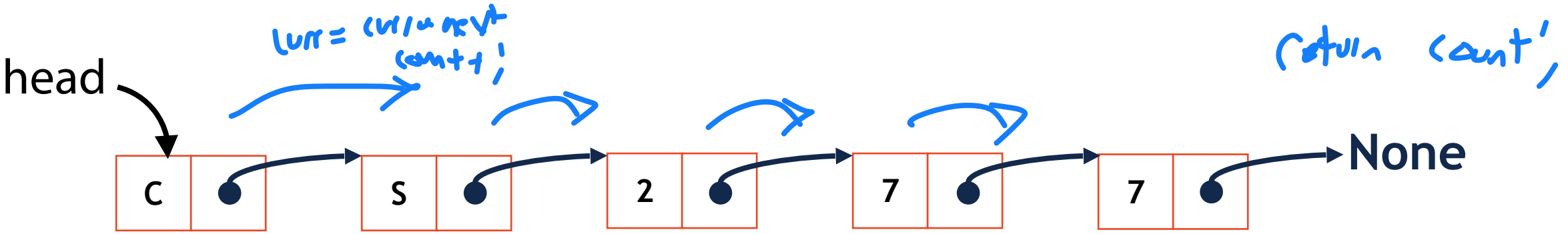
Thinking critically about lists: tradeoffs

The implementations shown are foundational (simple).

Can we make our lists better at some things? What is the cost?

Thinking critically about lists: tradeoffs

Getting the size of a linked list has a Big O of: O(n) → O(1)



List
↳ ListNode* head
↳ int size;
↳ modifying insert / remove
size++ / size--

ListNode
↳ T & data
↳ ListNode * next;

Goal: O(n) → O(1) size()

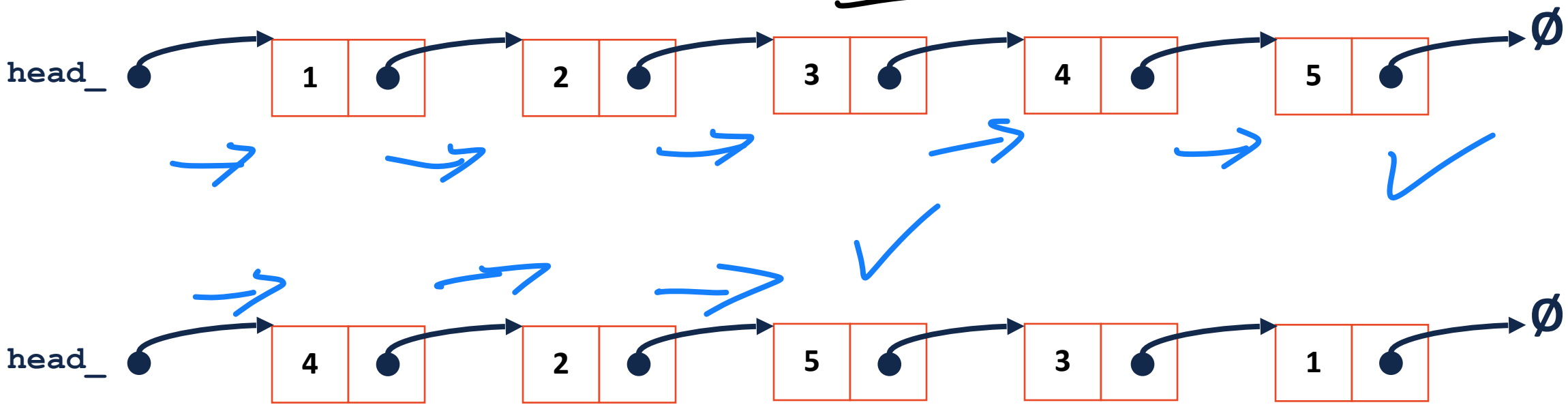
Cost: One single int in memory
↳ An O(1) increase in insert & remove

