

Data Structures

Array Lists

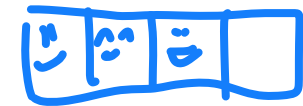
CS 225

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vs



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Learning Objectives

Discuss data variables for implementing array lists

Review array list implementations

Discuss amortized analysis

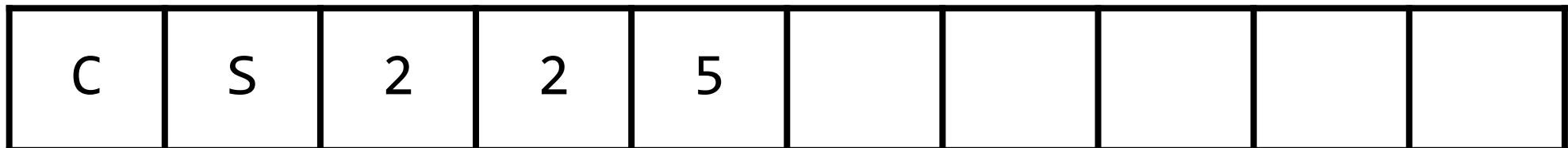
Consider extensions to lists

List Implementations

1. Linked List



2. Array List



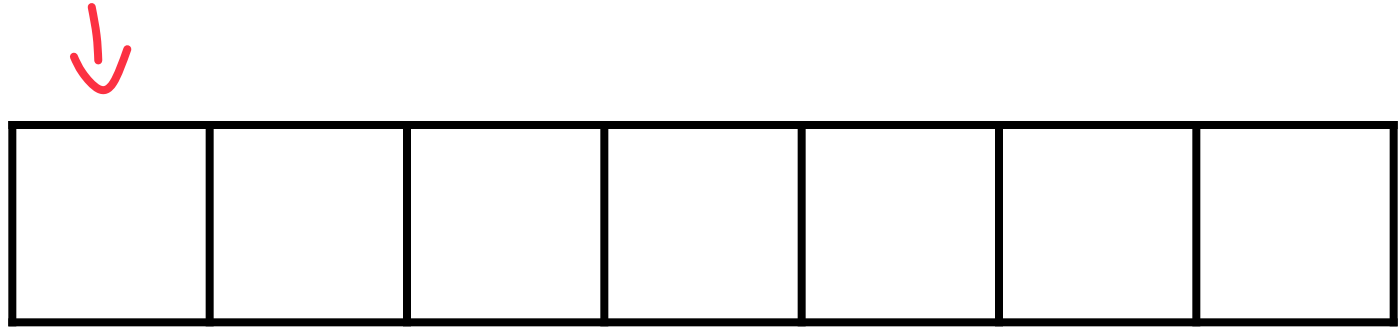
Linked List Runtimes



Random Access

	@Front	@RefPointer	@Index <i>Some arbitrary index</i>
Insert	$O(1)$	$O(1)$	$O(n)$
Delete	$O(1)$	$O(1)$	$O(n)$

Array List

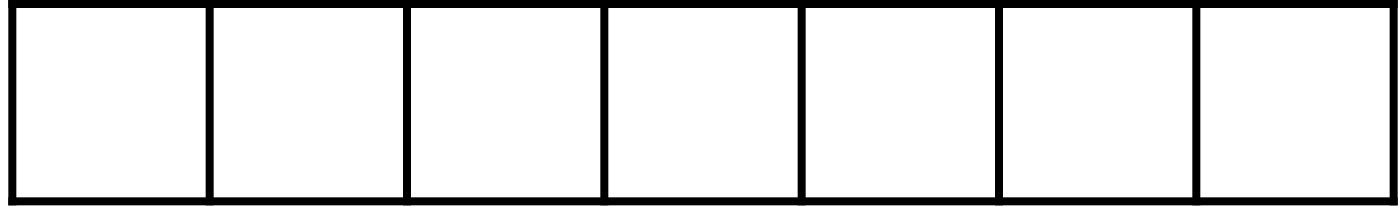


An array is allocated as continuous memory.

