Data Structures
Tree Traversal

CS 225
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September 17, 2025

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Computer Science

Extra Credit Reminder

MP submission on PL has two separate submissions

The extra credit portion will only test part 1

Completion of the extra credit portion by the following Monday is worth 4 points

MP_stickers feedback form out now!

Anonymous feedback form worth 2 points **collectively**

Learning Objectives

Review and expand on foundational tree terminology

Discuss the tree ADT

Explore tree implementation details

Binary Tree

Lets define additional terminology for different **types** of binary trees!

1. Full Tree

2. Perfect Tree

3. Complete Tree

Binary Tree: full

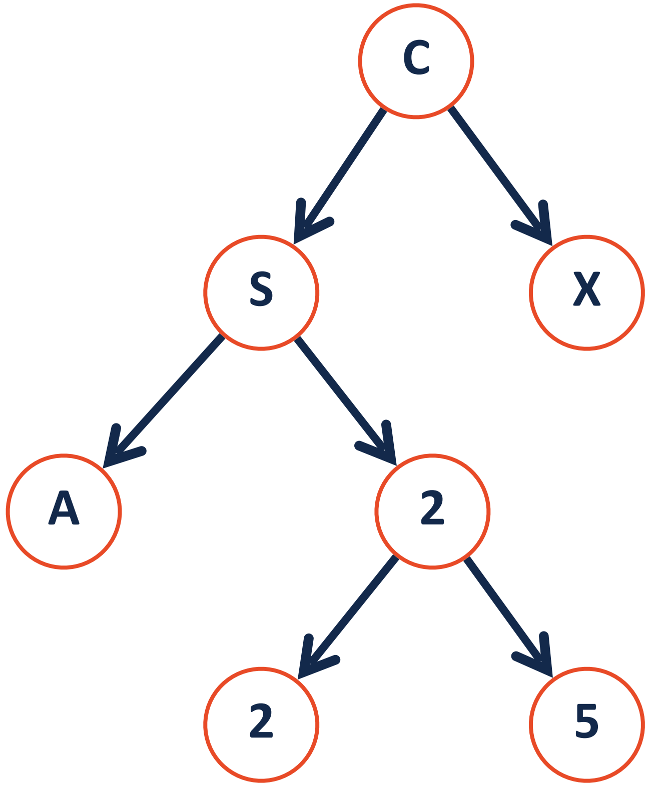
A **full tree** is a binary tree where every node has either 0 or 2 children

A tree **F** is **full** if and only if:

1.

2.

3.



Binary Tree: full

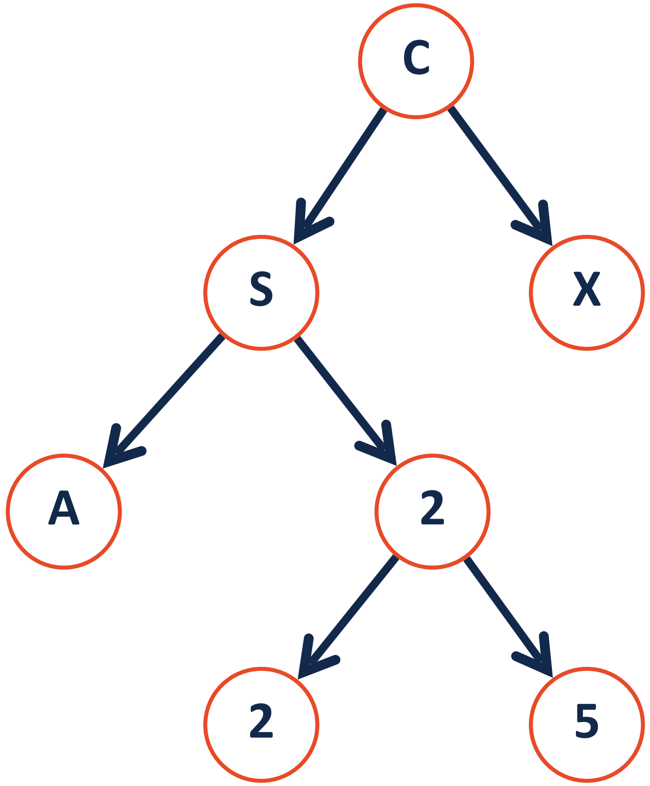
A **full tree** is a binary tree where every node has either 0 or 2 children

A tree **F** is **full** if and only if:

1. $F = \emptyset$

2. $F = (data, \emptyset, \emptyset)$

3. $F = (data, F_l \neq \emptyset, F_r \neq \emptyset)$



Full binary tree : Size

- Question - Which of the following are possible sizes (# of nodes) for a full binary tree?

  a) 2 b) 5 c) 7 d) 8

  A) 2 and 8

  B) 5 and 7

  C) 7 only

  D) All of these

  E) None of these

Join Code : 225

Binary Tree: perfect

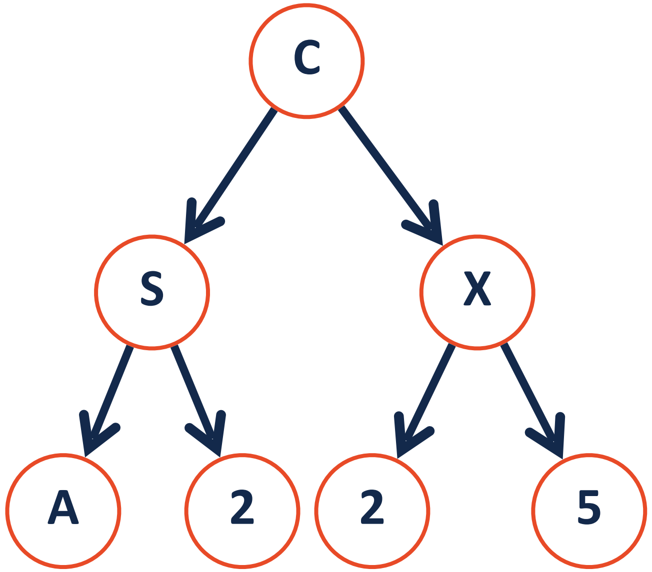
A **perfect tree** is a binary tree where…

Every internal node has 2 children and all leaves are at the same level.

A tree **P** is **perfect** if and only if:

1.

2.



Binary Tree: perfect

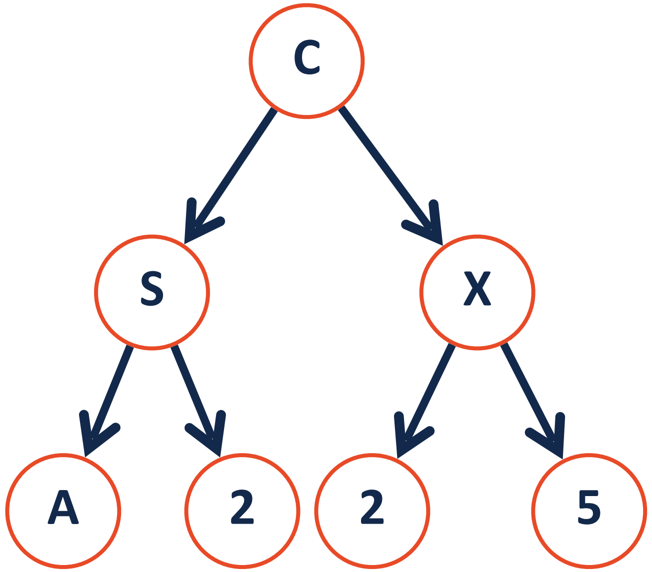
A **perfect tree** is a binary tree where…

Every internal node has 2 children and all leaves are at the same level.