Thinking critically about lists: tradeoffs

5

Sorted linked list

A BAD trade off



Gain: faster search on sorted

↳ Problem: Linked list has no random access

Cost: Insert always O(n) now

A situation where you needed to filter a list in smallest → largest

Thinking critically about lists: tradeoffs

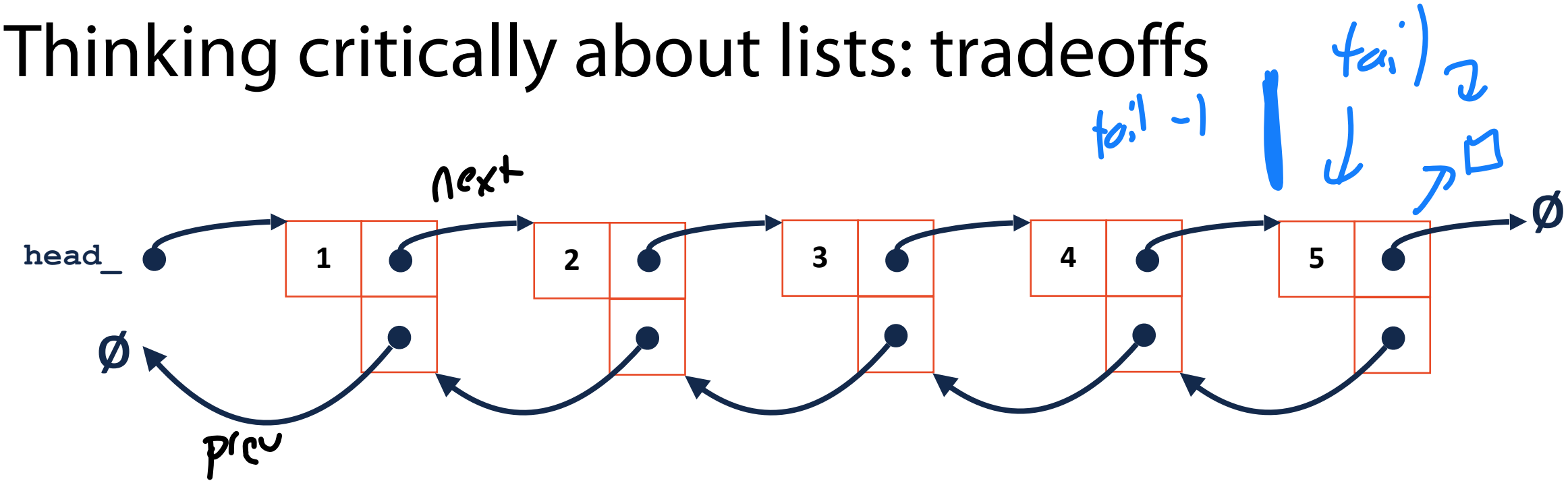
2	7	5	9	7	14	1	0	8	3

0	1	2	3	5	7	7	8	9	14

Gain: O(log n) search

Cost: No O(1) insertBack

Thinking critically about lists: tradeoffs



Gain: ???

Cost: 1 pointer per node

Combination of tail & prev lets us removeBack() O(1)!

Thinking critically about lists: tradeoffs

As we progress in the class, we will see that $O(n)$ isn't very good.

Take searching for a specific list value:

2	7	5	9	7	14	1	0	8	3

0	1	2	3	5	7	7	8	9	14

Thinking critically about lists: tradeoffs

Can we make a 'list' that is O(1) to insert and remove?

↳ special case exceptions!

Stack Data Structure

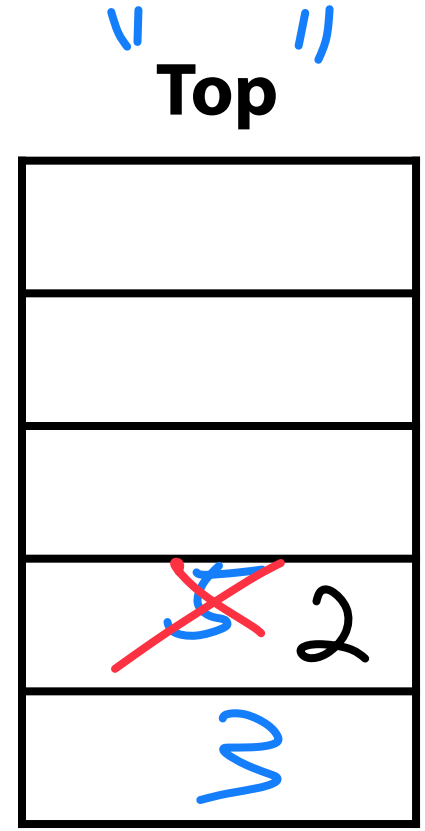
A **stack** stores an ordered collection of objects (like a list)

