Heaps Analysis

CS 225 - Fall 2025

Mattox Beckman

Objectives

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Demonstrate the time complexity of insert and delete

Implement the heapify function

Implement heap sort

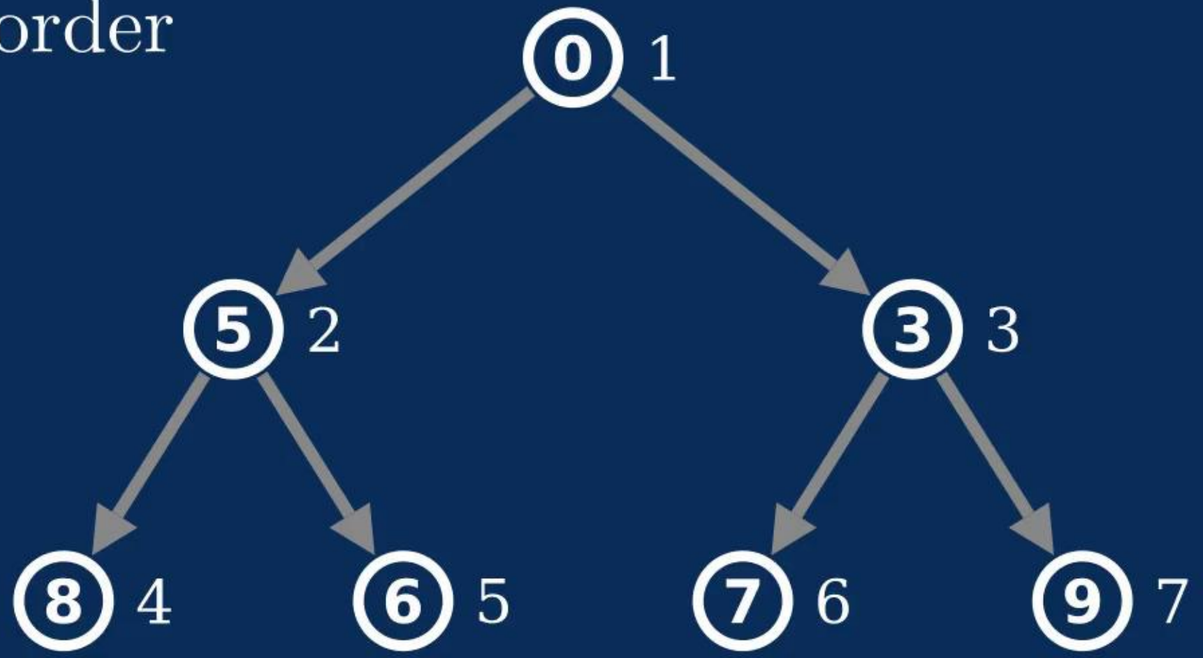
Nodes are placed in complete tree order

- Filled level by level
- Filled from left to right

Node priority higher than children

Nodes are numbered

We use an array for storage



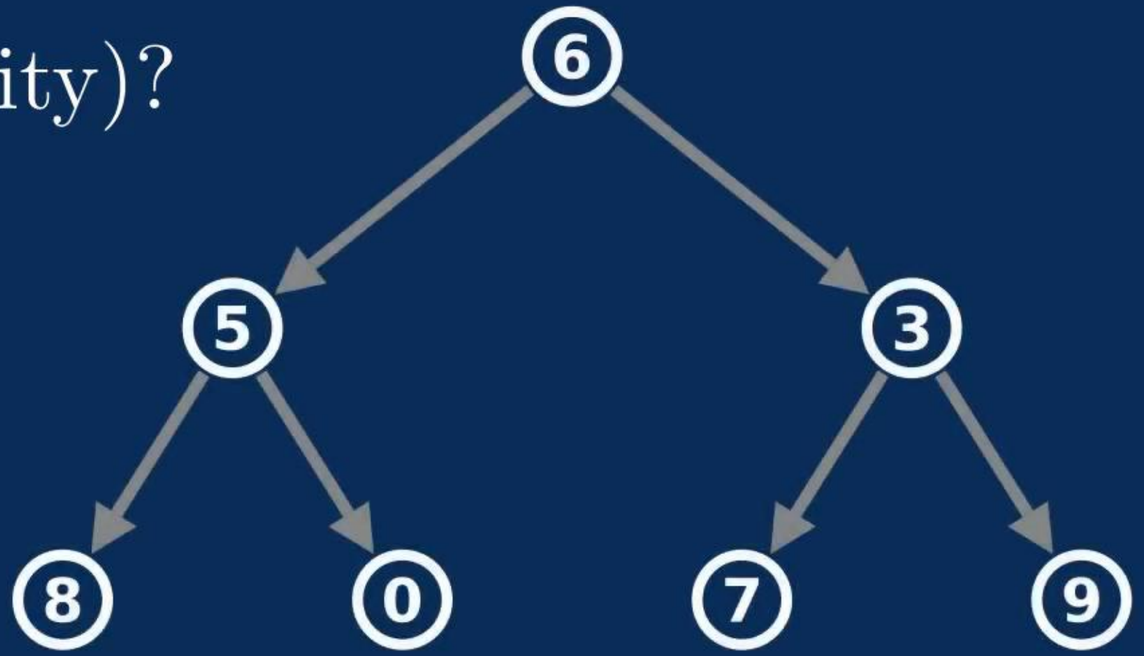
0	1	2	3	4	5	6	7
S	0	5	3	8	6	7	9

Percolate Up

Node n and parent p

Is $n < p$ (higher priority)?

- swap(n,p)
- recurse up.



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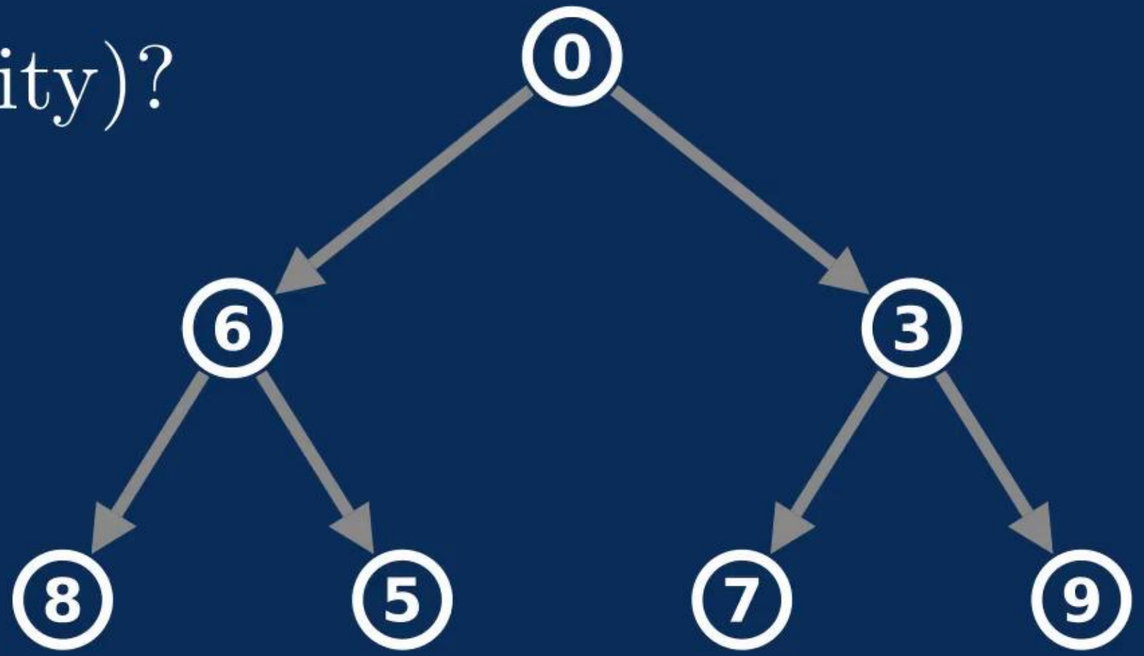
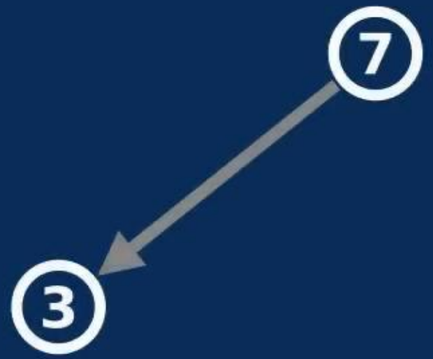


Percolate Down

If a child is smaller, swap with smallest child



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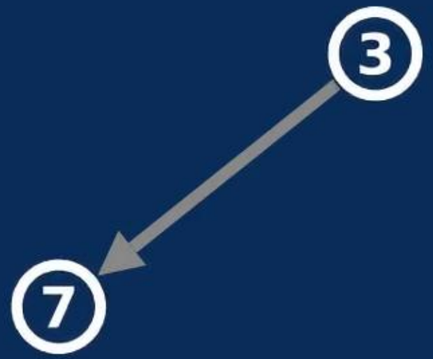

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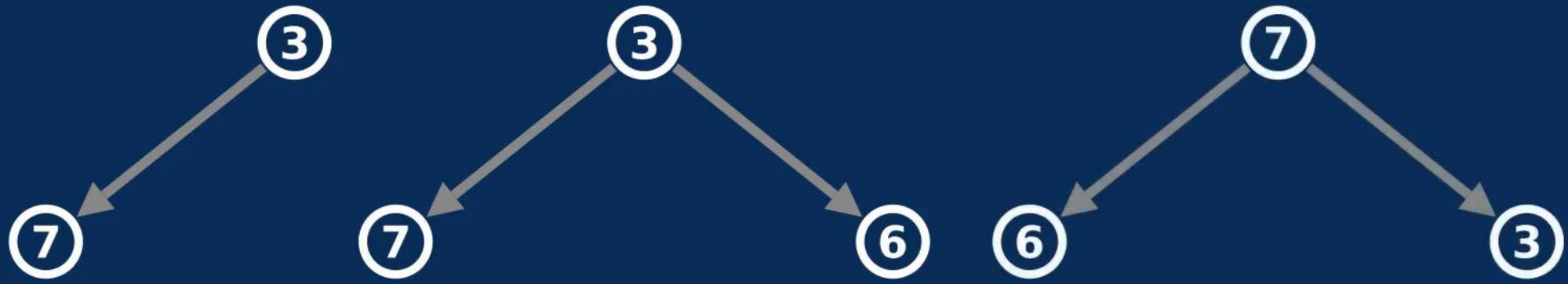

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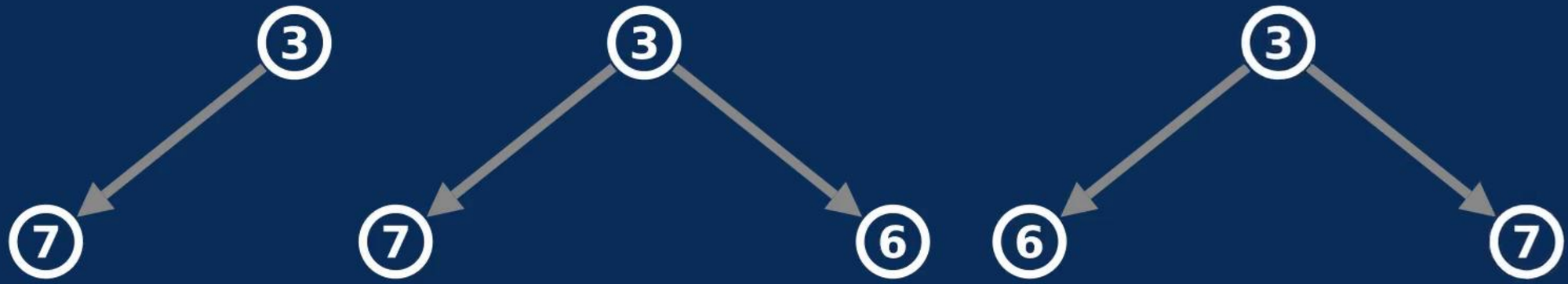

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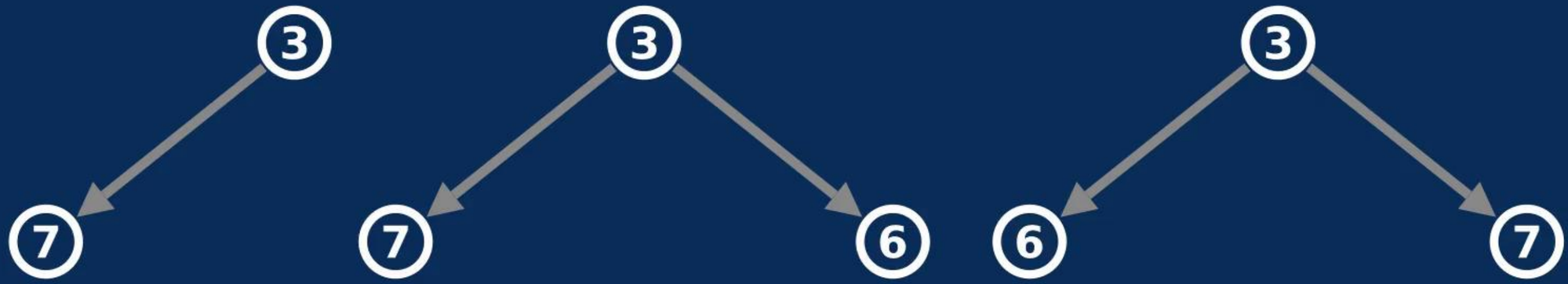
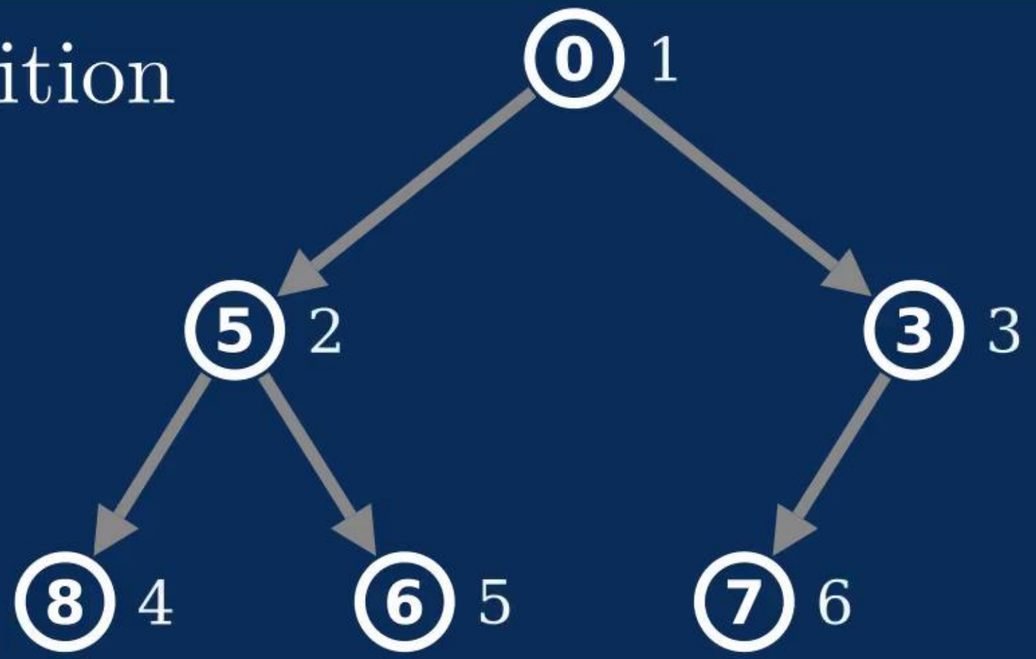



```
 1 while (true) {
 2    int left = idx * 2; int right = idx * 2 + 1; int smallest = idx;
 3    if (left < heap.size() &&
 4        heap[left] < heap[idx]) smallest = left;
 5    if (right < heap.size() &&
 6        heap[right] < heap[smallest]) smallest = right;
 7    if (smallest == idx) break;
 8    swap(heap[idx],heap[smallet]);
 9    idx = smallest;
10 }}
```

Insert

Place new element in last position

Call `percolate up` on it.