A tree **P** is **perfect** if and only if:

1. $P_h = (data, P_{h-1}, P_{h-1})$

2. $P_0 = (data, \emptyset, \emptyset) \equiv \; P_{-1} = \emptyset$



Perfect binary tree : Size

- Question - Which of the following are possible sizes (# of nodes) for a perfect binary tree?

a) 2 b) 5 c) 7 d) 8

A) 2 and 8

B) 5 and 7

C) 7 only

D) All of these

E) None of these

Join code : 225

Binary Tree: complete

A **complete tree** is a B.T. where…

All levels except the last are completely filled.

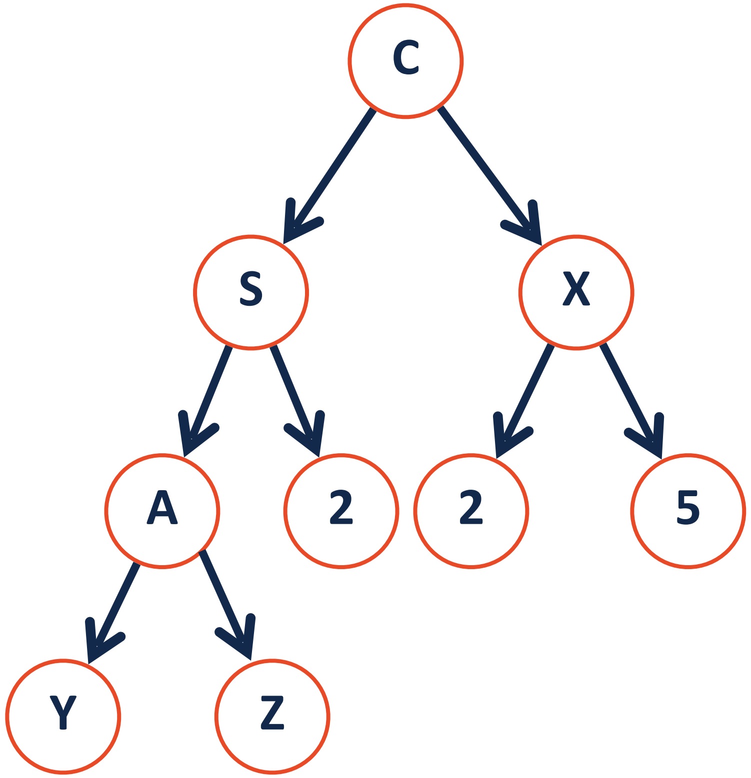
The last level contains at least one node (and is pushed to left)

A tree **C** is **complete** if and only if:

1.

2.

3.



Binary Tree: complete

A **complete tree** is a B.T. where…

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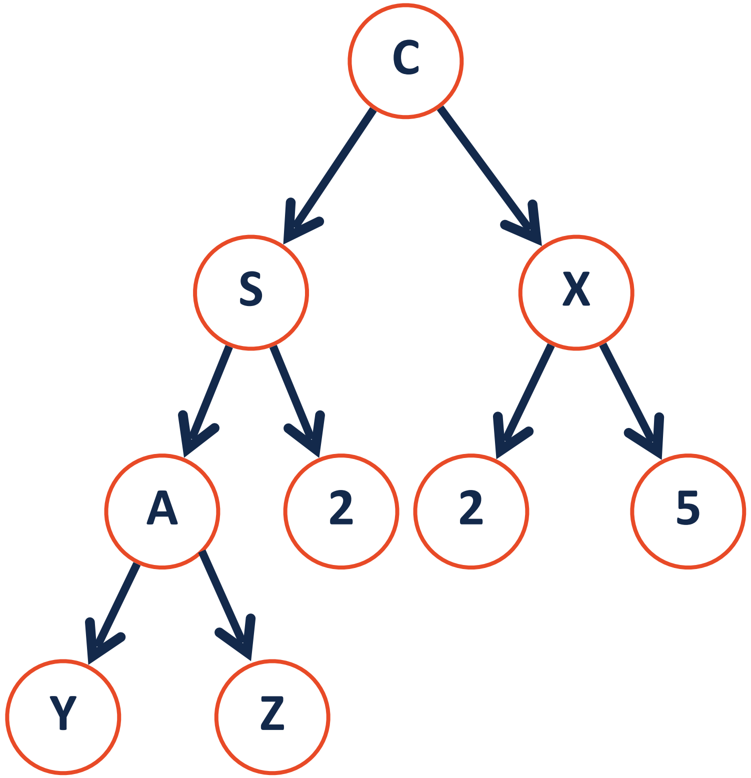
A tree **C** is **complete** if and only if:



1. $C_h = (data, C_{h-1}, P_{h-2})$

2. $C_h = (data, P_{h-1}, C_{h-1})$

3. $C_{-1} = \emptyset$

Binary Tree: complete

A **complete tree** is a B.T. where…

All levels except the last are completely filled.

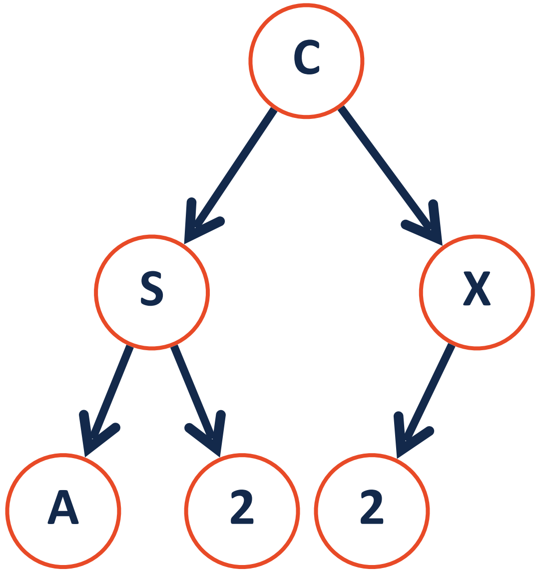
The last level contains at least one node (and is pushed to left)

A tree **C** is **complete** if and only if:

1. $C_h = (data, C_{h-1}, P_{h-2})$

2. $C_h = (data, P_{h-1}, C_{h-1})$

3. $C_{-1} = \emptyset$



Complete binary tree : Size

- Question - Which of the following are possible sizes (# of nodes) for a complete binary tree?

  a) 2 b) 5 c) 7 d) 8

  A) 2 and 8

  B) 5 and 7

  C) 7 only

  D) All of these

  E) None of these

Join code : 225

Binary Tree

Why do we care?

1. Terminology instantly defines a particular tree structure

2. Understanding how to think ‘recursively’ is very important.

Binary Tree: Thinking with Types

Is every **full** tree **complete**?

Is every **complete** tree **full**?