However you can only do three* operations:

Push: Put an item on top of the stack

Pop: Remove the top item of the stack

Top: Return the top item of the stack




```
push(3); push(5); pop(); push(2)
```

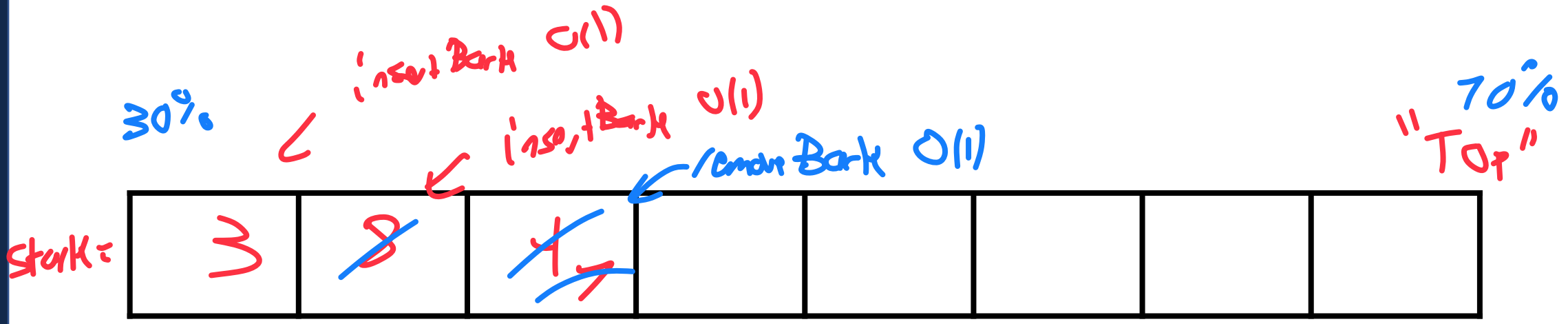
Stack Data Structure

C++ has a built-in stack

Underlying implementation is vector or deque

Is the 'top' the first or last item in an array?

```
1  #include <stack>
2  int main() {
3    stack<int> stack;
4    stack.push(3);
5    stack.push(8);
6    stack.push(4);
7    stack.pop();
8    stack.push(7);
9    stack.pop();
10   stack.pop();
11 }
```



Stack Data Structure

Push(X) is equivalent to …

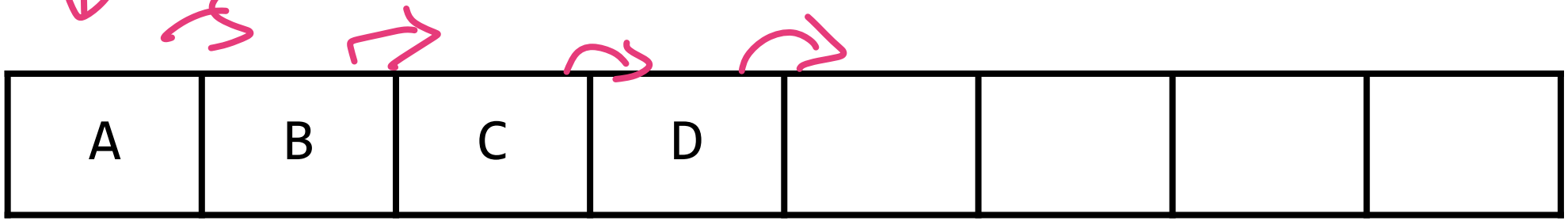
After class Q: why not "front as top"

add( )

X

Worst case would not best case!