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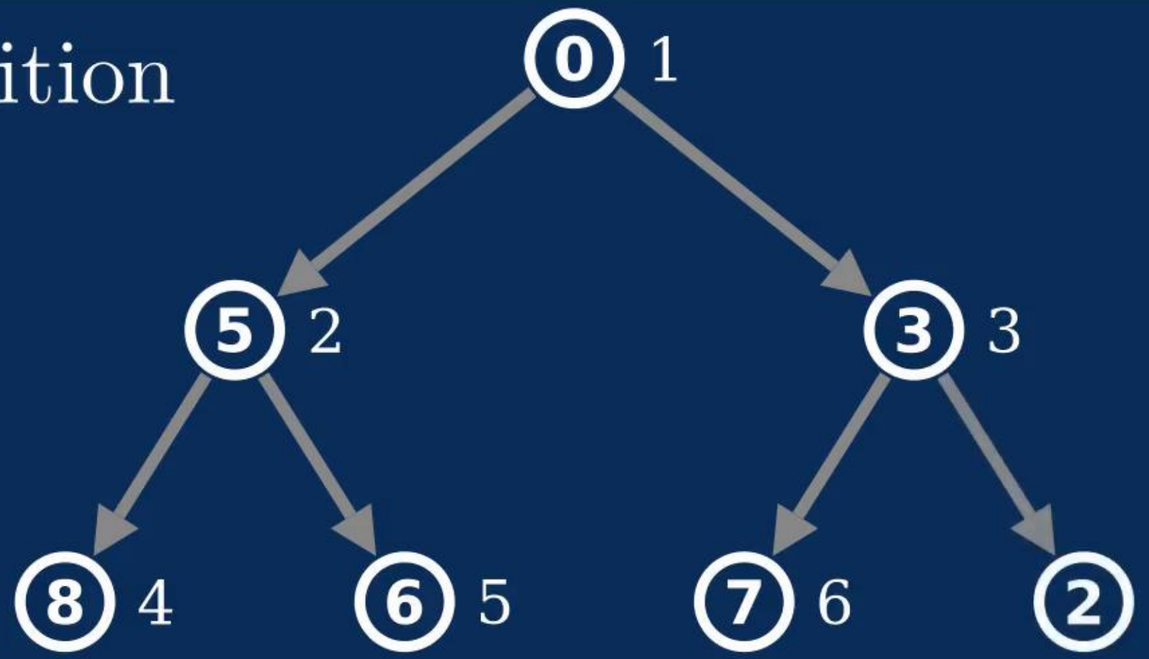

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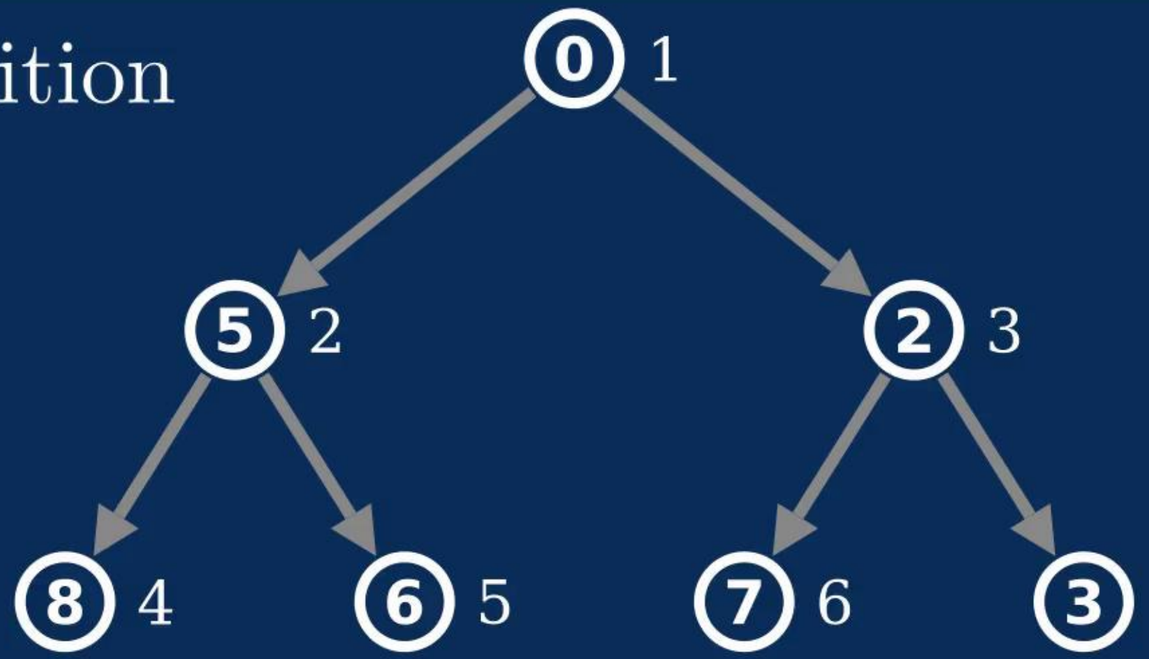
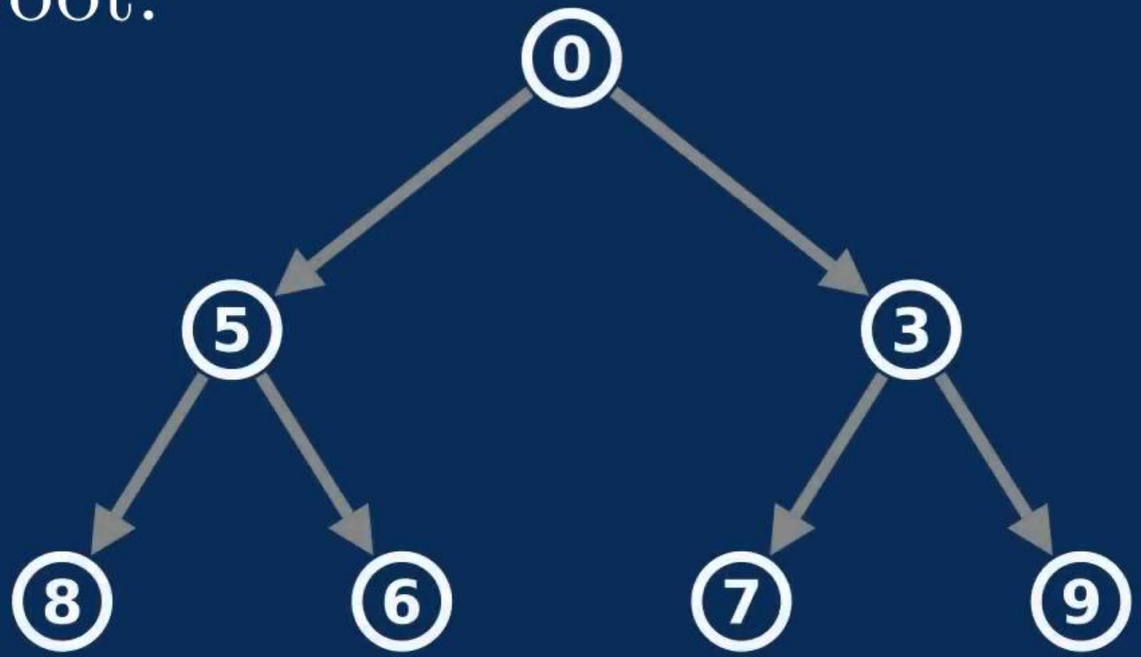


Delete Min

Overwrite last element with root.

Call `percolate down` on it.



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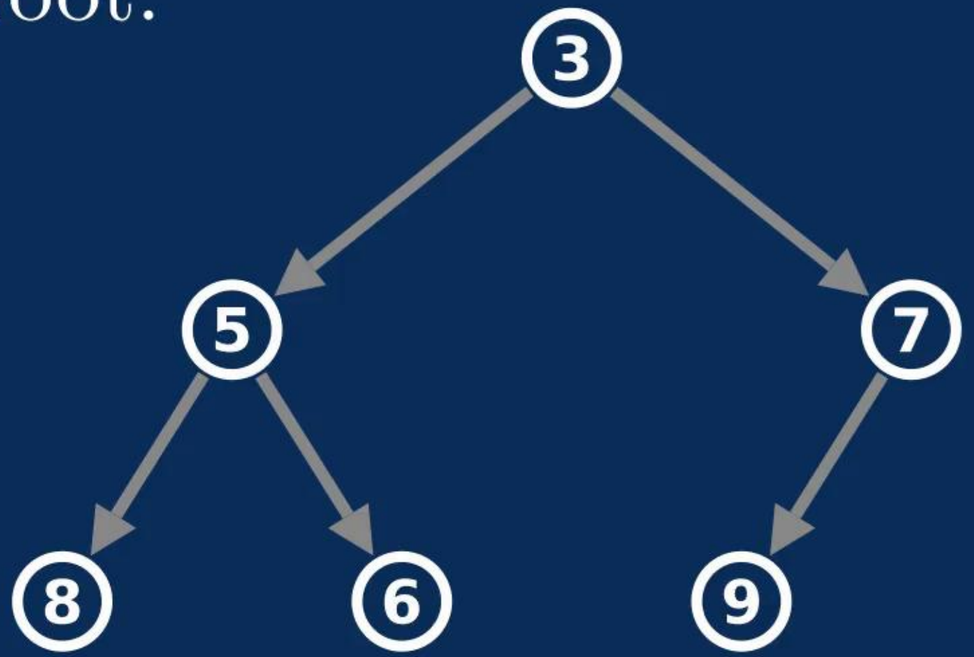
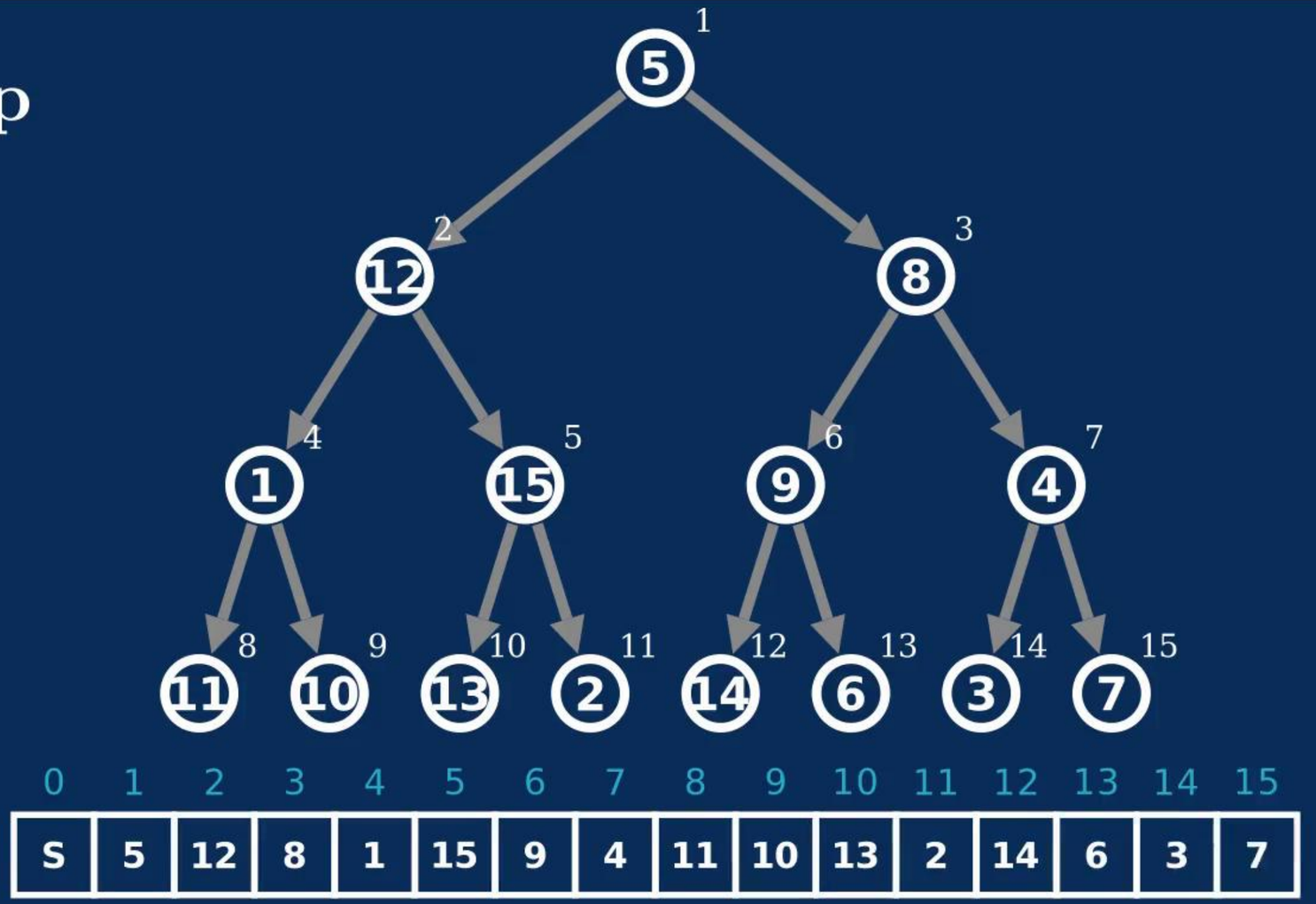


Heapify

Creating a Heap

How to start?