^{x 4}
Three values are necessary for efficient array usage:

- 1) Start location of array
- 2) # of items in array (currently) (size)
- 3) End of my allocated block (capacity / size limit)
- 4) Size of item in array $\rightarrow$ homogeneous (Data type)

Array List

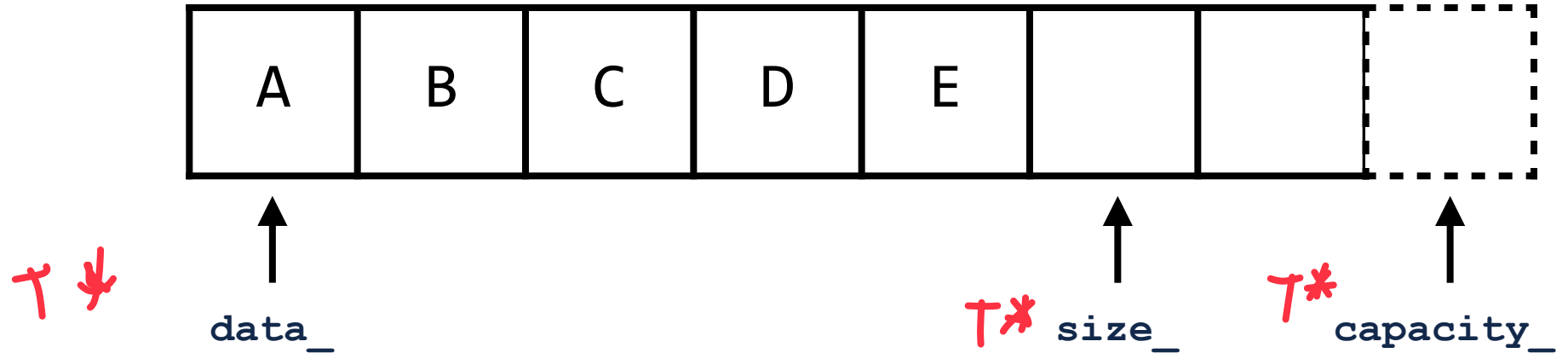


An array is allocated as continuous memory.

Three values are necessary for efficient array usage:

- 1) The start location of the array
- 2) The number of items currently stored in the array
- 3) The maximum capacity of the array

Array List



In C++, vector is implemented as:

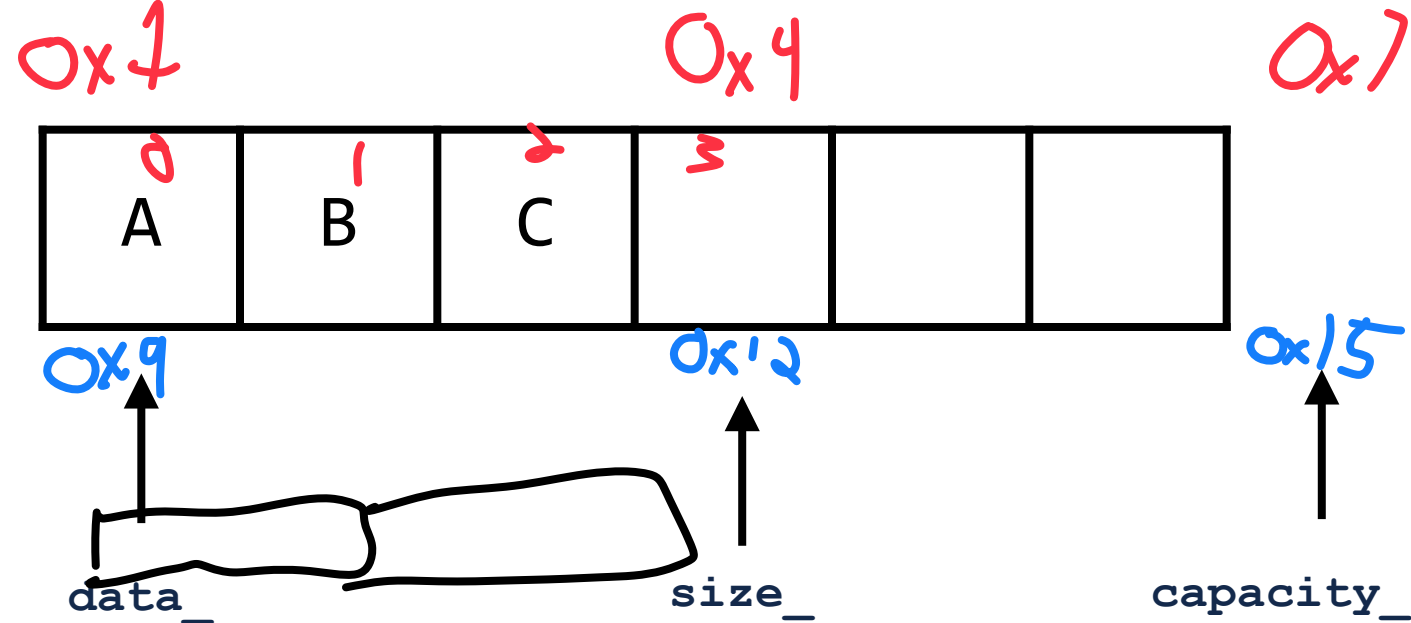
- 1) **Data:** Stored as a pointer to array start
- 2) **Size:** Stored as a pointer to the next available space
- 3) **Capacity:** Stored as a pointer past the end of the array