O(n) to move to the spot



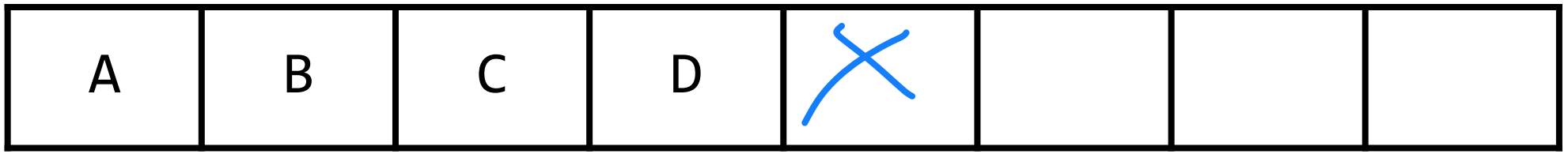
Stack Data Structure

`Push(X)` is equivalent to `insertBack(X)`

```
*size = X;
size++;
```

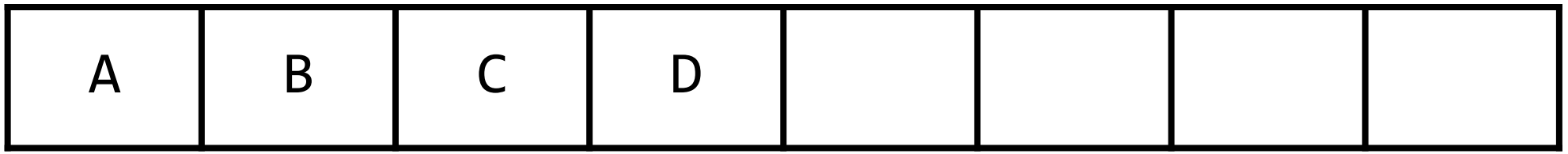
O(1)

size



Stack Data Structure

Pop( ) is equivalent to…



Stack Data Structure

`Pop()` is equivalent to `removeBack()`

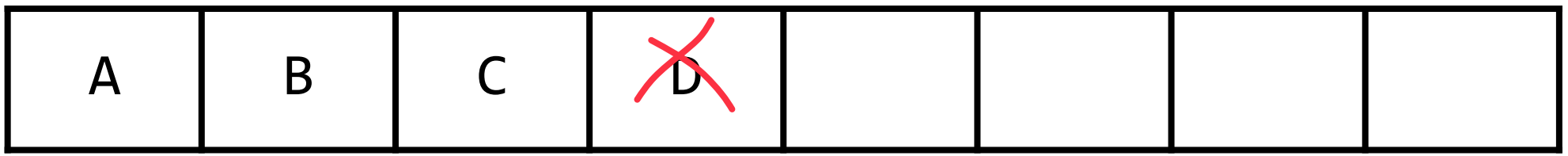
```
size--;
```

O(1)

```
remove(size);
```

O(1)