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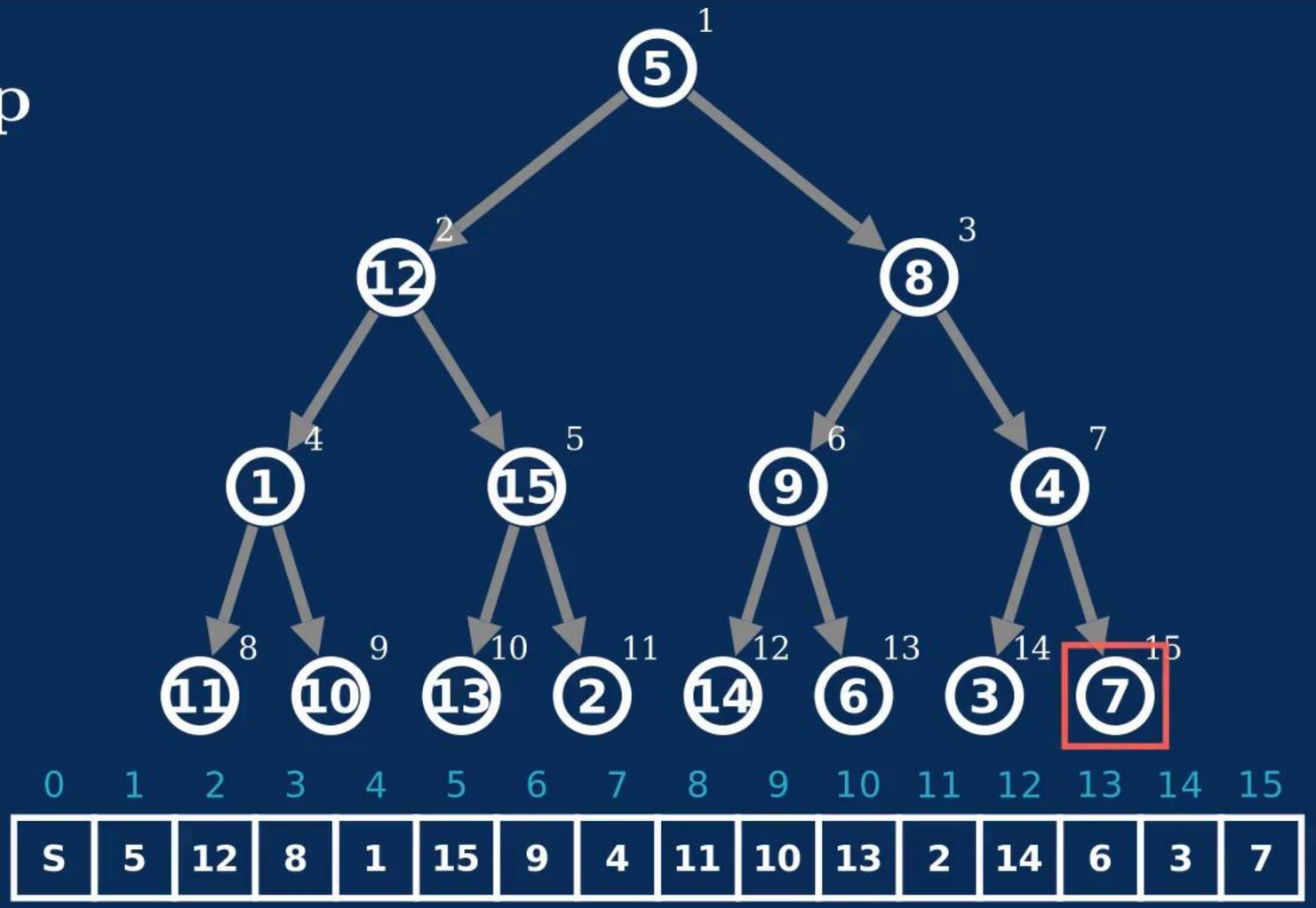

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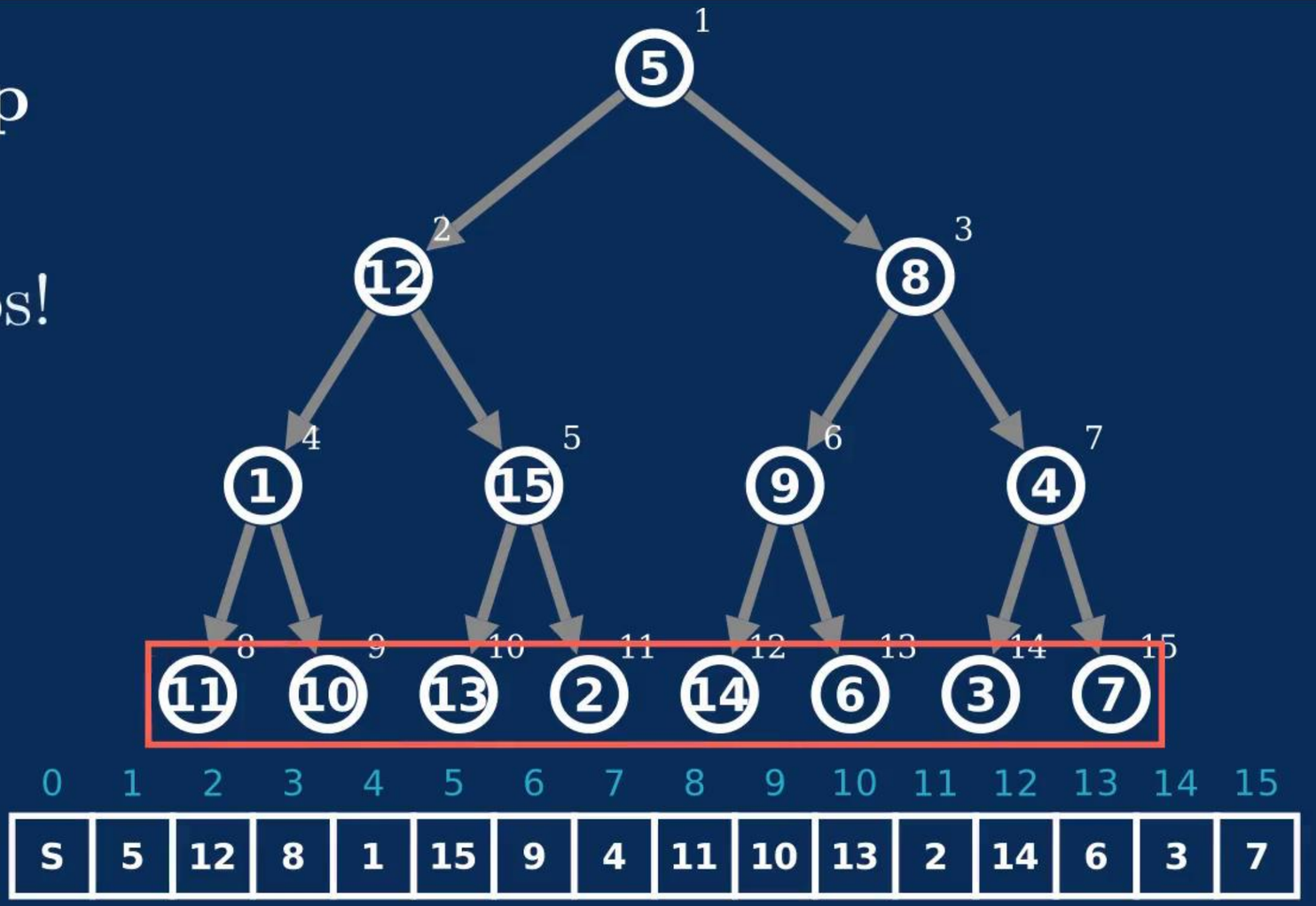

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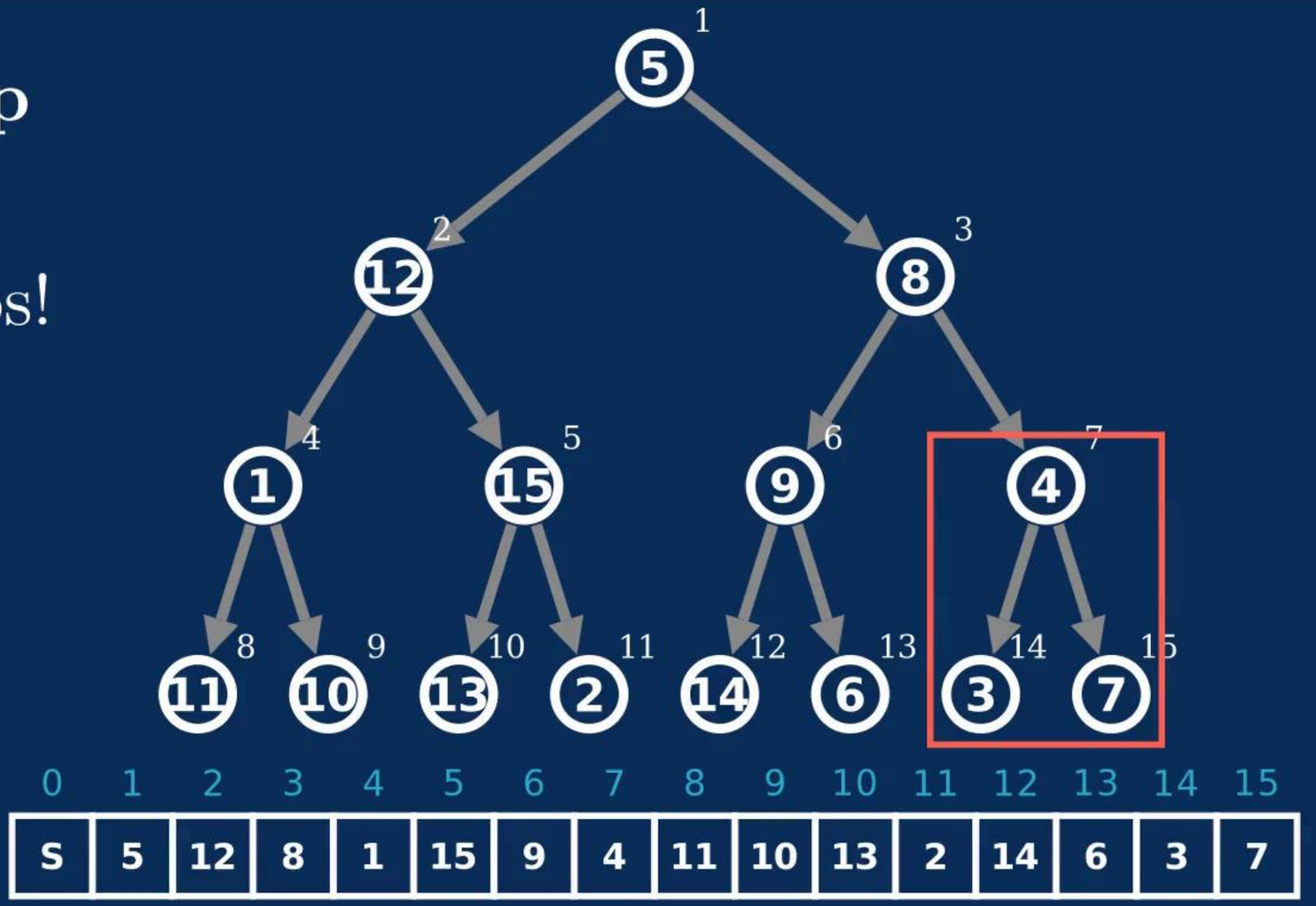

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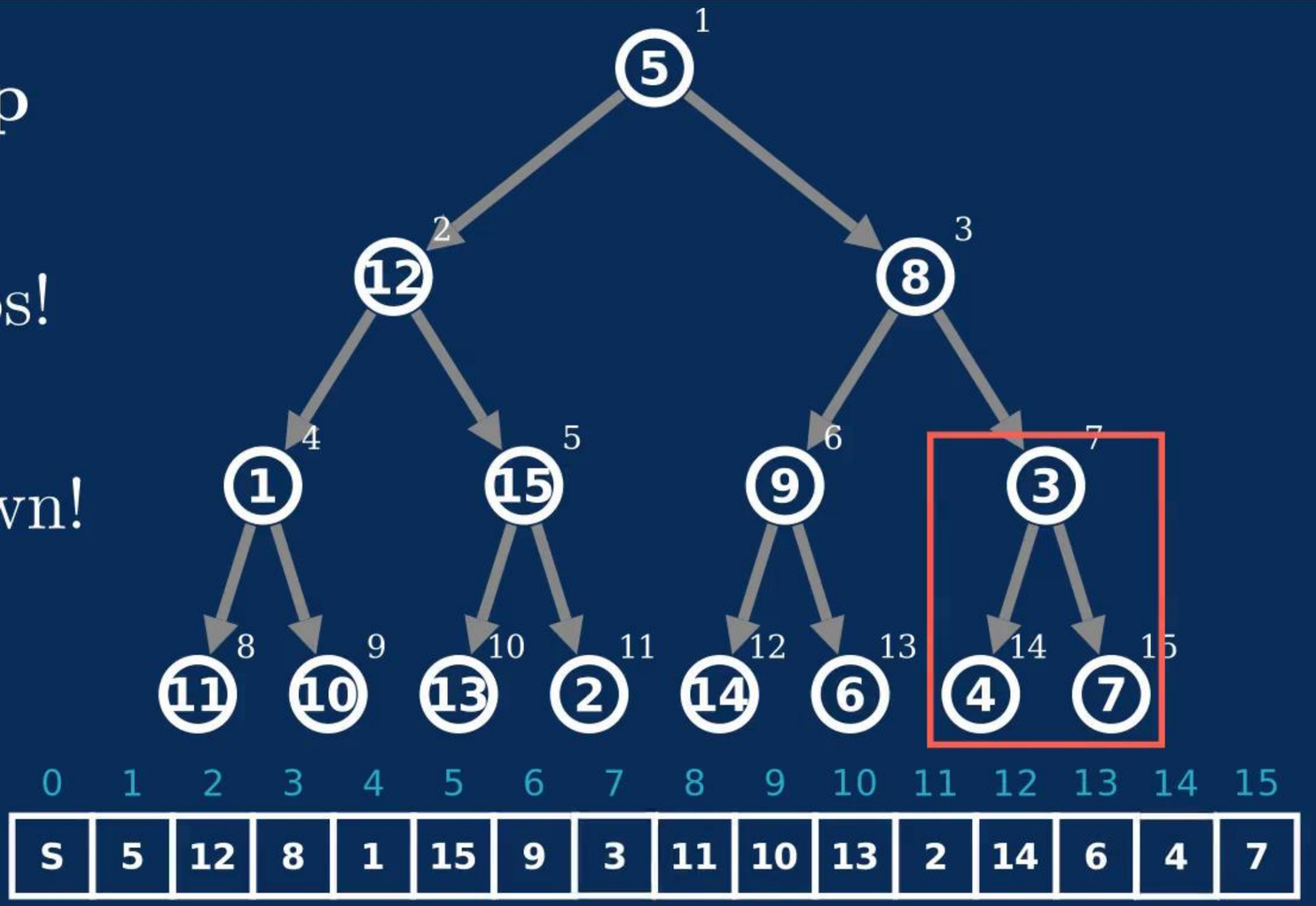