Tree ADT

Insert

Remove

Traverse

Find

Constructor

List.h

```
#pragma once

template <typename T>
class List {
  public:
    /* ... */
  private:
    class ListNode {
      T & data;

      ListNode * next;



      ListNode(T & data) :
       data(data), next(NULL) { }
    };



    ListNode *head_;
    /* ... */
};
```

Tree.h

```
#pragma once

template <typename T>
class BinaryTree {
  public:
    /* ... */
  private:











    };

    TreeNode *root_;
    /* ... */
};
```

List.h

```
#pragma once

template <typename T>
class List {
  public:
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    };



    ListNode *head_;
    /* ... */
};
```
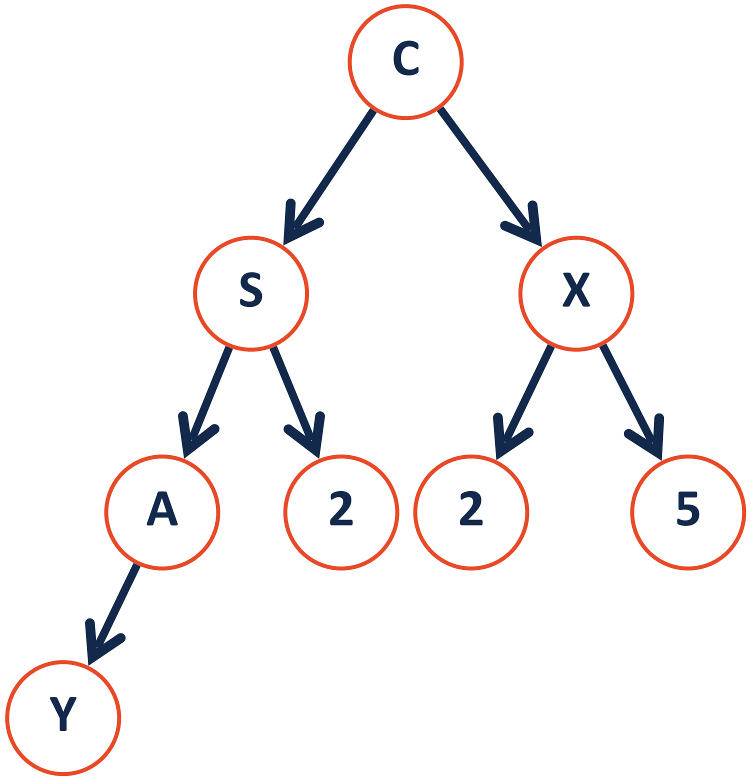
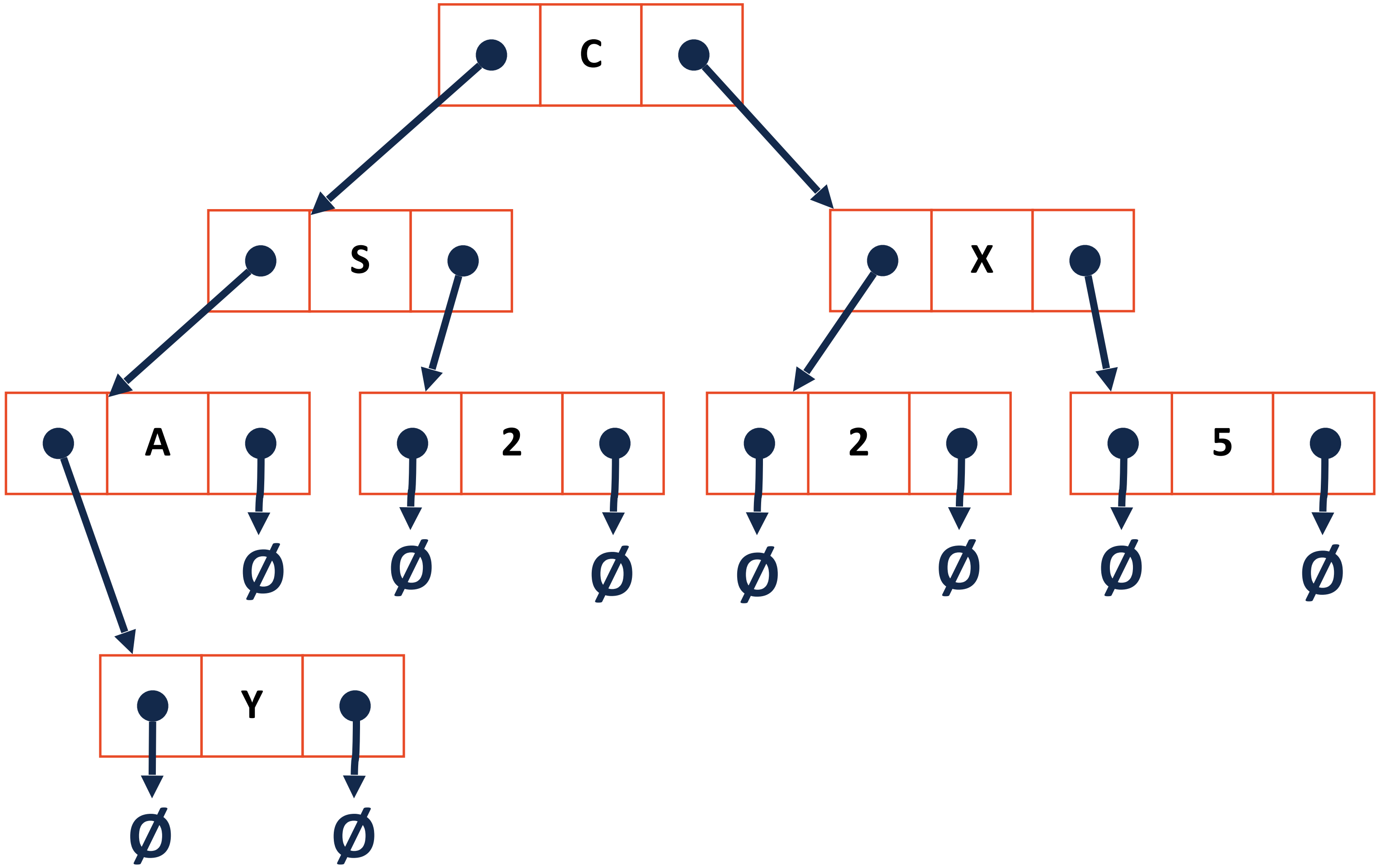

Tree.h

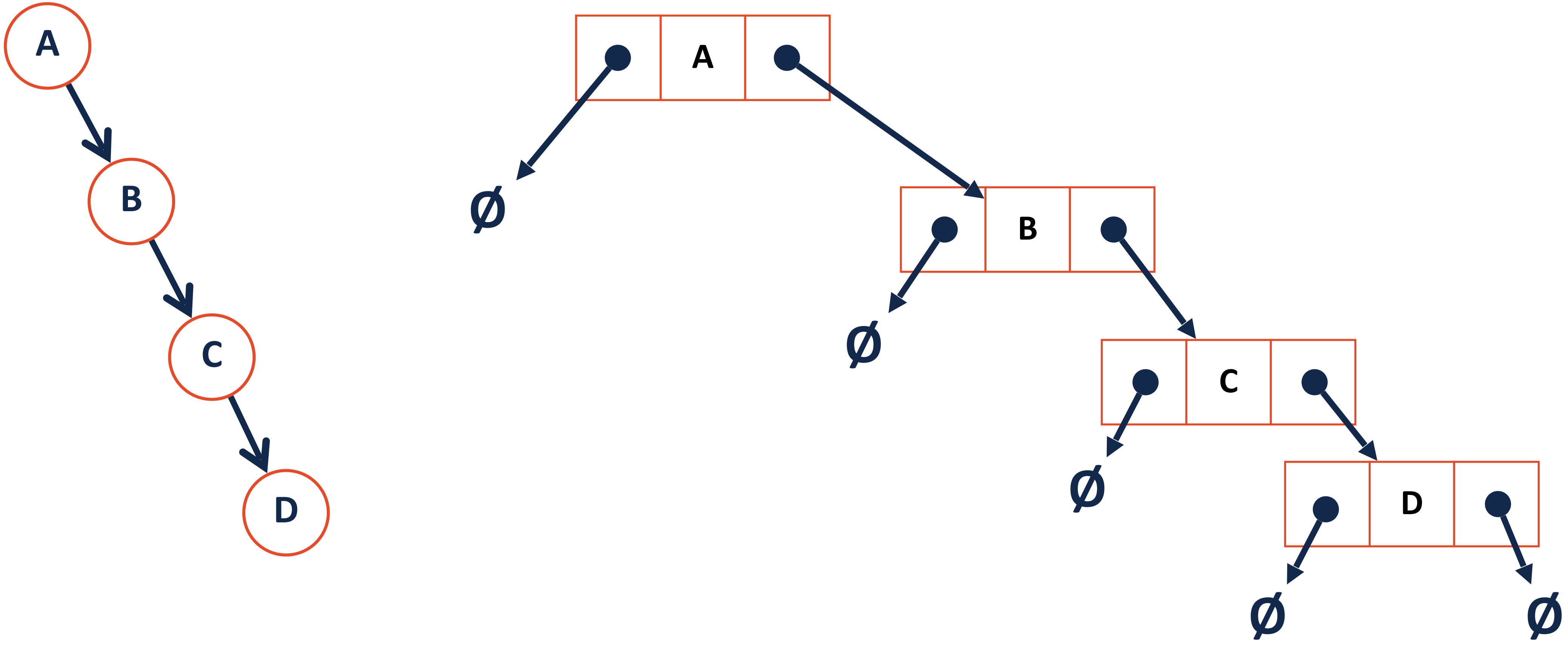
```
#pragma once

template <typename T>
class BinaryTree {
  public:
    /* ... */
  private:
    class TreeNode {
      T & data;

      TreeNode * left;

      TreeNode * right;

      TreeNode(T & data) :
       data(data), left(NULL),
right(NULL) { }

    };

    TreeNode *root_;
    /* ... */
};
```

Visualizing trees



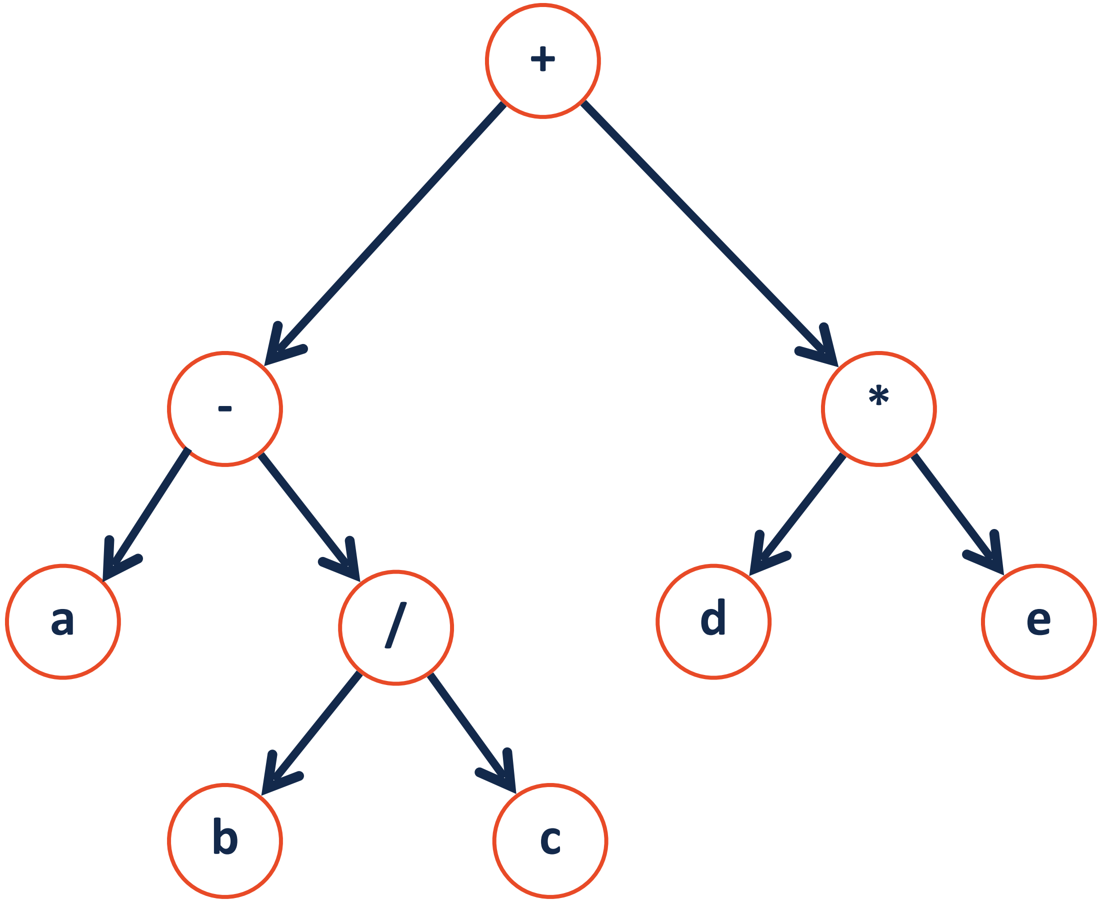


Tree Insert / Remove acts like Linked List

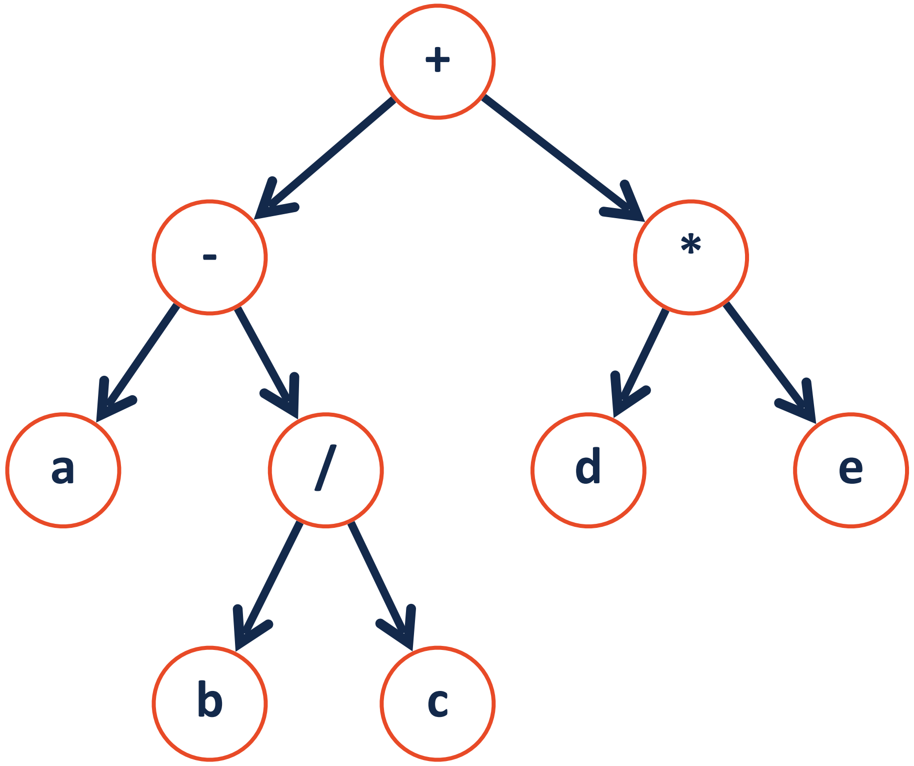


Tree Traversal

A **traversal** of a tree T is an ordered way of visiting every node once.

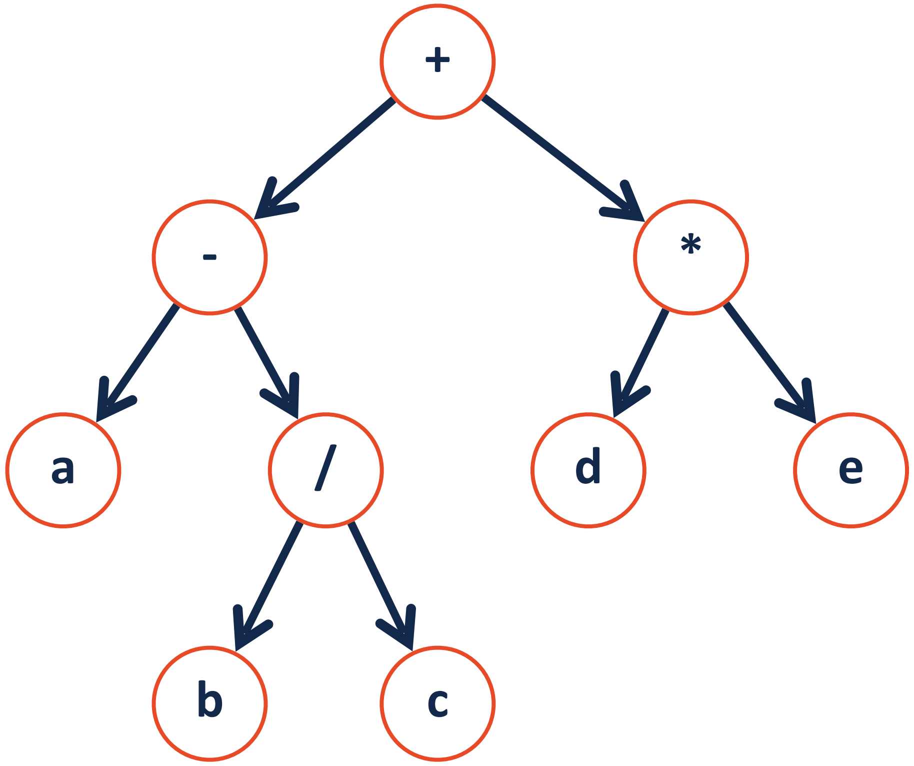


Traversals



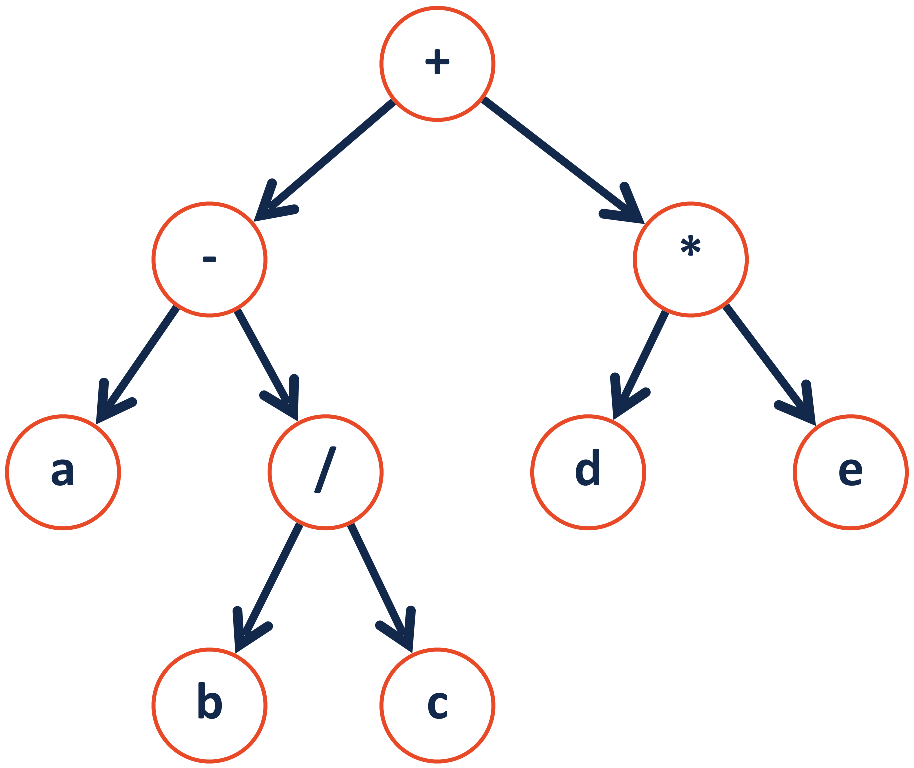

```
template<class T>
void BinaryTree<T>::_____Order(TreeNode * root)
{
}
```

Traversals



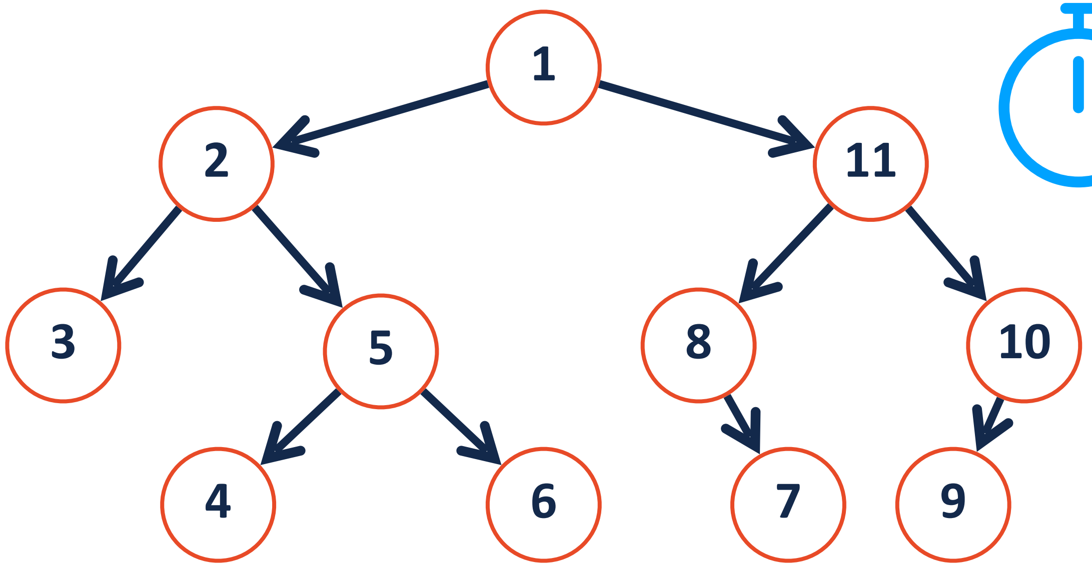

```
template<class T>
void BinaryTree<T>::_____Order(TreeNode * root)
{
  if (root) {
      _____________________;
      _____Order(root->left);
      _____________________;
      _____Order(root->right);
      _____________________;
  }
}
```

Traversals




```
template<class T>
void BinaryTree<T>::_____Order(TreeNode * root)
{

  if (root) {

      ______________________;

      _____Order(root->left);

      ______________________;

      _____Order(root->right);

      ______________________;

  }



}
```

Tree Traversals

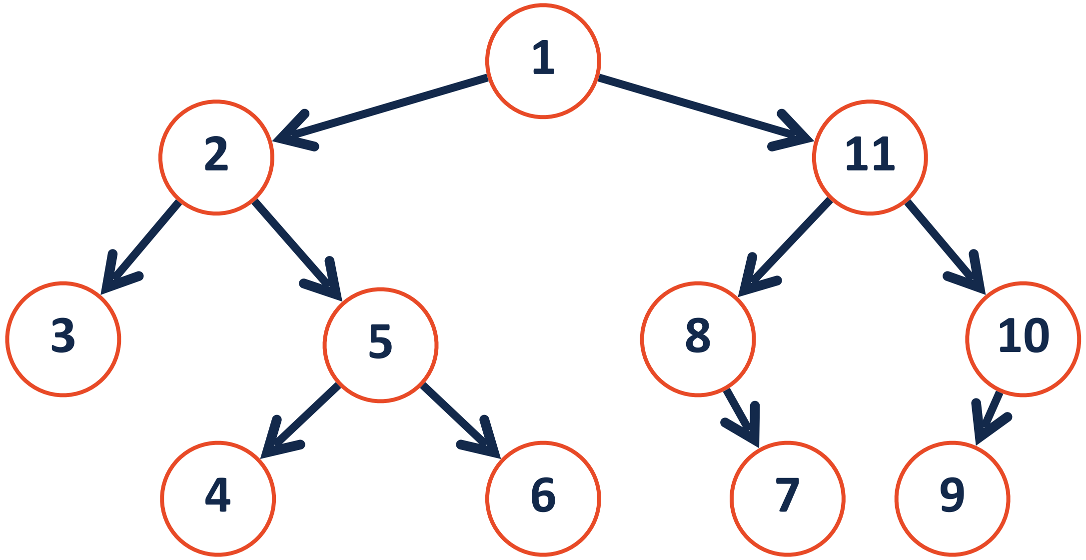


Pre-order:

In-order:

Post-order:

Tree Traversals (Solution)



Pre-order: **1** 2 3 5 4 6 11 8 7 10 9

Tip: Preorder always starts with root!

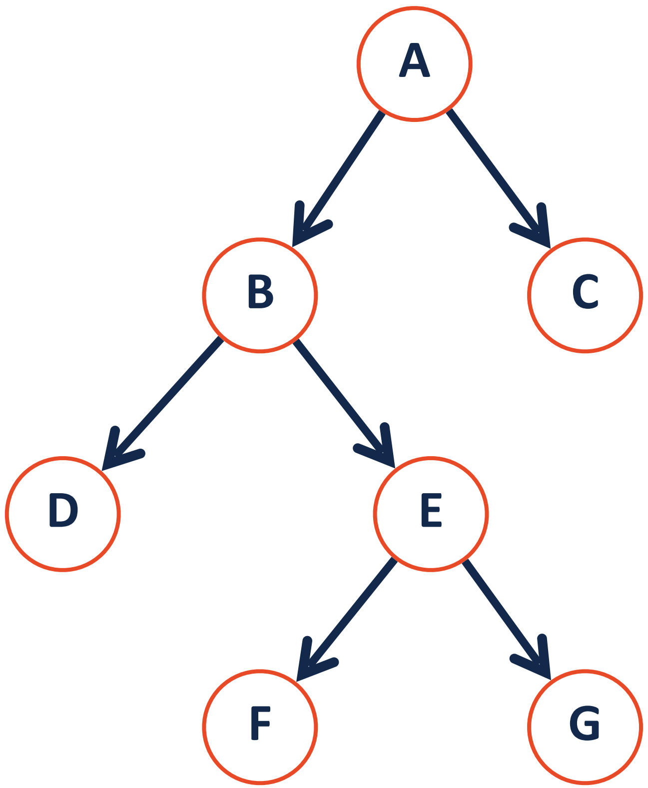
In-order: **3** 2 4 5 6 **1** 8 7 11 9 10

Tip: Inorder always starts with leftmost node. Root is after all left nodes!

Post-order: **3** 4 6 5 2 7 8 9 10 11 **1**

Tip: Post always starts with leftmost node. Root is always LAST node!

Traversal vs Search

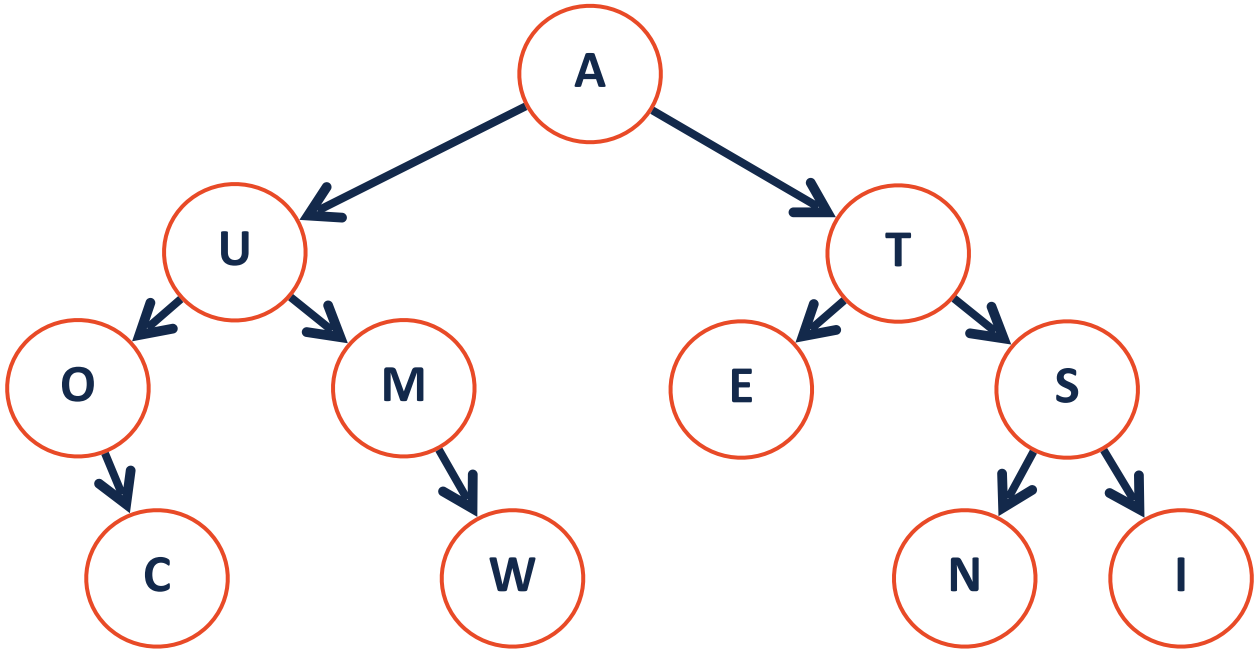


Traversal

Search

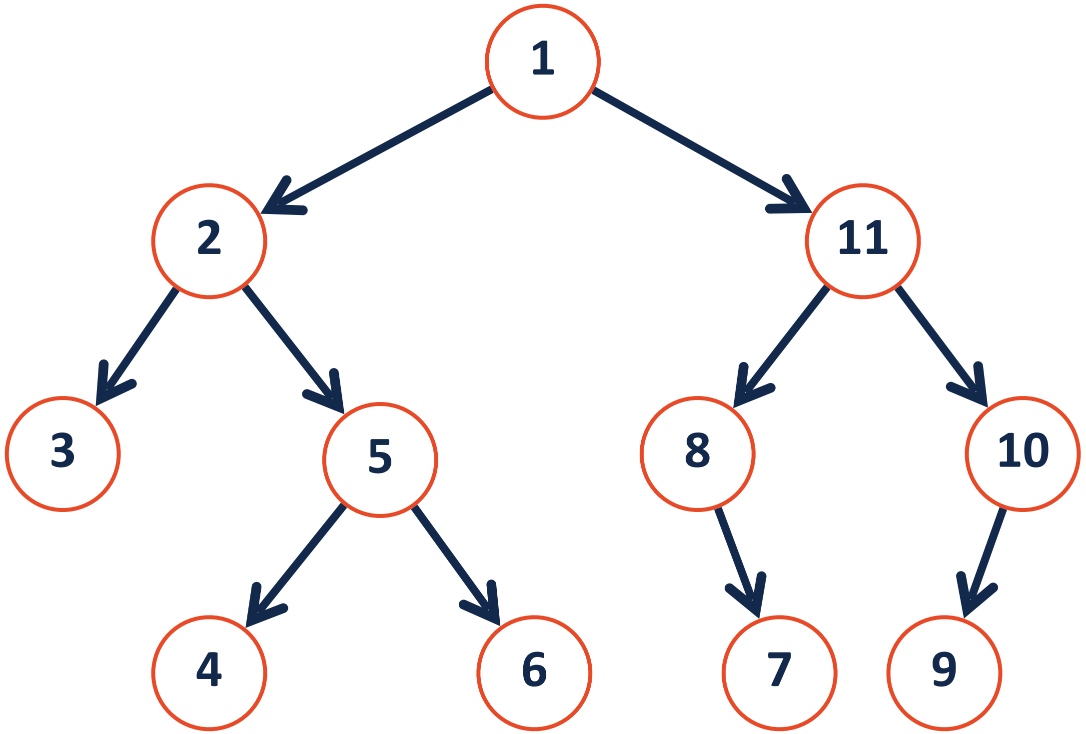
Tree Search

There are two main approaches to searching a binary tree:



Depth First Search

Explore as far along one path as possible before backtracking



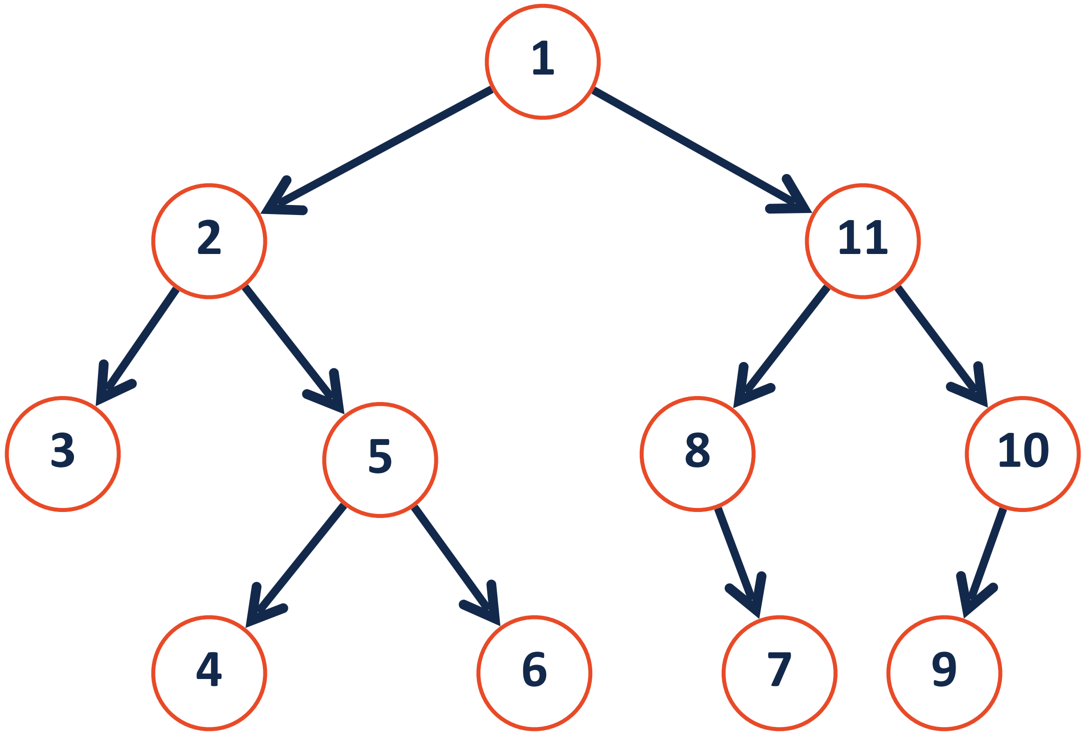
Depth First Search

Explore as far along one path as possible before backtracking

Make a stack (initialized with root)

While stack not empty:

```
tmp = stack.pop()
print(tmp)
stack.push(tmp->right)
stack.push(tmp->left)
```

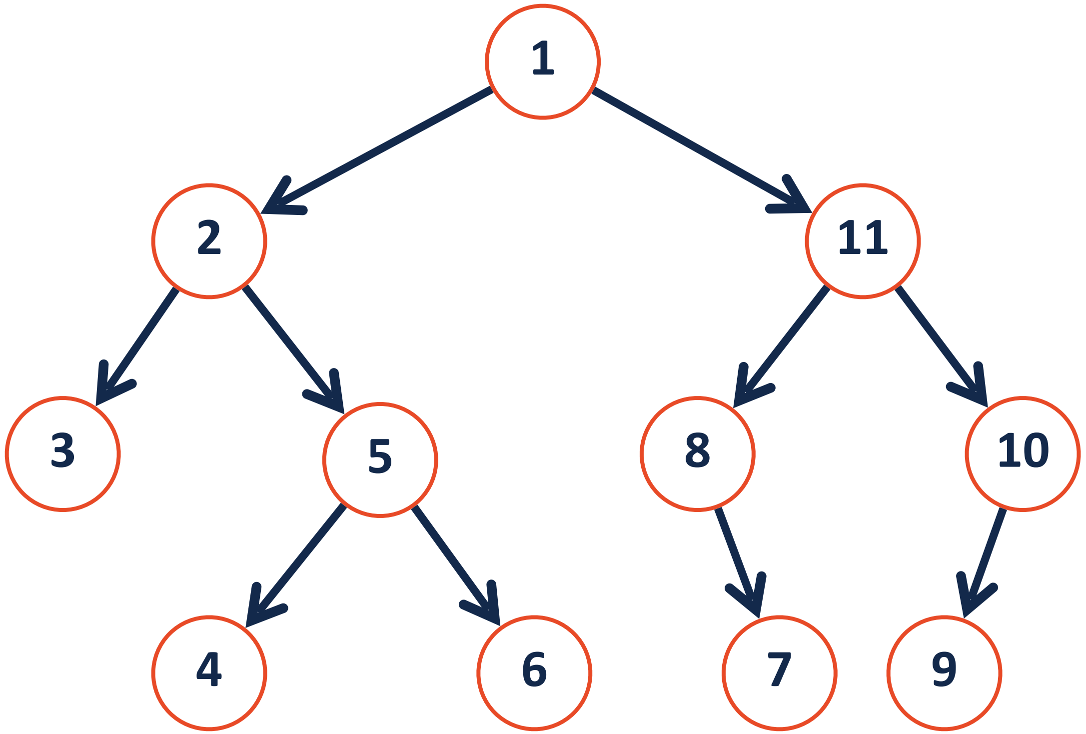


Stack:

Print:

Breadth First Search

Fully explore depth i before exploring depth i+1



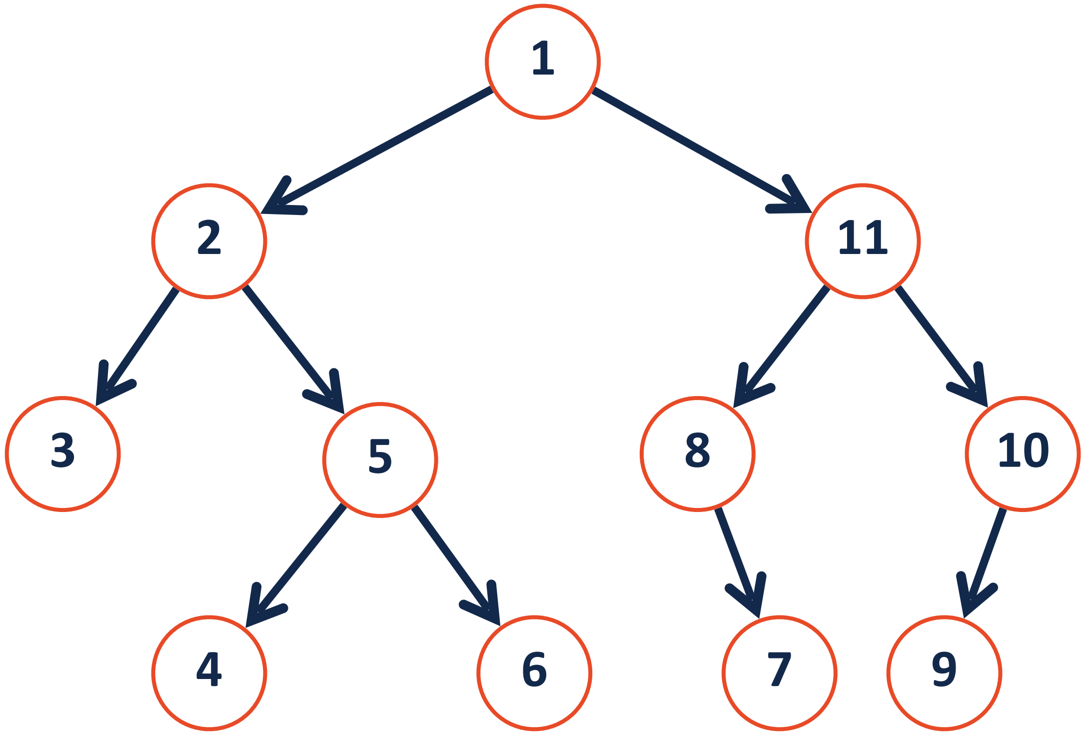
Breadth First Search

Fully explore depth i before exploring depth i+1

Make a queue (initialized with root)

While queue not empty:

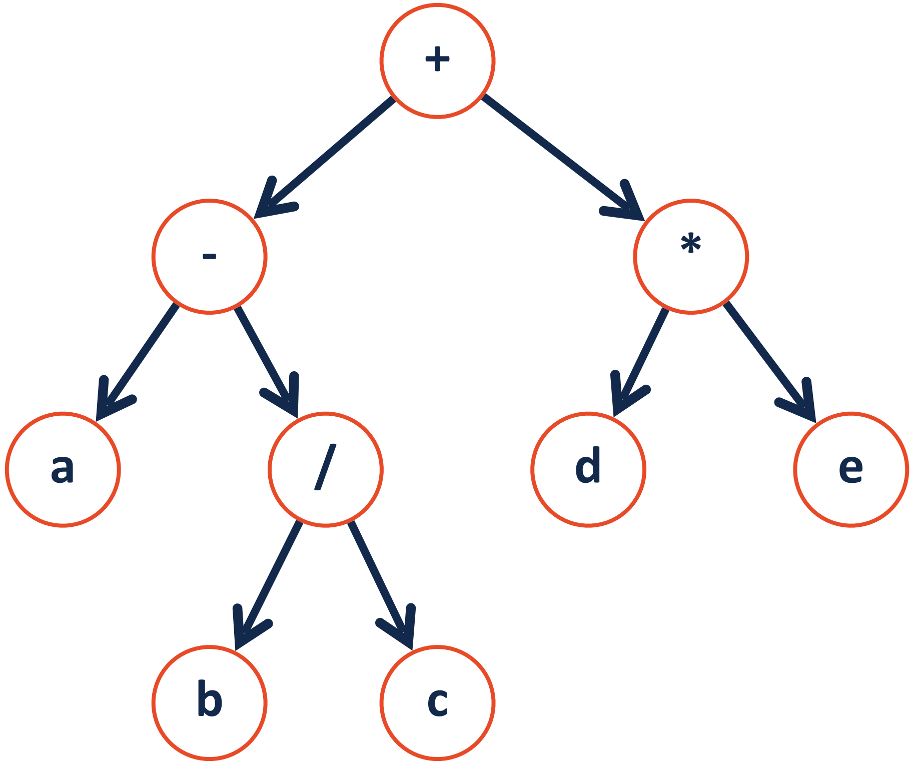
```
tmp = queue.dequeue()
print(tmp)
queue.enqueue(tmp->left)
queue.enqueue(tmp->right)
```



Queue:

Print:

Level-Order Traversal




```
template<class T>
void BinaryTree<T>::lOrder(TreeNode * root)
{

    Queue<TreeNode*> q;
    q.enqueue(root);

    while( q.empty() == False){

        TreeNode* temp = q.head();
        process(temp);

        q.dequeue();

        q.enqueue(temp->left);
        q.enqueue(temp->right);

    }
}
```

Tree Search

How can we improve our ability to search a binary tree?

What do we trade in order to do so?