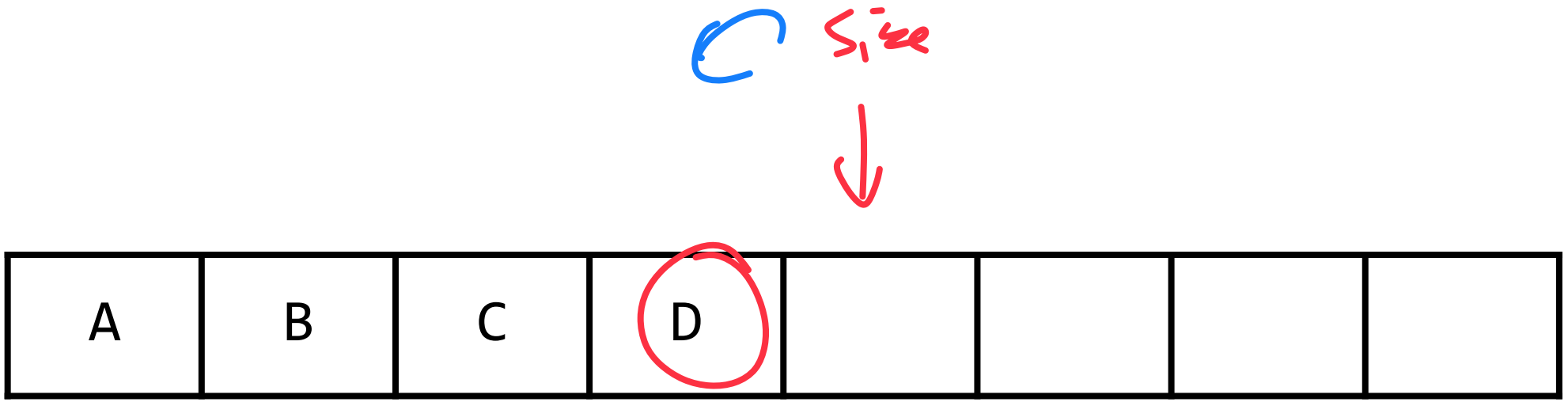
size



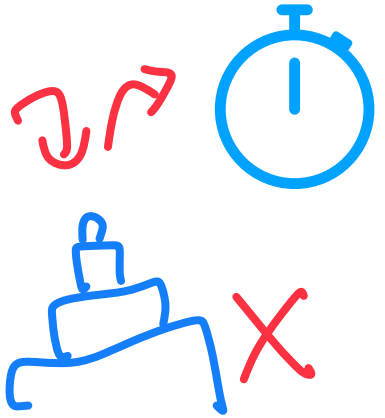
Stack Data Structure

Top() is tricky — remember size points to next available space!

```
return *(size - 1);
```



Stack ADT



- [Order]: last in first out (Lifo)
- [Implementation]: As array (top is last item)
  As linked list (first item)
- [Runtime]: O(1)* always for all ops

* array not full

Queue Data Structure

A **queue** stores an ordered collection of objects (like a list)

However you can only do three* operations:

Enqueue: Put an item at the back of the queue

Dequeue: Remove the front item of the queue

Front: Return the front item of the queue

```
enqueue(3); enqueue(5); dequeue(); enqueue(2)
```

Queue Data Structure

The queue is a **first in — first out** data structure (FIFO)

What data structure excels at removing from the front?

Can we make that same data structure good at inserting at the end?

Queue Data Structure

The C++ implementation of a queue is also a vector or deque — why?

Engineering vs Theory Efficiency

	Time x1 billion	Like
L1 cache reference	0.5 seconds	Heartbeat 💓
Branch mispredict	5 seconds	Yawn 😯
L2 cache reference	7 seconds	Long yawn 😯😯😯
Mutex lock/unlock	25 seconds	Make coffee ☕
Main memory reference	100 seconds	Brush teeth
Compress 1K bytes	50 minutes	TV show 📺
Send 2K bytes over 1 Gbps network	5.5 hours	(Brief) Night's sleep 🛌
SSD random read	1.7 days	Weekend
Read 1 MB sequentially from memory	2.9 days	Long weekend
Read 1 MB sequentially from SSD	11.6 days	2 weeks for delivery 📦
Disk seek	16.5 weeks	Semester
Read 1 MB sequentially from disk	7.8 months	Human gestation 🐣
Above two together	1 year	🌍☀️
Send packet CA->Netherlands->CA	4.8 years	Ph.D. 🎓

(Care of https://gist.github.com/hellerbarde/2843375)

Engineering vs Theory Efficiency

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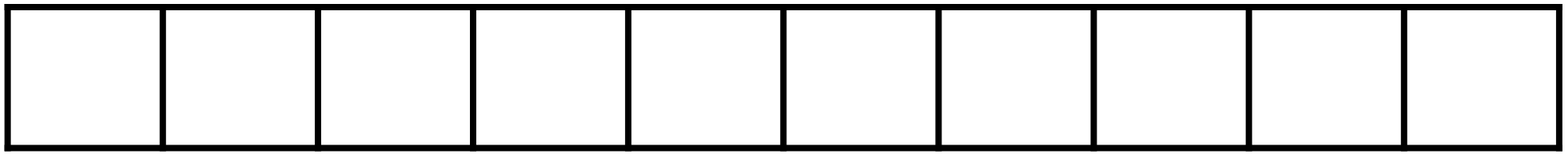
(Care of https://gist.github.com/hellerbarde/2843375)

Queue Data Structure

```
q.enqueue(8);
q.enqueue(4);
q.dequeue();
```

What do we need to track to maintain a queue with an array list?