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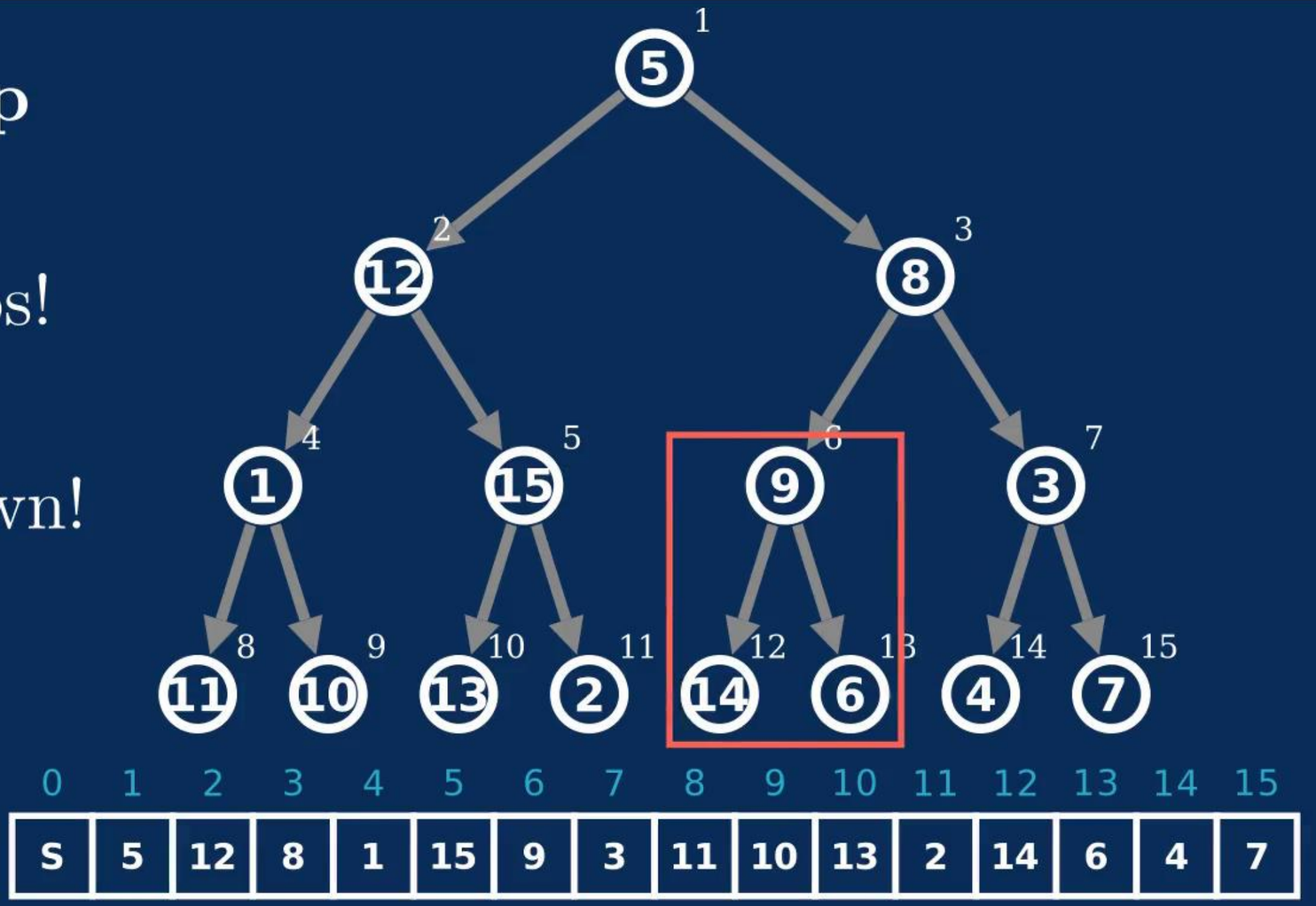

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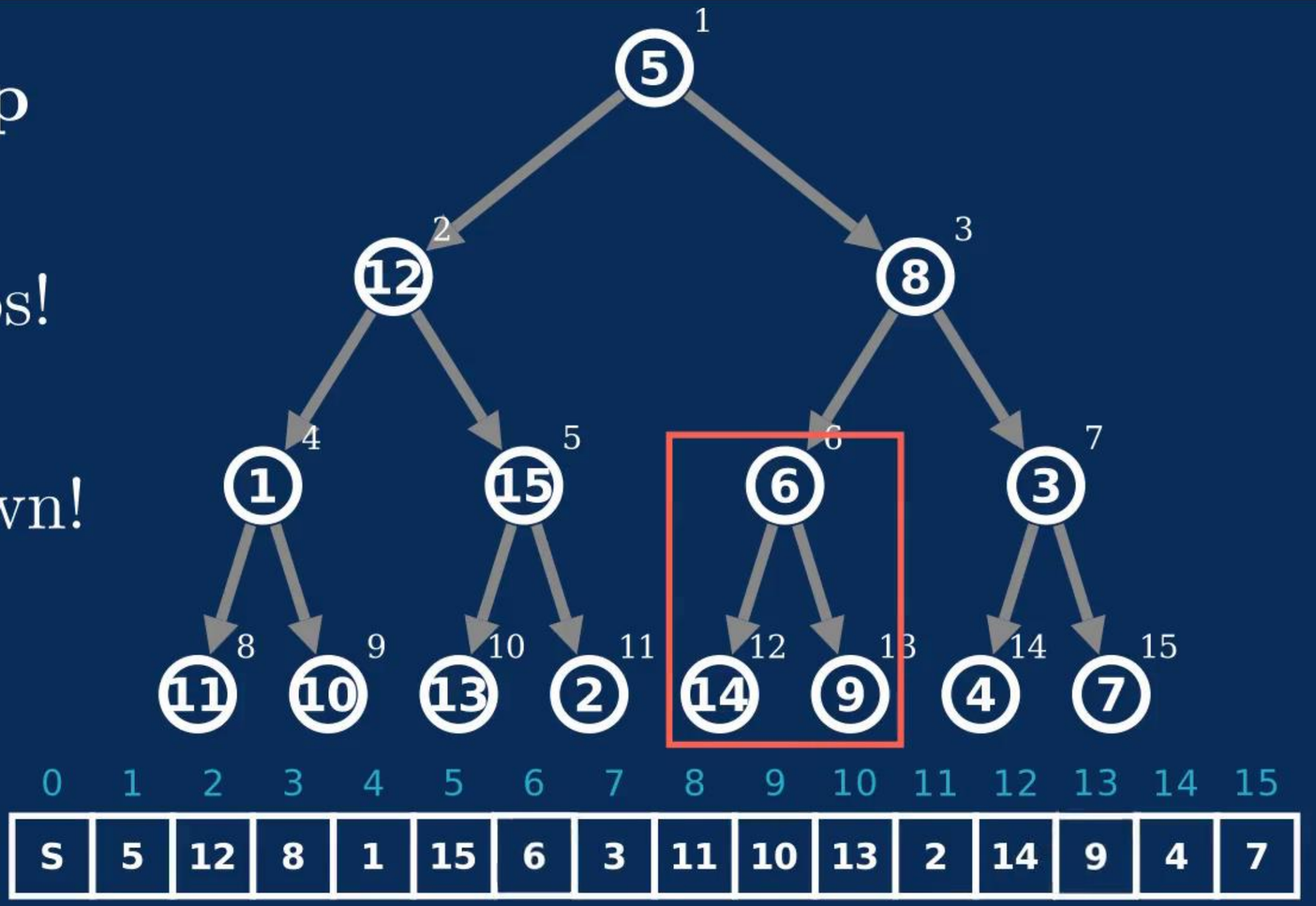

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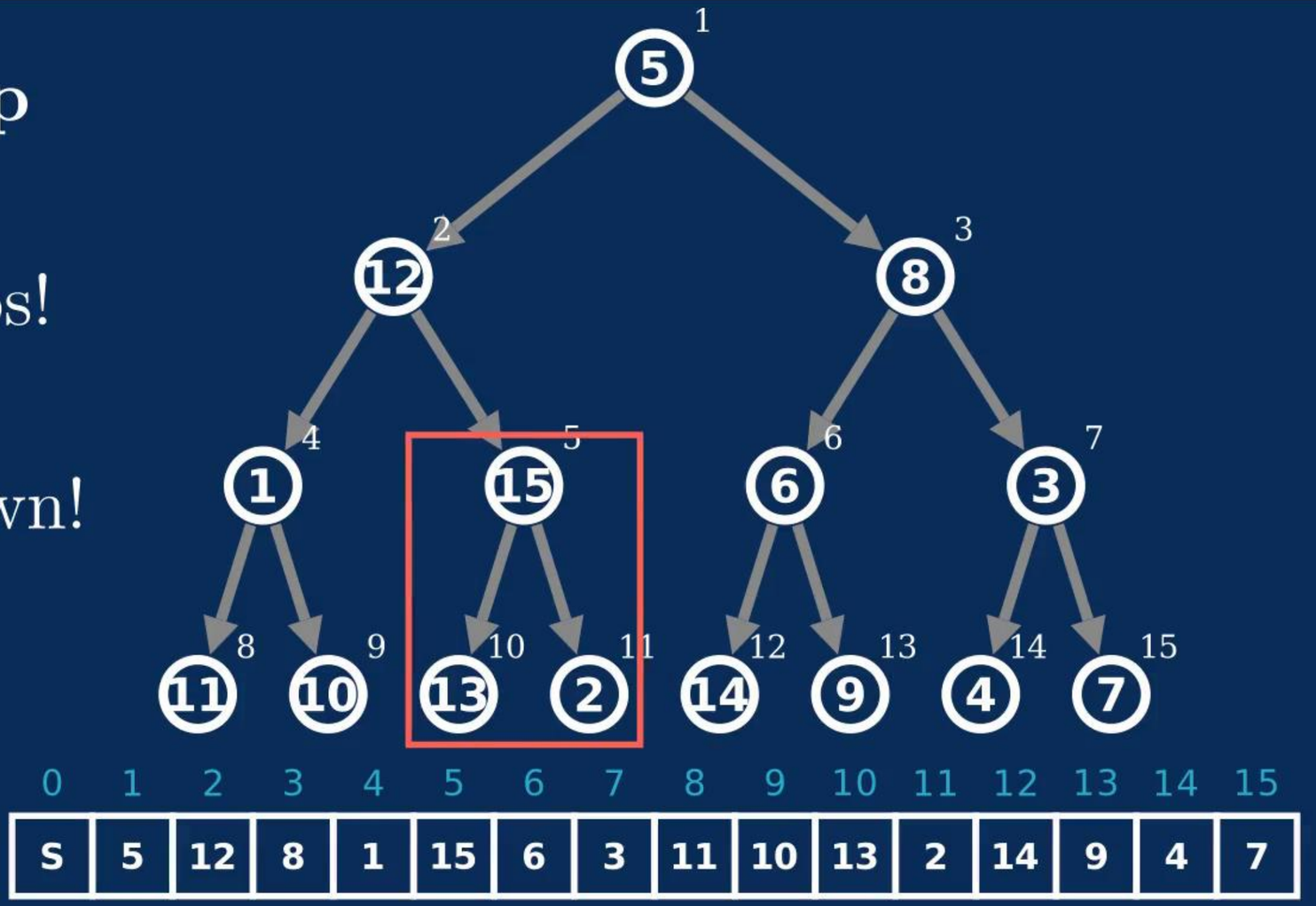

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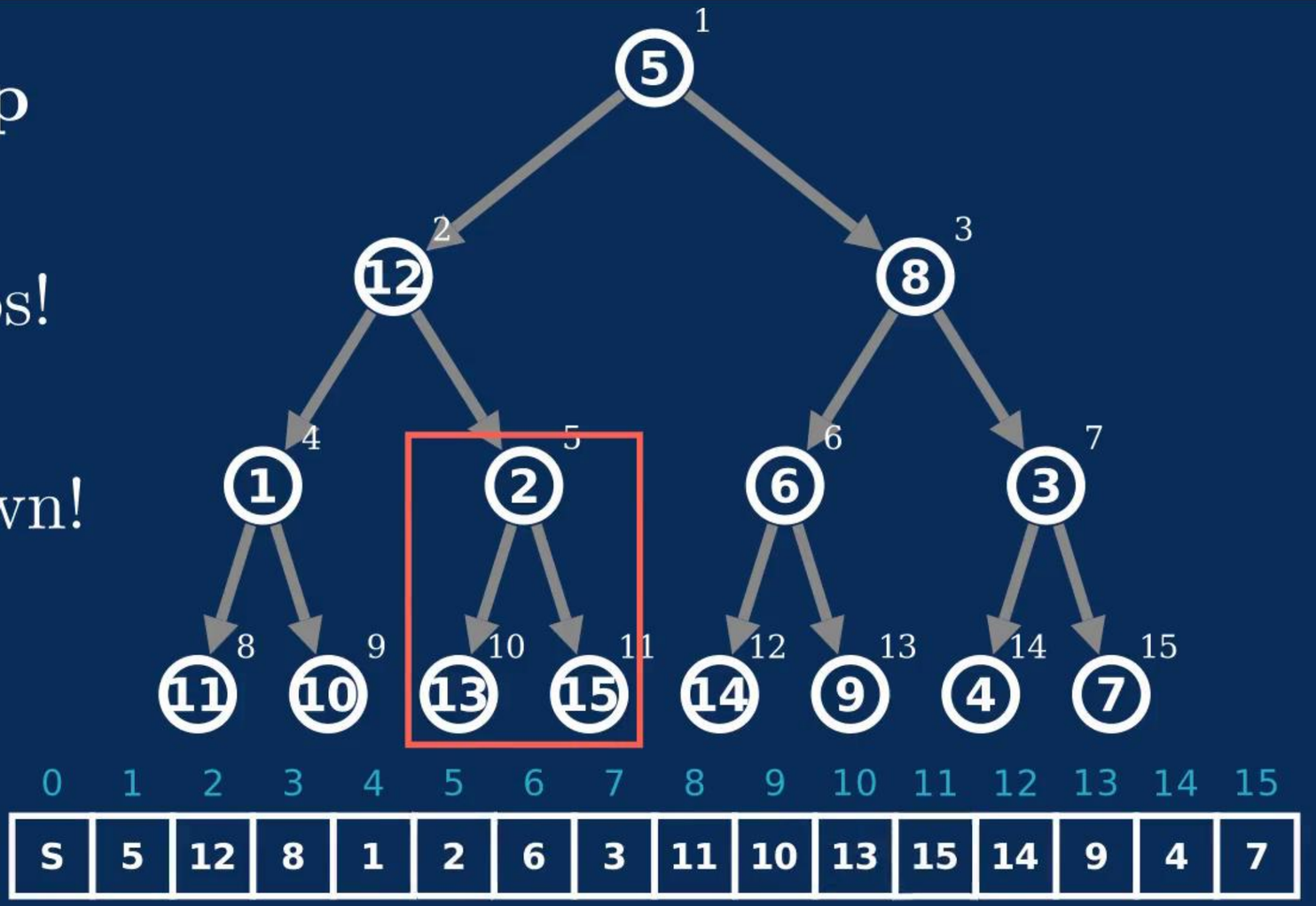

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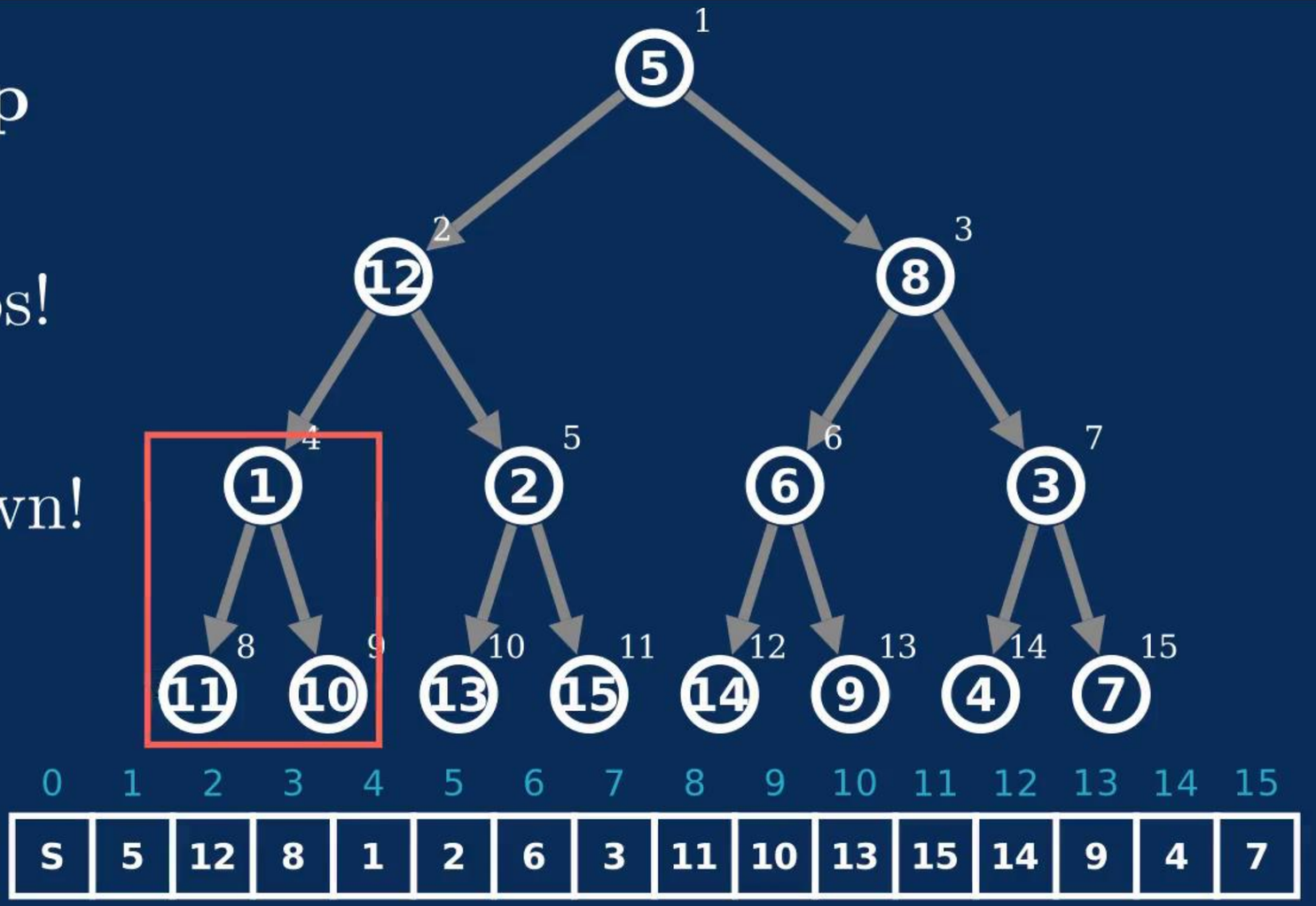

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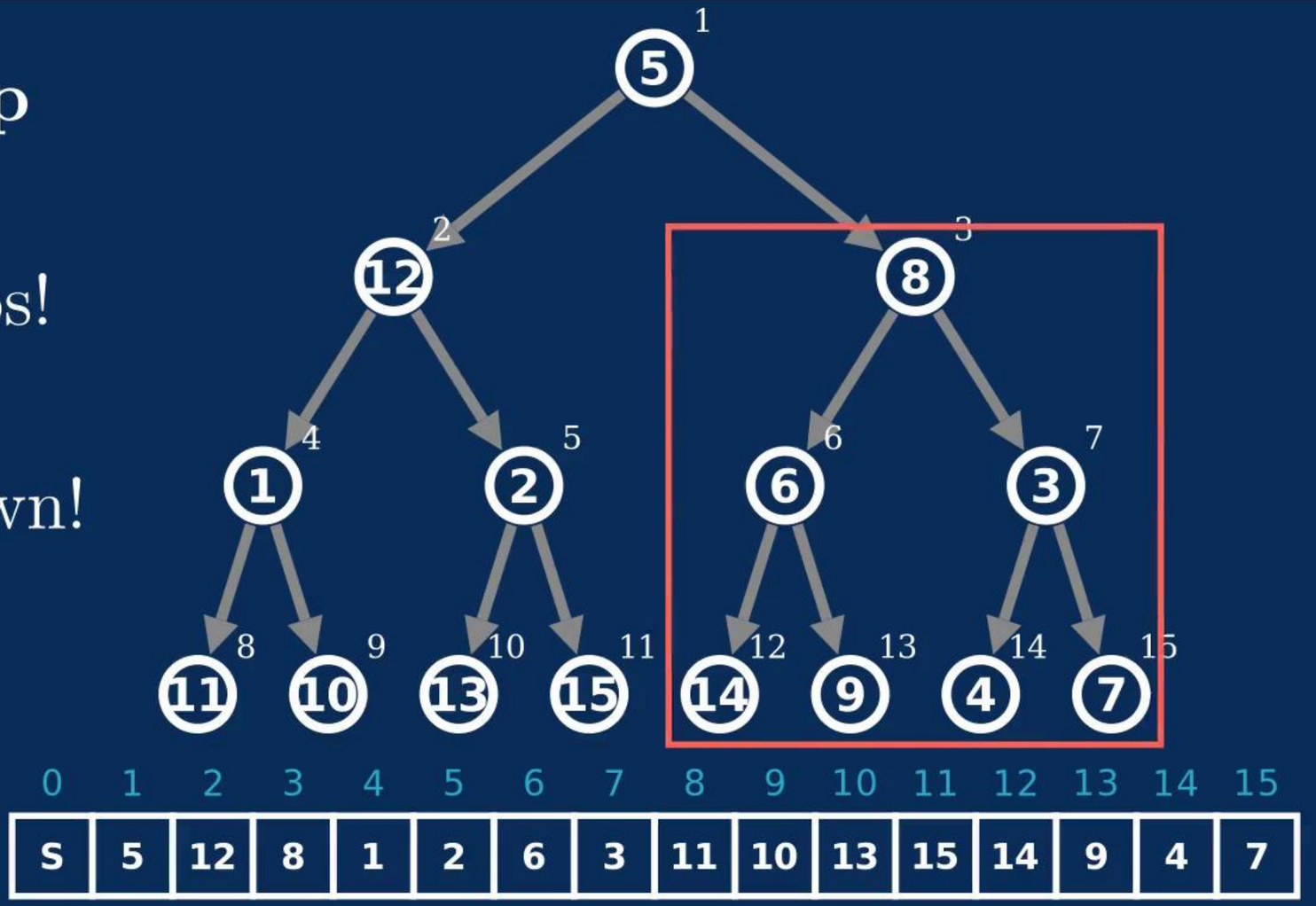

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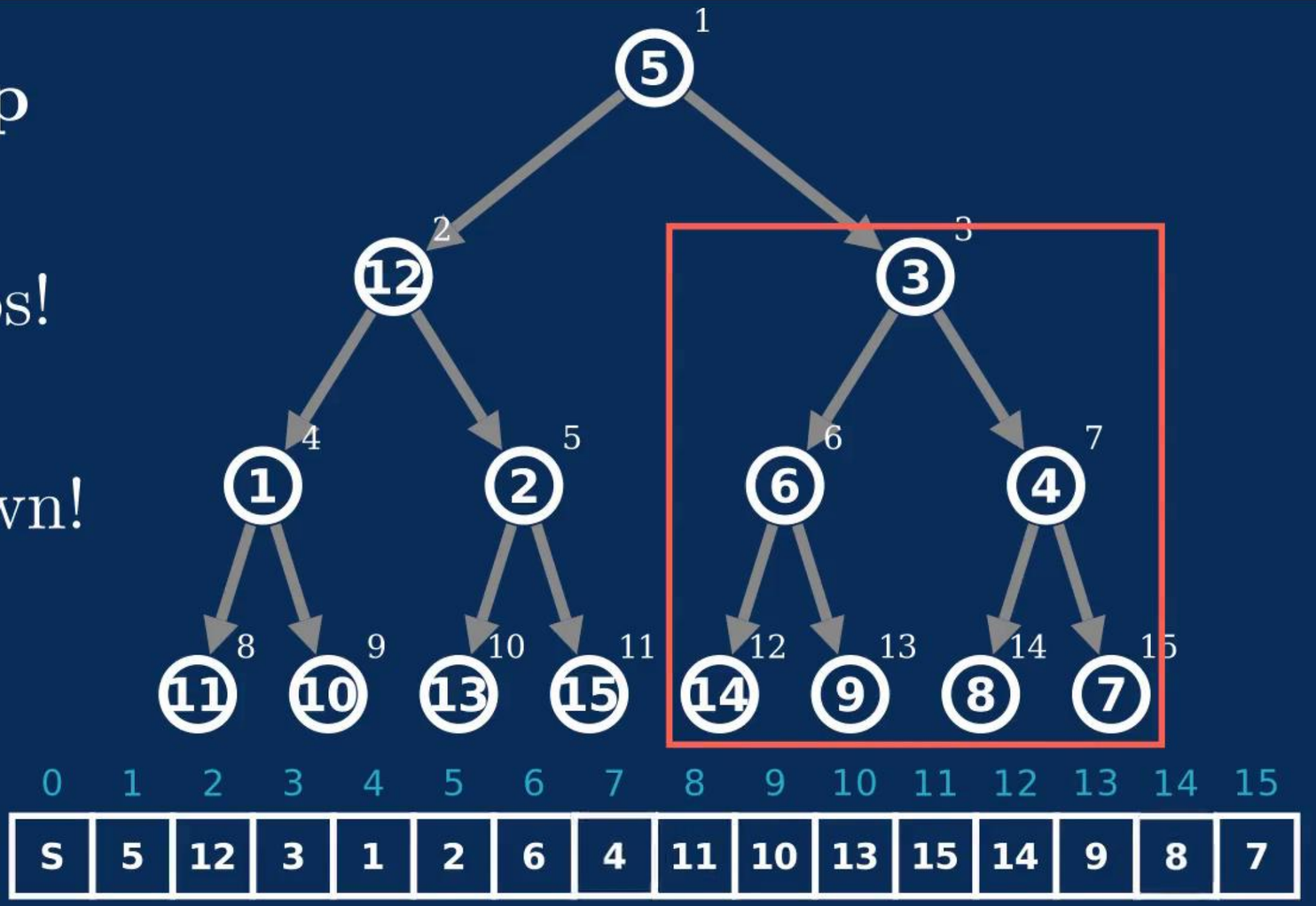

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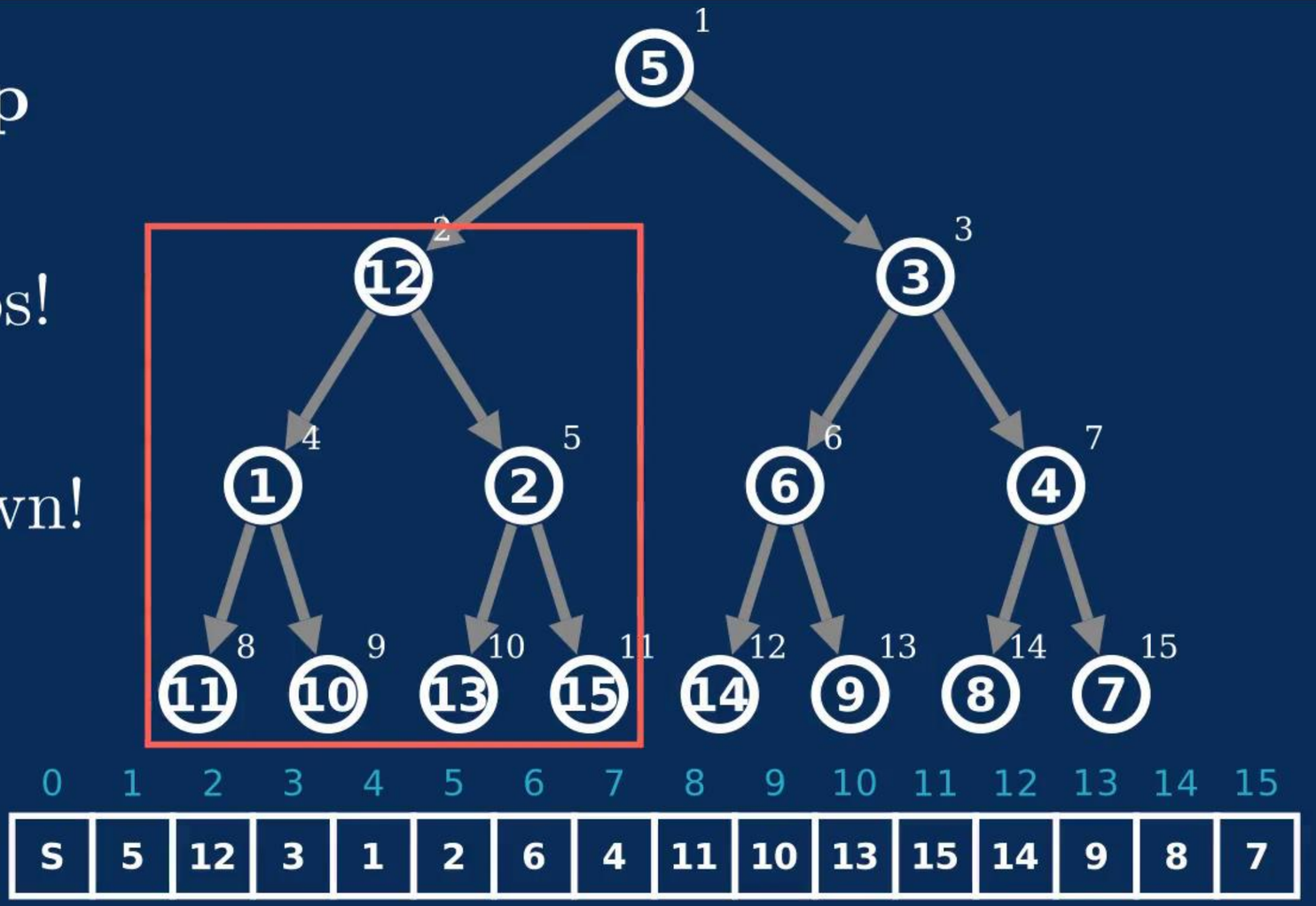

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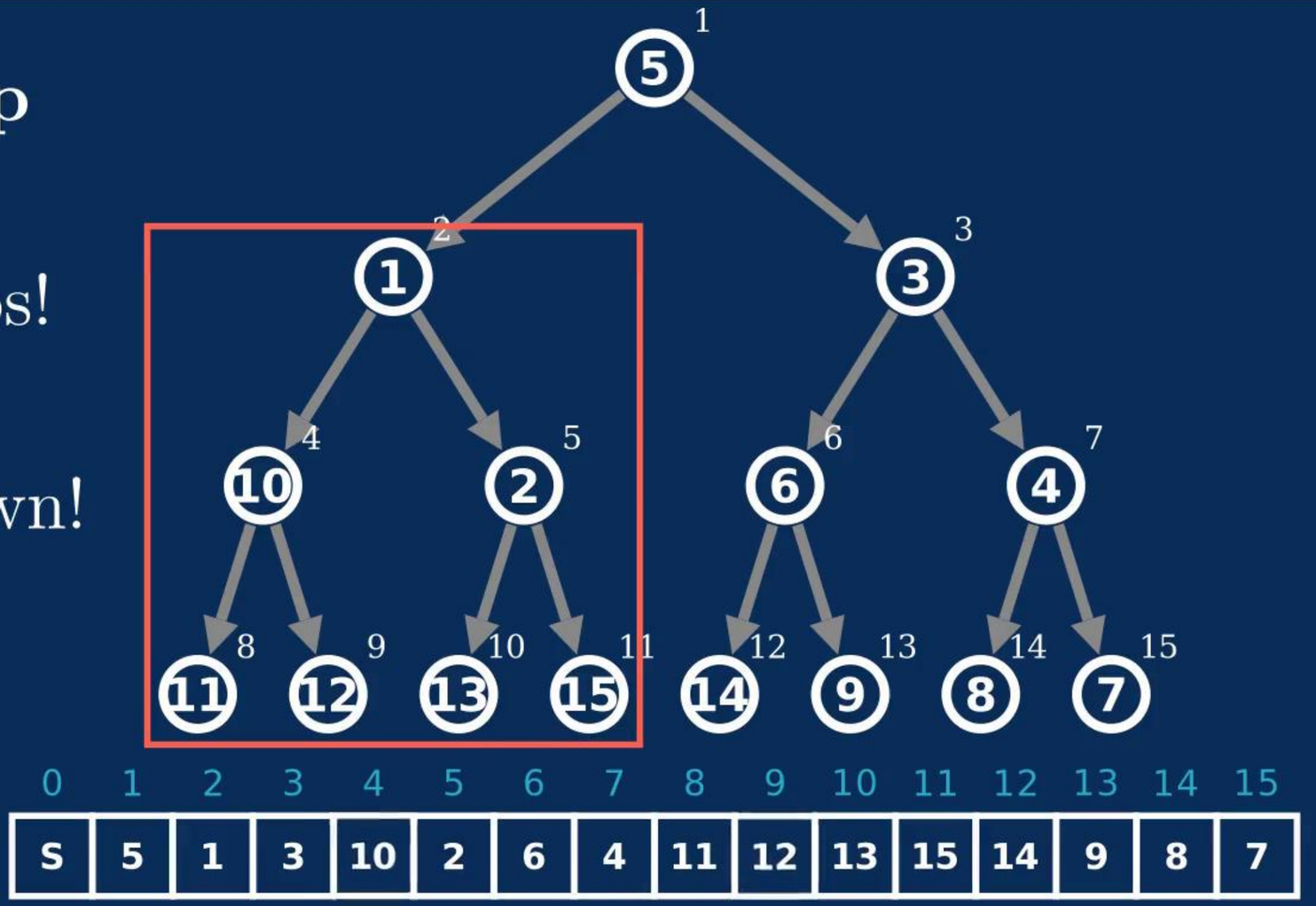

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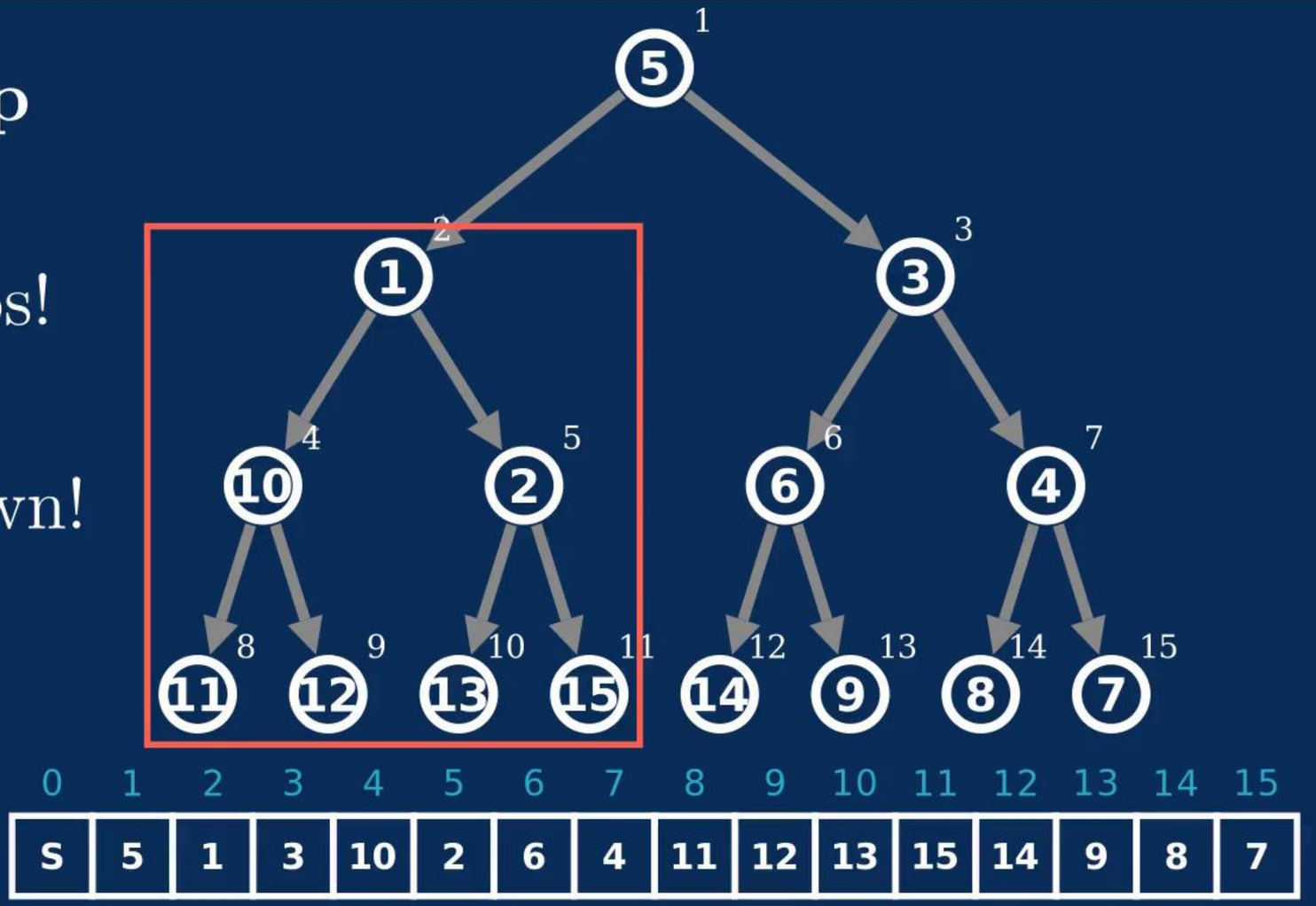

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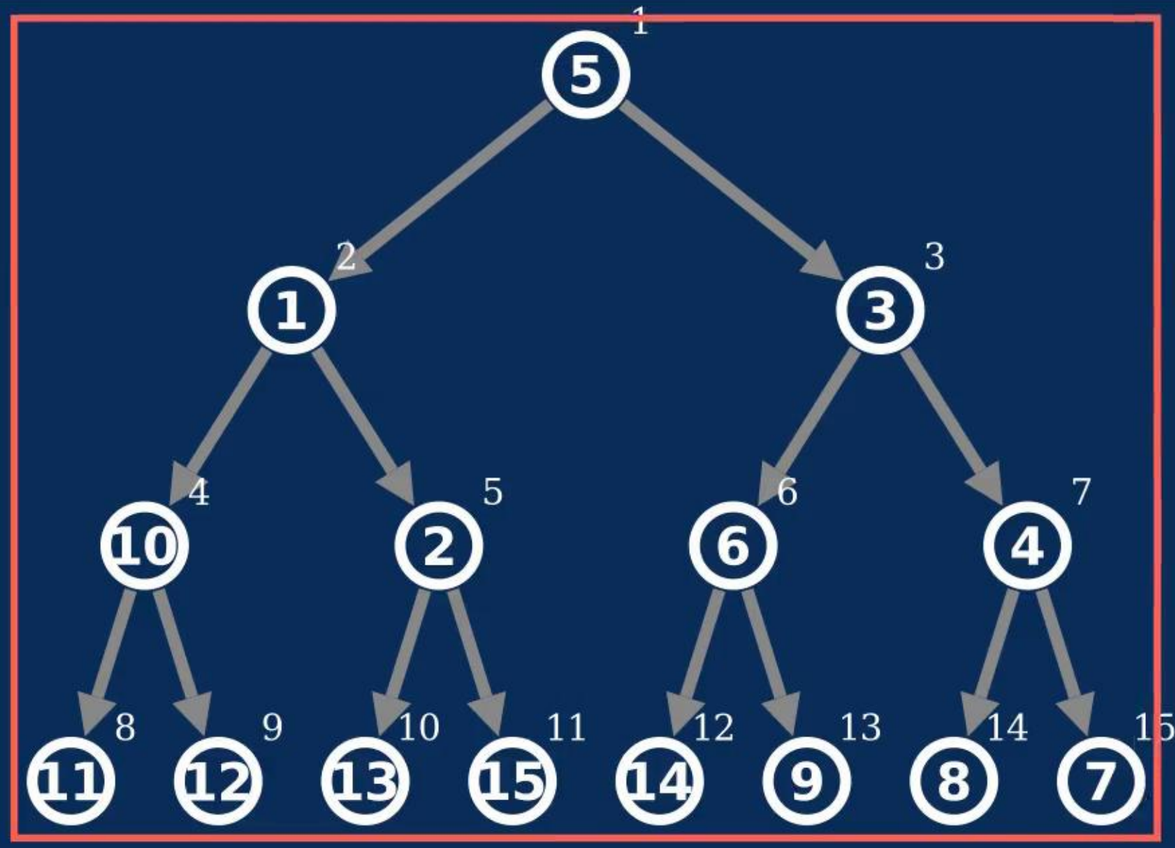

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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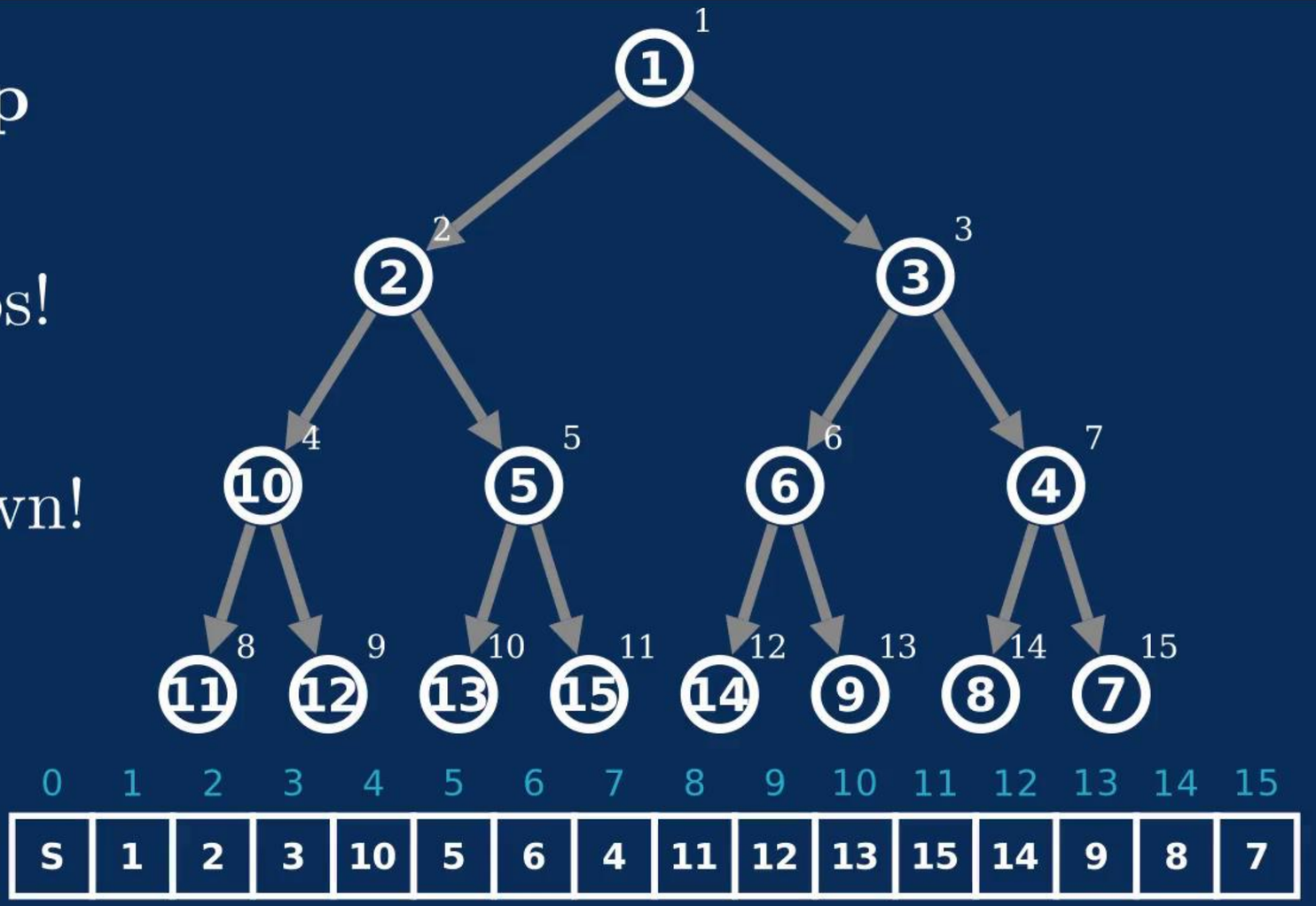

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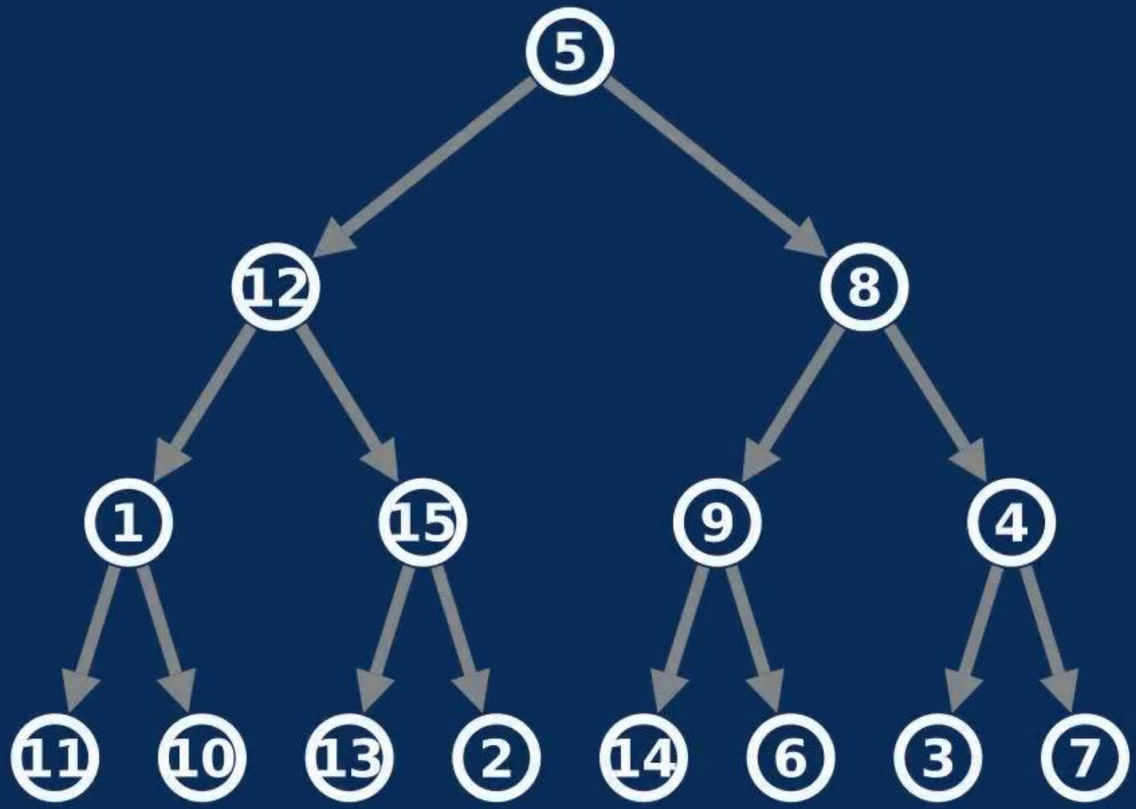

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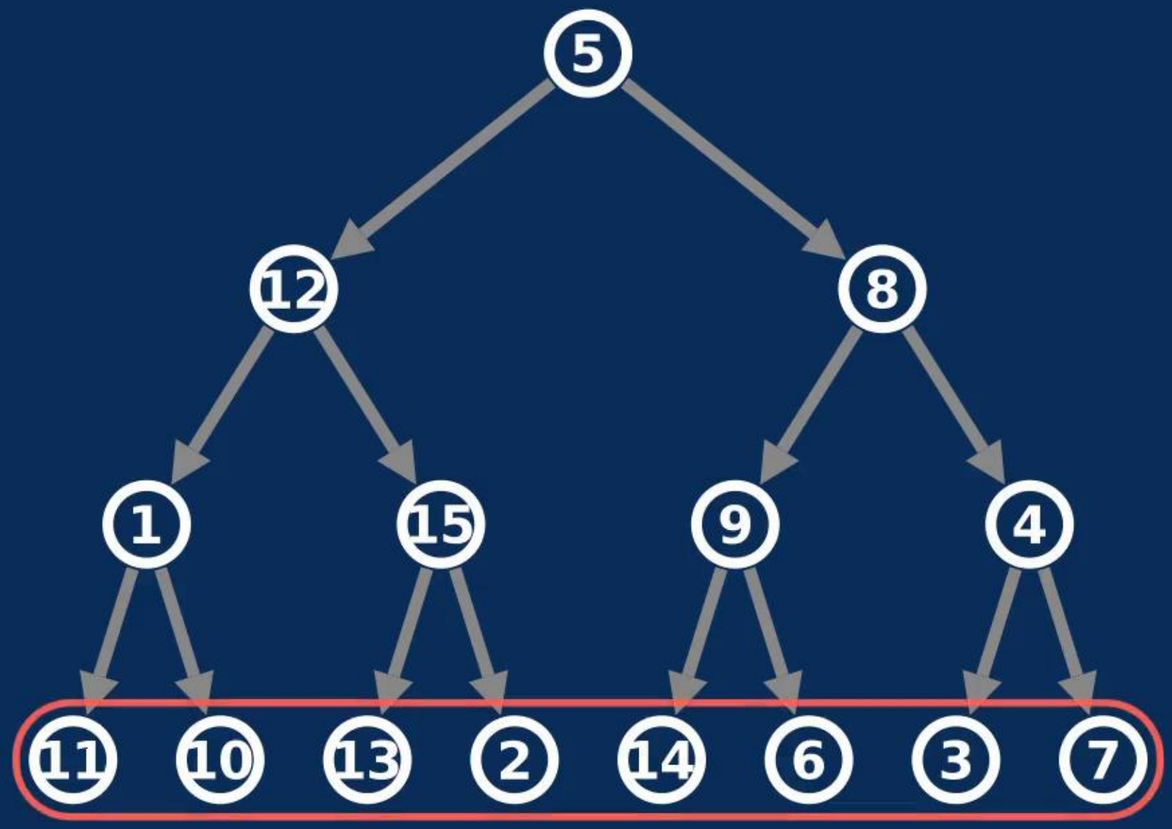
$h = 3$ (leaves): Free!



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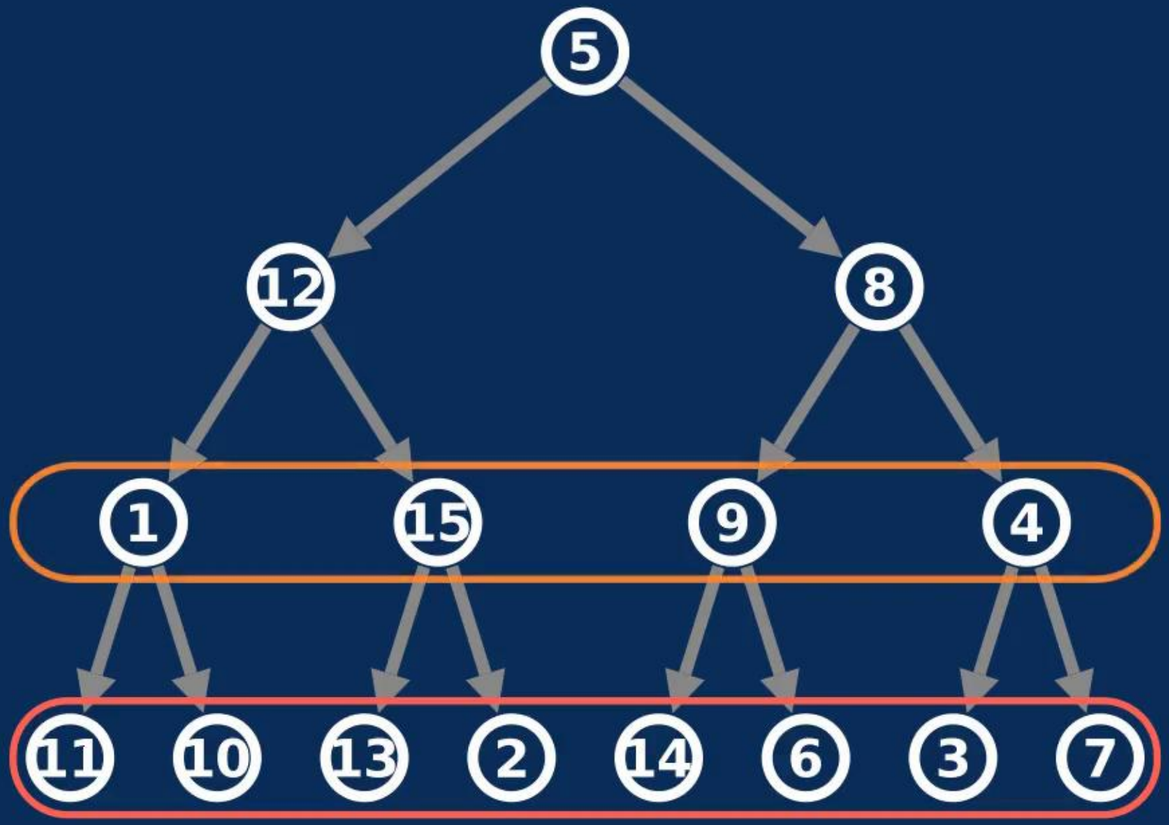

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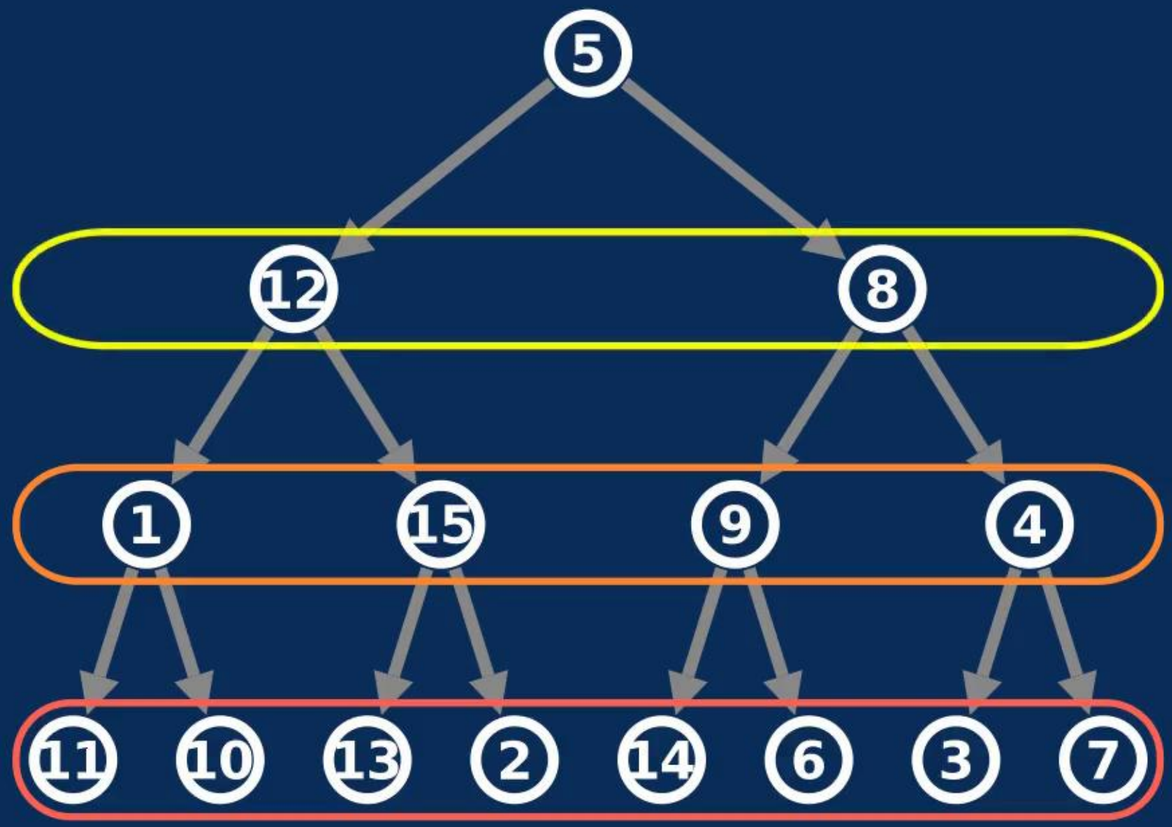

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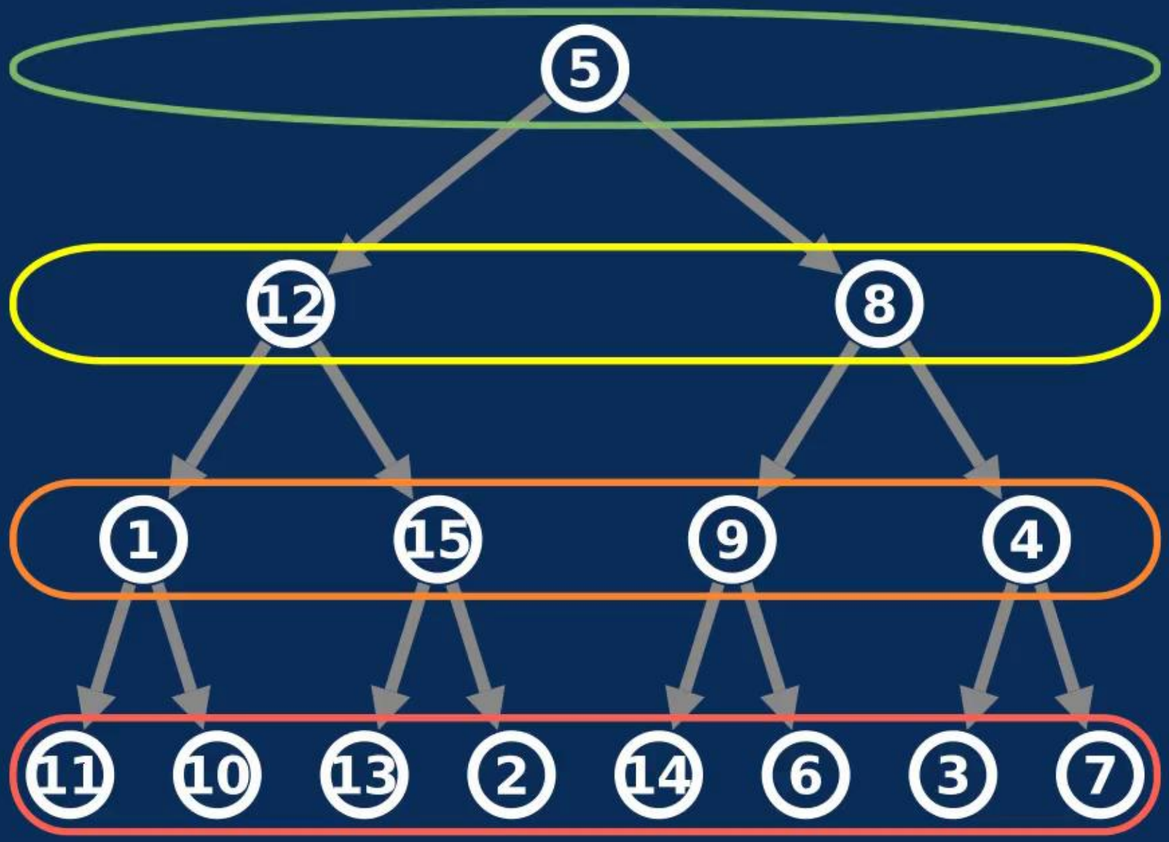

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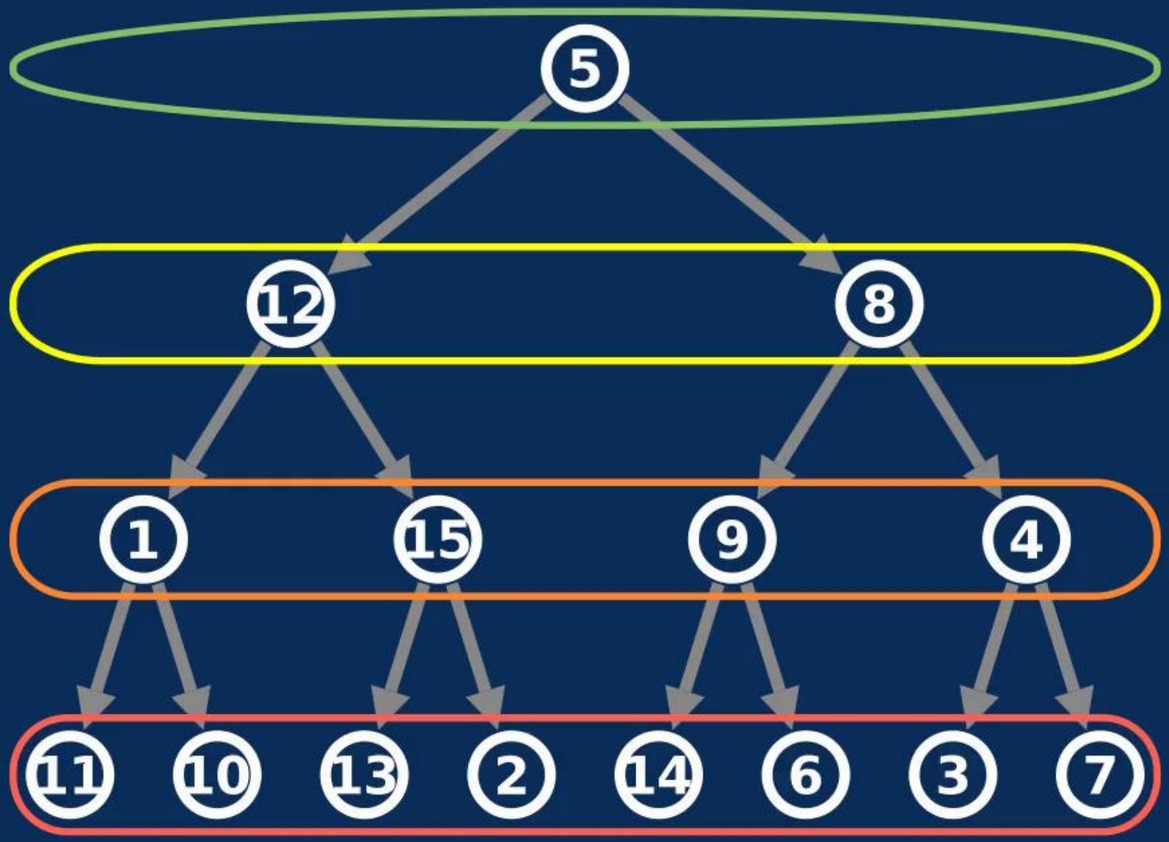

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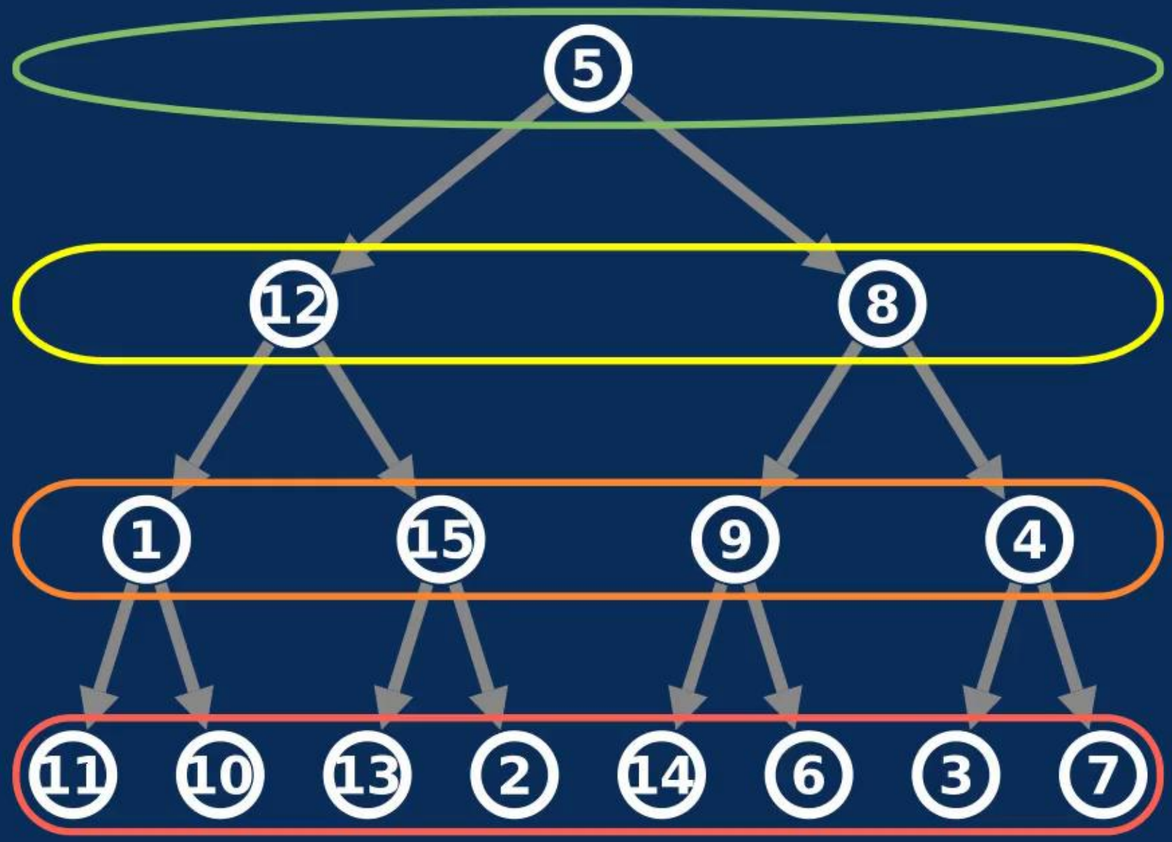
For this: 11 swaps.



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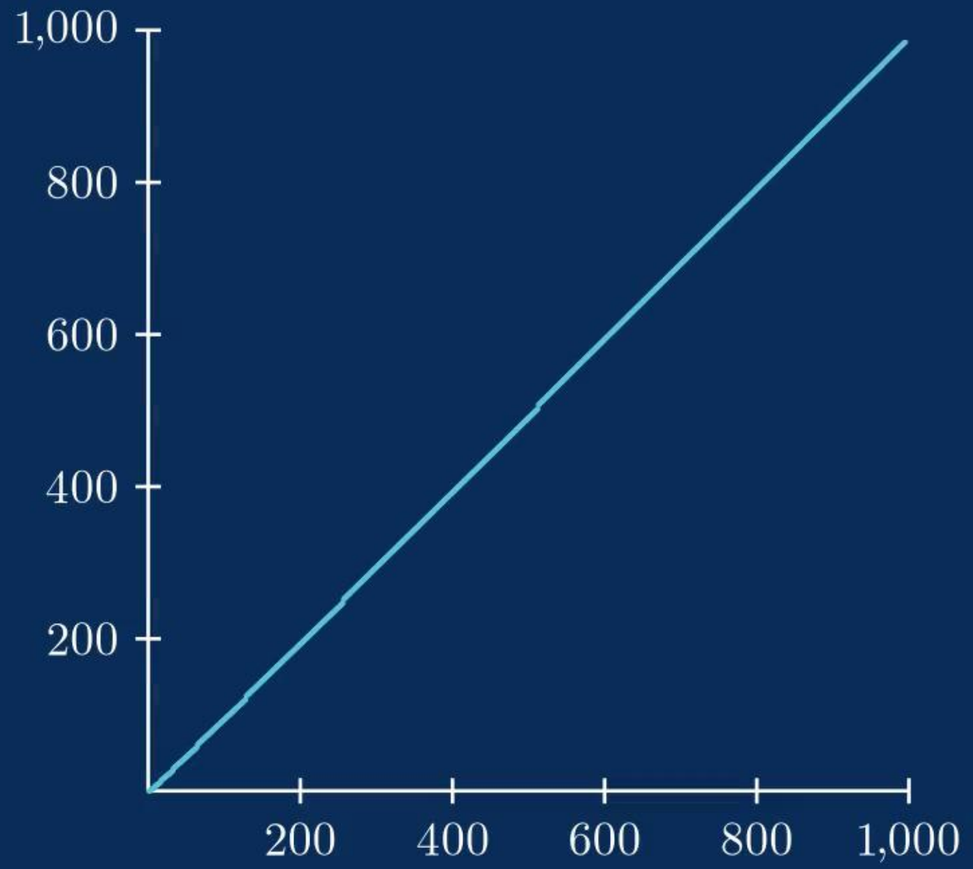

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