List.h

```

1  #pragma once
2
3  template <typename T>
4  class List {
5  public:
6      /* --- */
7  ...
8  private:
9      T *data_;
10
11     T *size_;
12
13     T *capacity_;
14
15     /* --- */
16 };

```



$$0x13 - 0x9 = 4 / \text{size of } (T)$$

If I want to know the number of items in the array:

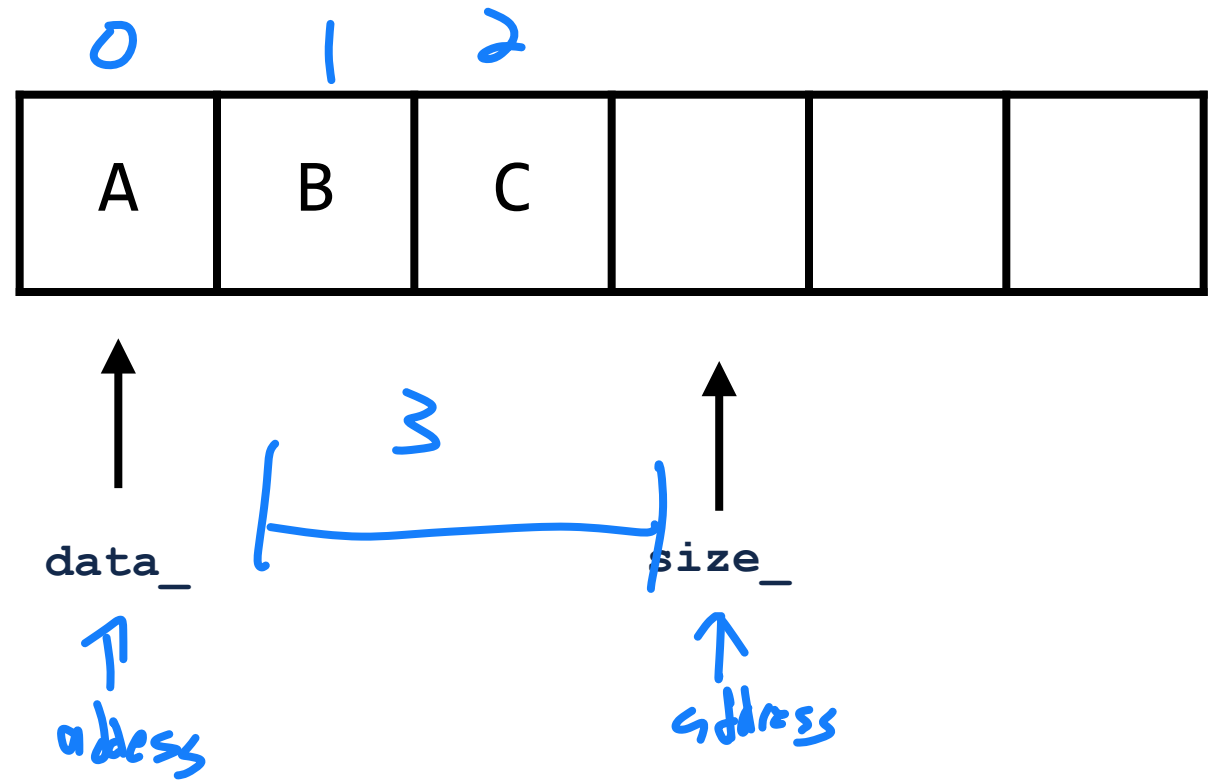
↳ Size of (T) is needed?

(++ pointer math does this for you!

Size_ - data_

List.h

```
1  #pragma once
2
3  template <typename T>
4  class List {
5  public:
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7  private:
8      T *data_;
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14     /* --- */
15 };
```



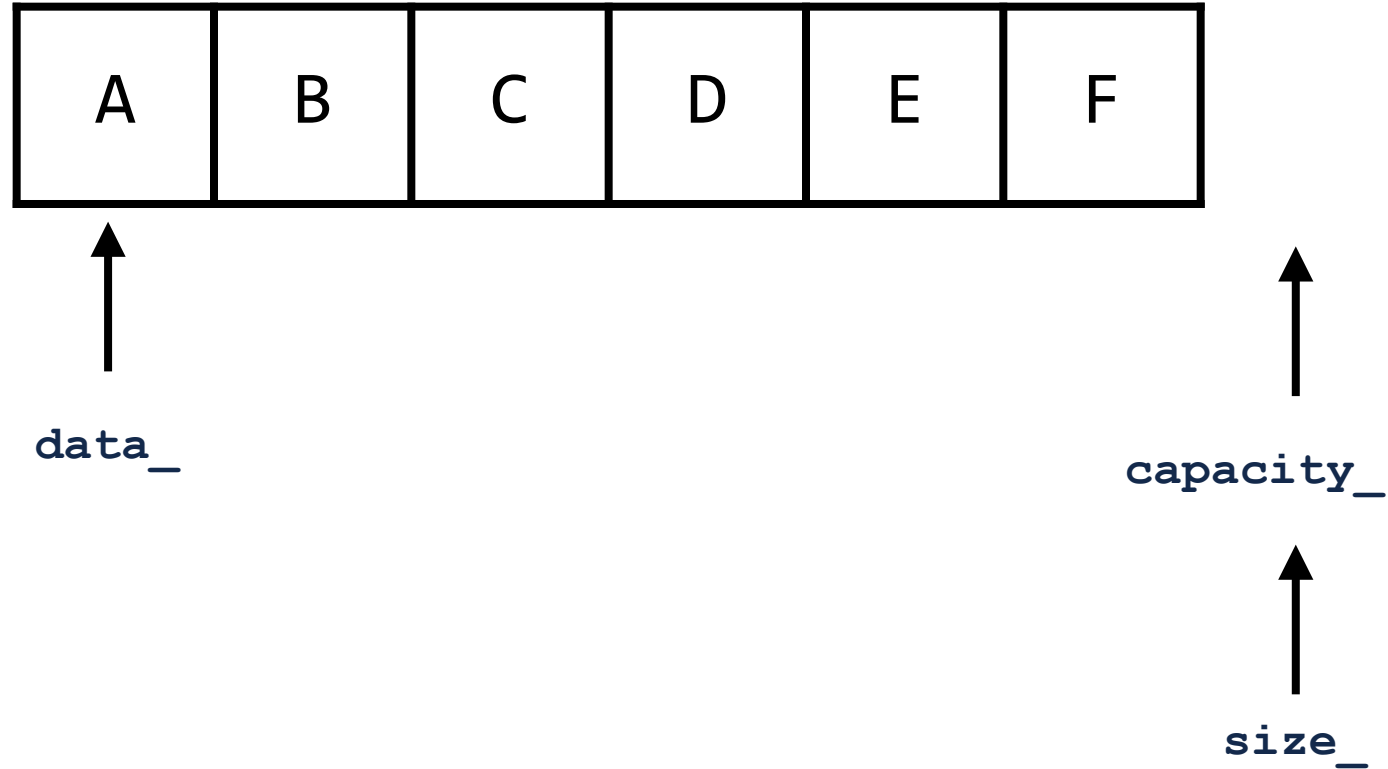
If I want to know the number of items in the array: **size_ - data_**

↳ why P++ (p is type T*) works!

$P = P + 1$ — size of(T)

List.h

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```

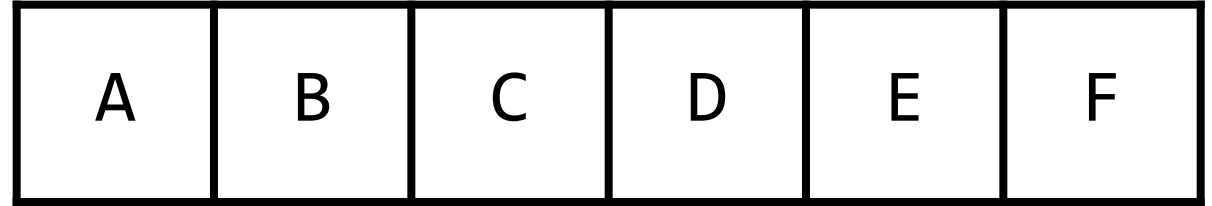


How do I know if I'm at capacity? $(capacity_ - size_ = 0)$

$< 0 \text{ capacity} = size_$

List.h

```
1  #pragma once
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13     T *capacity_;
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15     ...
16     /* --- */
17 };
```



↑
capacity_
↑
size_

How do I know if I'm at capacity?

size_ == capacity_

Array List: []

$L[i]$

$i = 2$

$L[i] = 10$
 $x = L[i]$

C	S	2	2	5					
---	---	---	---	---	--	--	--	--	--

↳ data_ → data_ → data_ →
↳ size_ → away
↳ capacity_

↑

✓

return

data_ + i

return type is

↗

| :f

return type T or T*

↳

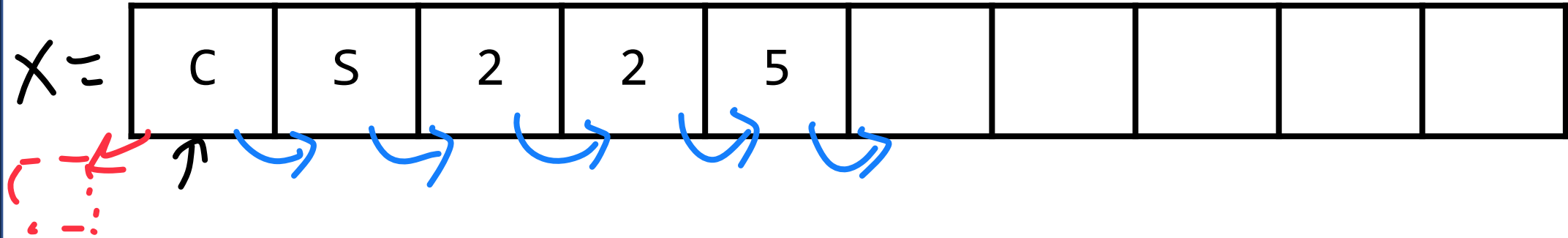
$*(data_ + i)$

Random access!

$O(1)$

T*

Array List: insertFront(data)



(cont odd)
↓ 12.3

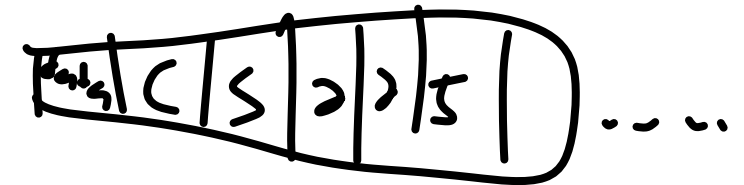
1

c: this!

1) Move everything over by one address $O(n)$

$O(n)$
overall

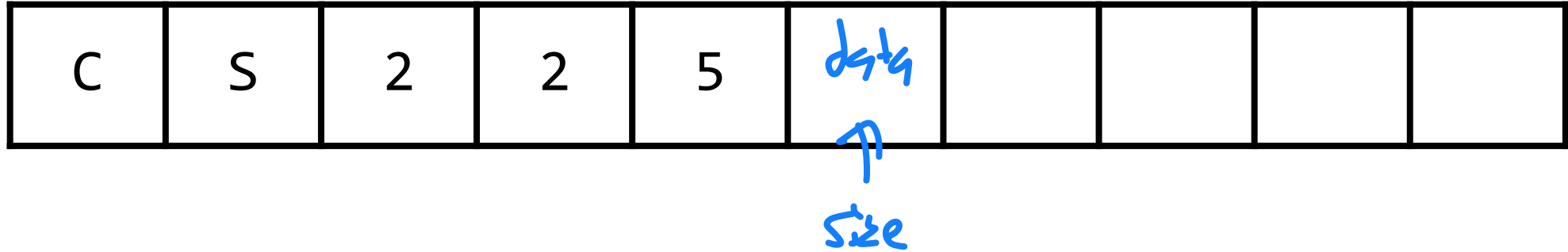
2) Add data to index 0 $O(1)$



Why not claim mem address data - 1?

↳ We don't own and cannot request memory before or after array

Array List: insertBack(data)



- 1) Check if $size == capacity \rightarrow$ resize Then insert
- 2) size points to next available space!
↳ $*size = data;$
- 3) $size++;$

Array List: insert(data, index)

C	S	2	2	5					
---	---	---	---	---	--	--	--	--	--

↑
insert front
 $O(n)$

↑
insert Back
 $O(1)$ * not at capacity

↑
insert(data, 0)

$O(n)$

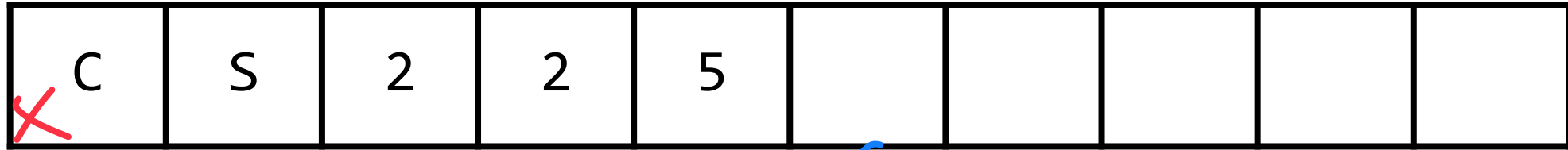
- 1) Shift all values at & after index
- 2) Insert data @ index
- 3) size ++;

Array List: Not at capacity

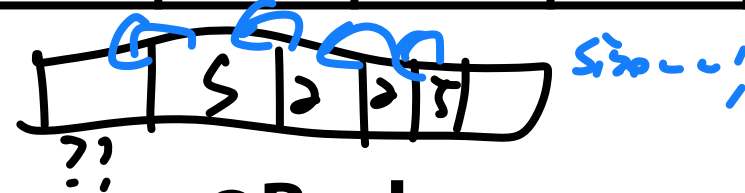
Join Code: 225



data it to front of allocated memory!



Memory
leak!



@Front

@Back

@Index

Insert

$O(n)$

$O(1)^*$

$O(n)$

Delete



$O(n)$

Array List: addspace(data)

N	O	S	P	A	C	E
---	---	---	---	---	---	---

Size == capacity

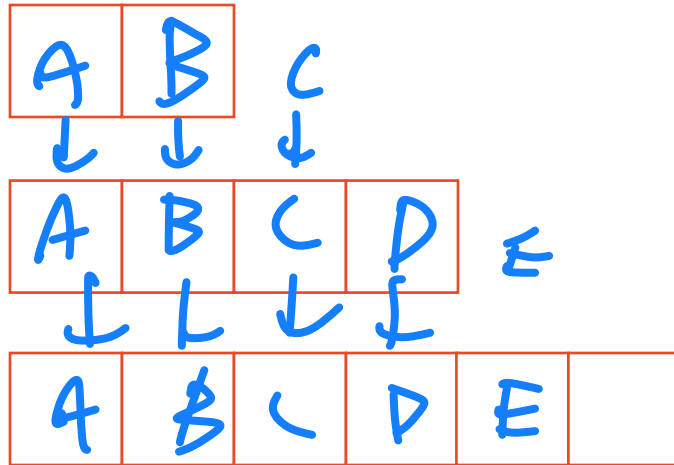
We cannot assume ownership of next mem address!

↳ we must allocate new list

↳ and copy items!

N	O	S	P	A	C	E	??
---	---	---	---	---	---	---	----

Resize Strategy: +2 elements every time



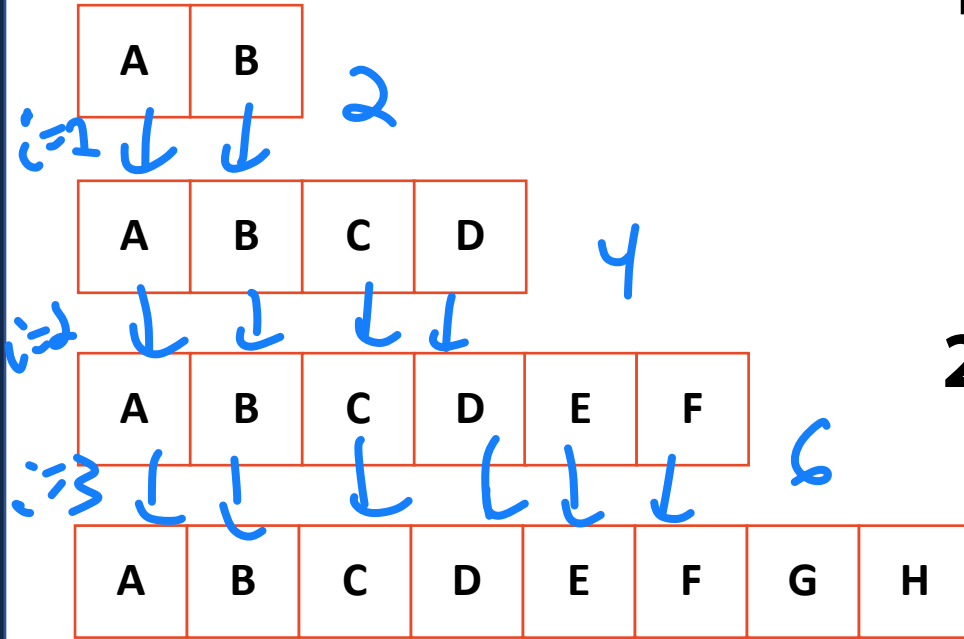
Resize Strategy: +2 elements every time

1) How many copy calls per reallocation?

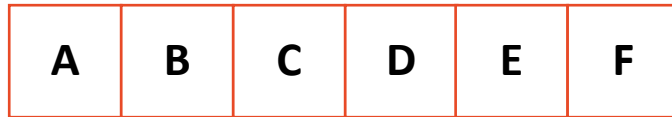
for all i , 2^i copies

2) Total reallocations for N objects?

$N/2$ times to add total of N capacity



Resize Strategy: +2 elements every time



Total number of copy calls:

1) How many copy calls per reallocation?

For reallocation i , $2i$ copy calls are made

2) Total reallocations for N objects?

Let k be the number of reallocs, $k = \frac{N}{2}$

$$\sum_{i=1}^k 2i = k(k+1) = k^2 + k$$

Resize Strategy: +2 elements every time



1) How many copy calls per reallocation?

For reallocation i , $2i$ copy calls are made

2) Total reallocations for N objects?

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Total number of copy calls:

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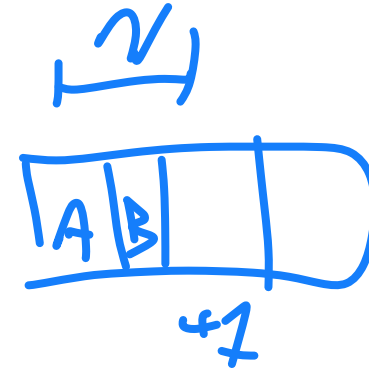
... For N objects: $\frac{N^2 + 2N}{4}$

Total # of operations done to add N items

Resize Strategy: +2 elements every time



Total copies for N inserts: $\frac{N^2 + 2N}{4}$



This is Bad!

Amortized:

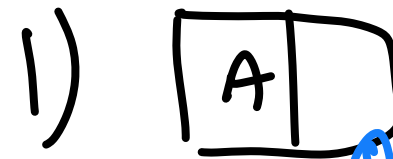
Look at $\frac{\text{total runtime}}{\text{\# of inserts}} \approx N$

"Average performance over N inserts"

$$\frac{N^2 + 2N}{4} / N \approx N \text{ ops per insert}$$

Big O: $O(n)$

2 types of insertions



Open position at end of array
 $O(1)$ add to opening



Reallocation insert
 $O(n)$ has to copy all prev values

Resize Strategy: x2 elements every time



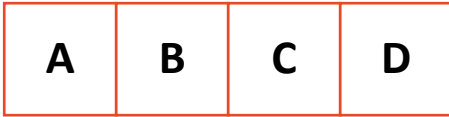
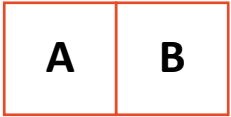
Consider: why might this be better?



Resize Strategy: x2 elements every time

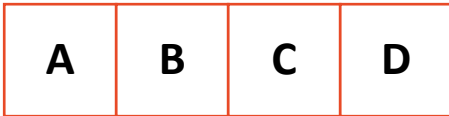


1) How many copy calls per reallocation?



2) Total reallocations for N objects?

Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

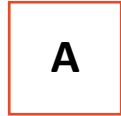
For reallocation i , 2^i copy calls are made

2) Total reallocations for N objects?

$k = \text{final realloc needed} = \lceil \log_2 N \rceil$

Total number of copy calls:

Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

For reallocation i , 2^i copy calls are made

2) Total reallocations for N objects?

$k = \text{final realloc needed} = \lceil \log_2 N \rceil$

Total number of copy calls:

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

... For N objects: $2N - 1$

Resize Strategy: x2 elements every time

Total copies for n inserts: $2N - 1$

Amortized:

Big O:

Resize Strategy: x2 elements every time

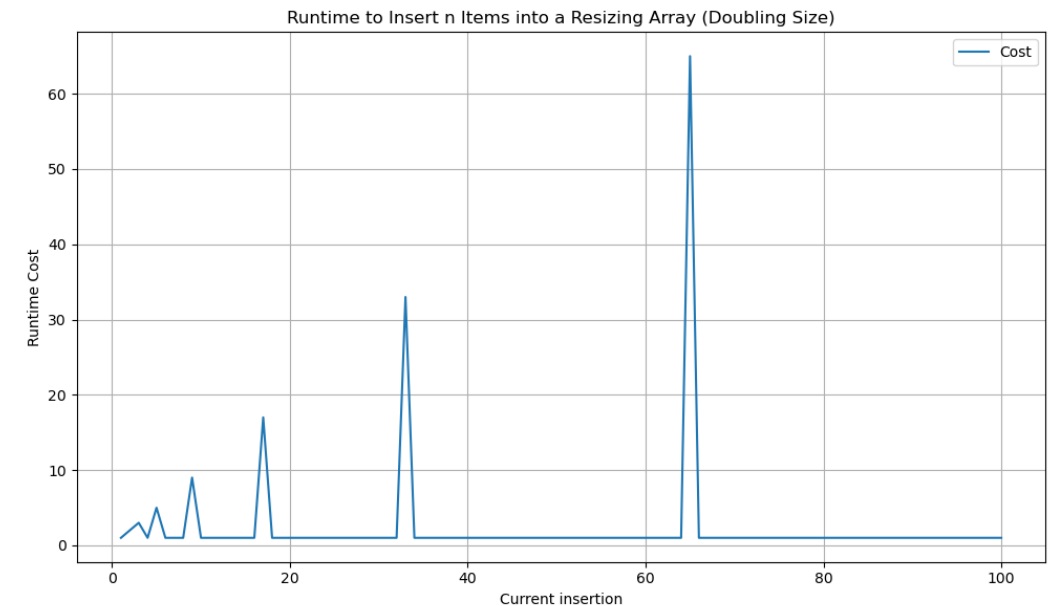
Total copies for n inserts: $2N - 1$

Amortized:

Precise total work over N calls

Big O:

Upperbound on worst case



Distinguishing amortized vs Big O

- 1) Every insert operation has a Big O of $O(N)$
- 2) By totaling up the total work over N inserts, we see a different story:
 - The 'bad' insert steps happen roughly once every n inserts

Ex: $n = 2, 4, 8, 16, 32, \dots$

- The amount of work in the bad insert also grows linearly
- All other inserts are 'good' inserts each taking $O(1)$ time

When viewed in the lens of 'total work over N inserts' we get $O(1)$ *

List Implementation



	Singly Linked List	Array
Look up arbitrary location		
Insert after given element		
Remove after given element		
Insert at arbitrary location		
Remove at arbitrary location		
Search for an input value		

Thinking critically about lists: tradeoffs

The implementations shown are foundational (simple).

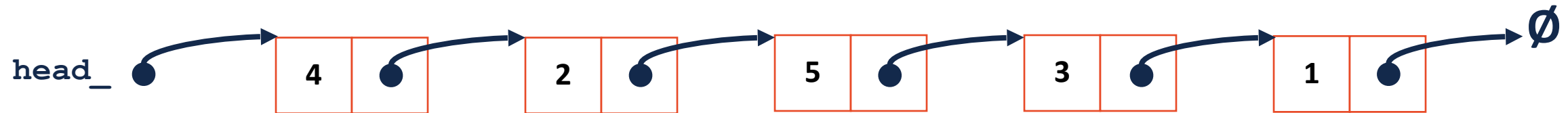
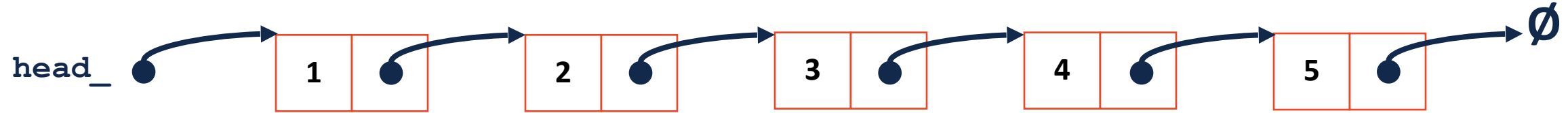
Can we make our lists better at some things? What is the cost?

Thinking critically about lists: tradeoffs

Getting the size of a linked list has a Big O of:



Thinking critically about lists: tradeoffs

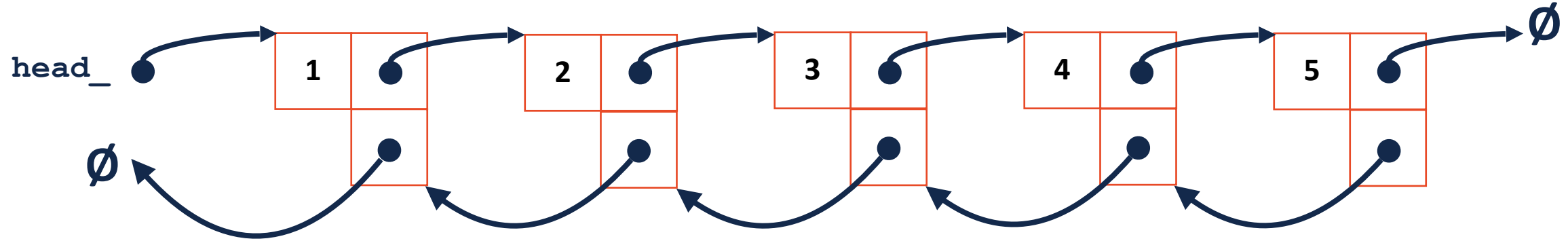


Thinking critically about lists: tradeoffs

2	7	5	9	7	14	1	0	8	3
---	---	---	---	---	----	---	---	---	---

0	1	2	3	5	7	7	8	9	14
---	---	---	---	---	---	---	---	---	----

Thinking critically about lists: tradeoffs



Thinking critically about lists: tradeoffs

When we discuss data structures, consider how they can be modified or improved!

Next time: Can we make a 'list' that is $O(1)$ to insert and remove? What is our tradeoff in doing so?