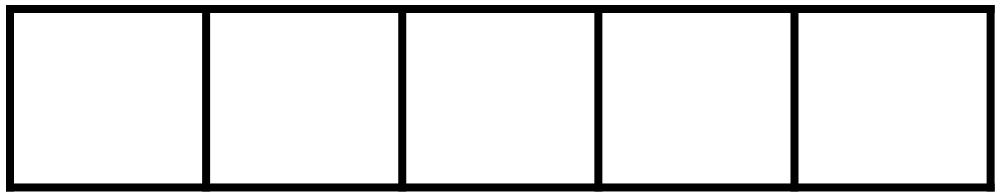


Queue Data Structure

Unlike the array list, it is easier to implement a Queue using unsigned ints

Queue.h

```
 1  #pragma once
 2
 3  template <typename T>
 4  class Queue {
 5    public:
 6      void enqueue(T e);
 7      T dequeue();
 8      bool isEmpty();
 9
10    private:
11      T *data_;
12      unsigned size_;
13      unsigned capacity_;
14      unsigned front_;
15  };
```



(Circular) Queue Data Structure

Queue.h

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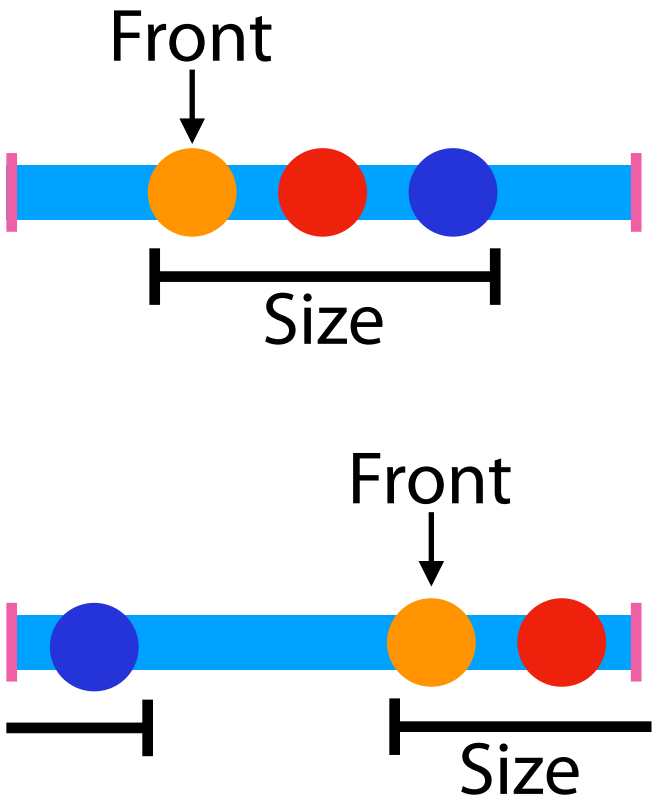
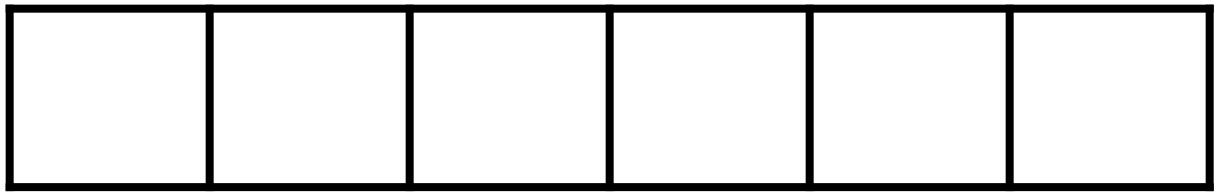
12     unsigned capacity_;
13     unsigned size_;
14     unsigned front_;
15 };
```





Enqueue(D):

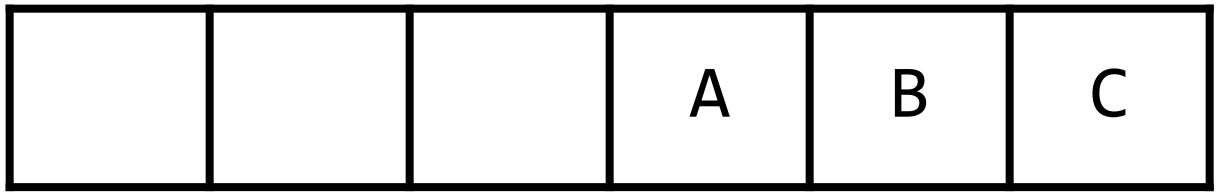
Dequeue():

Size:

Front:

Capacity:

```
Queue<int> q;
q.enqueue(3);
q.enqueue(8);
q.enqueue(4);
q.dequeue();
q.enqueue(7);
q.dequeue();
q.dequeue();
q.enqueue(2);
q.enqueue(1);
q.enqueue(3);
q.enqueue(5);
q.dequeue();
q.enqueue(9);
```




```
Queue<int> q;
...
q.enqueue(D);
q.dequeue();
q.dequeue();
q.dequeue();
q.dequeue();
q.enqueue(E);
```

Enqueue(D): Add data to ‘back’ of queue

Insert D at index **(size+front) % capacity**

size++ (as long as size != capacity)

Dequeue(): Remove data at index front

front = **(front+1) % capacity**

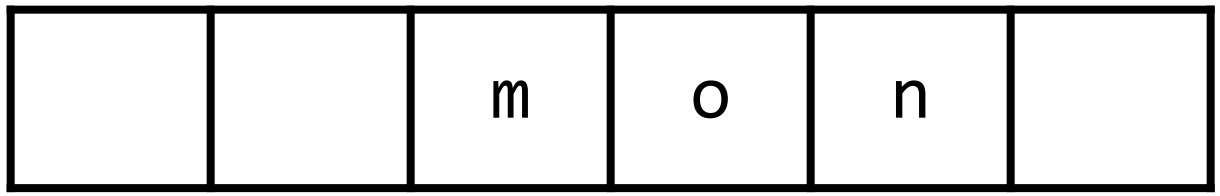
size-- (as long as size != 0)

Size: 3

Front: 3

Capacity: 6

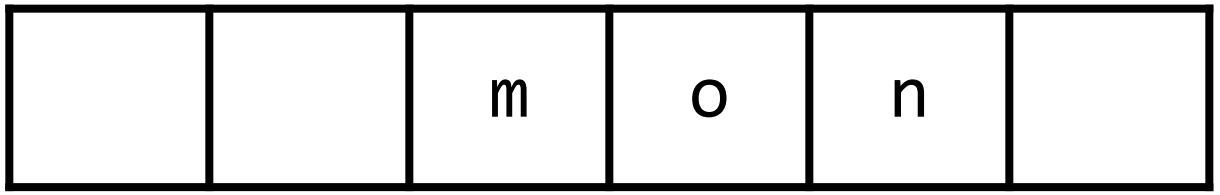
Queue Data Structure: Resizing

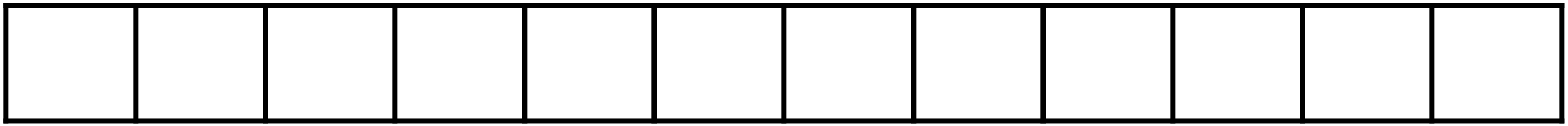



```
Queue<char> q;
…
q.enqueue(d);
q.enqueue(a);
q.enqueue(y);
q.enqueue(i);
q.enqueue(s);
```

Queue Data Structure: Resizing

```
Queue<char> q;
…
q.enqueue(d);
q.enqueue(a);
q.enqueue(y);
q.enqueue(i);
q.enqueue(s);
```





Queue ADT

- [Order]:
- [Implementation]:
- [Runtime]: