Data Structures

Graph Traversals

CS 225

Brad Solomon

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UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Department of Computer Science

Changes to Class based on IEF

Will defer ‘off-topic’ questions until after class

Will use announcements channel more.

Will try out weekly ‘bonus videos’ summarizing content

Further reduced MP timeout to 6 hours

The return of the ‘hangout’ channel on Discord

MP release dates are now on lecture’s page

Learning Objectives

Discuss graph traversal algorithms

$|V| = n, |E| = m$

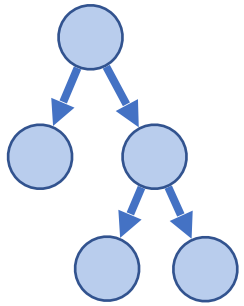
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1*	n*	1*
removeVertex(v)	n+m	n	deg(v)
insertEdge(u, v)	1	1	1*
removeEdge(u, v)	m	1	min(deg(u), deg(v))
incidentEdges(v)	m	n	deg(v)
areAdjacent(u, v)	m	1	min(deg(u), deg(v))

Graph Traversals

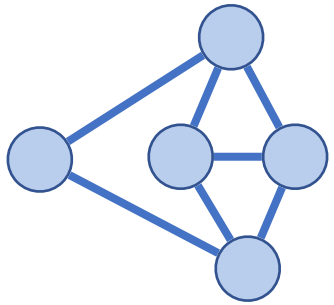
Objective: Visit every vertex and every edge in the graph.

How can we systematically go through a complex graph in the fewest steps?

Tree traversals won't work — lets compare:



- Rooted
- Acyclic
-



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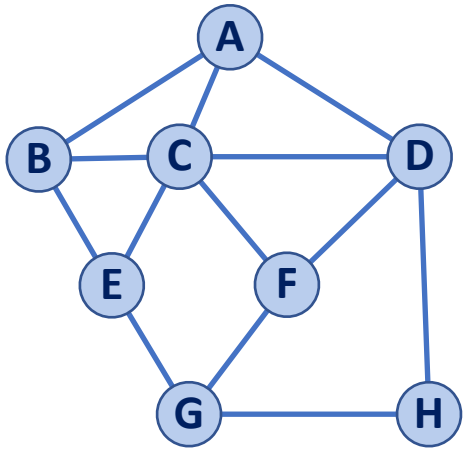
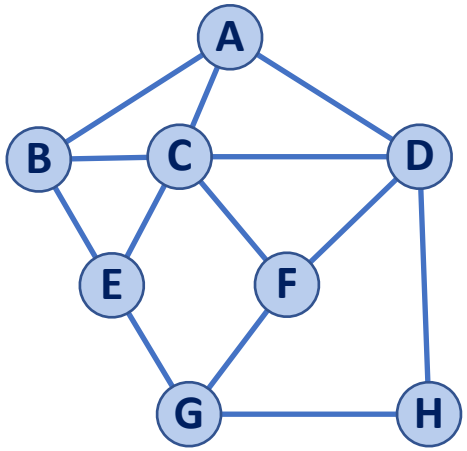
Traversal: BFS



Traversal: BFS



v	d	P	Adjacent Edges
A			B C D
B			A C E
C			A B D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

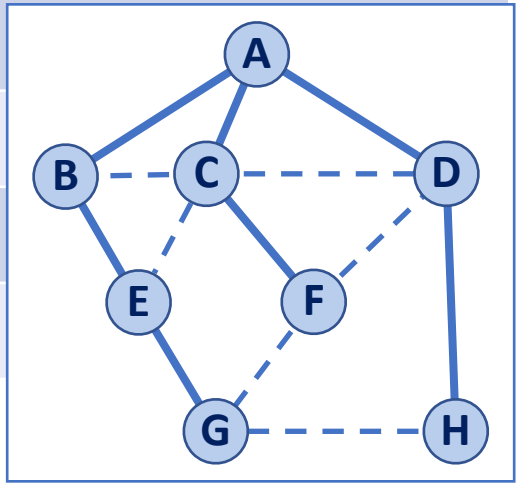
Traversal: BFS

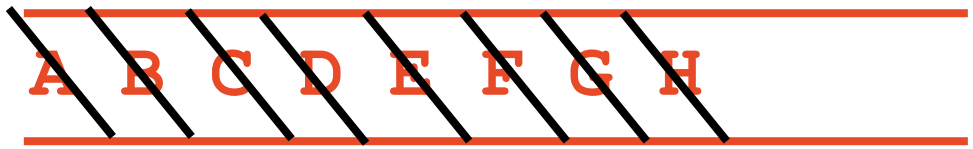
Initialize queue / depth / predecessor

While queue not empty:

- Remove front vertex of queue
- Check if edge connects to new vertex
  - Set dist / pred if new vertex
- Add unvisited edges to queue

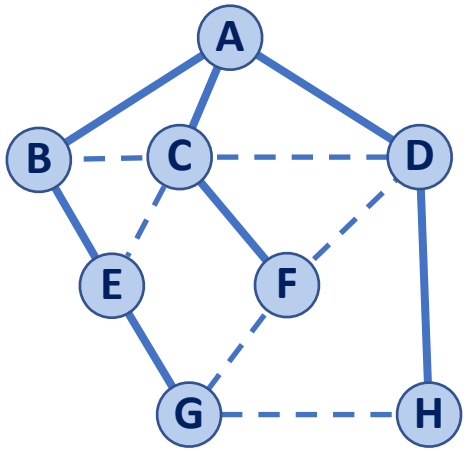
v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	1	A	A B D E F
D	1	A	A C F H
E	2	B	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



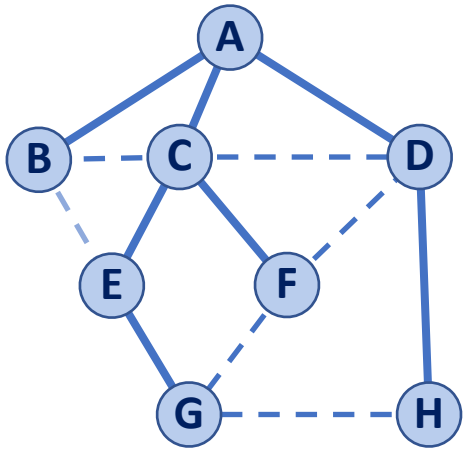


Traversal: BFS

Traversal depends on start position as well as order of edges at each node



v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	1	A	A B D E F



v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	A B D E F

Input: Graph, G

Output: A labeling of the edges in G as discovery or cross

```
 1 BFS(G):
 2   foreach (Vertex v : G.vertices()):
 3    setPred(v, NULL)
 4    setDist(v, -1)
 5
 6   foreach (Edge e : G.edges()):
 7     setLabel(e, UNEXPLORED)
 8
 9   foreach (Vertex v : G.vertices()):
10     if getDist(v) == -1:
11        BFS(G, v)
```

```
1  BFS(G):
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9    foreach (Vertex v : G.vertices()):
10     if getDist(v) == -1:
11        BFS(G, v)
```

```
12 BFS(G, v):
13   Queue q
14   setDist(v, 0)
15   q.enqueue(v)
16
17   while !q.empty():
18     v = q.dequeue()
19
20     foreach (Vertex w : G.adjacent(v)):
21        if( getDist(w) == -1):
22           setLabel((v, w), DISCOVERY)
23           setPred(w, v)
24           setDist(w, v + 1)
25           q.enqueue(w)
26        else:
27           setLabel((v, w), CROSS)
```

```
1  BFS(G):
2    foreach (Vertex v : G.vertices()):
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```

Count connected components?

Cycle Detection?

```
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23           setPred(w, v)
24           setDist(w, v + 1)
25           q.enqueue(w)
26        else:
27           setLabel((v, w), CROSS)
```

BFS Observations

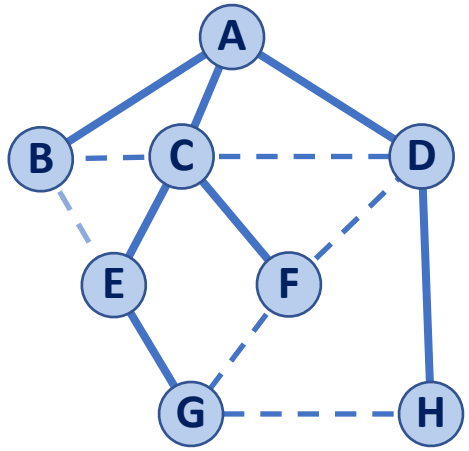
What is the shortest path from **A** to **H**?

What is the shortest path from **E** to **H**?

If my node has distance **d**, do I know anything about the nodes connected by a **cross edge**?

What structure is made from **discovery edges**?

v	d	P	Adjacent Edges
A	0	-	**C B D**
B	1	**A**	**A C E**
C	1	**A**	**B A D E F**
D	1	**A**	**A C F H**
E	2	**C**	**B C G**
F	2	**C**	**C D G**
G	3	**E**	**E F H**
H	2	**D**	**D G**



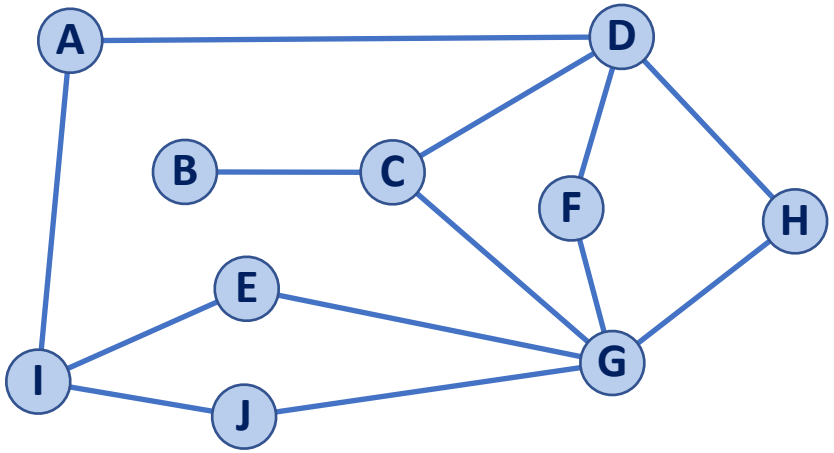
BFS Observations

1. BFS can be used to count components
2. BFS can be used to detect cycles

3. The BFS ‘distance’ value is always the shortest distance from source to any vertex (and the discovery edges form a MST)

4. The endpoints of a cross edge never differ in distance by more than 1 ($\mathbf{|d(u) - d(v)| = 1}$)

Traversal: DFS



```
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2    foreach (Vertex v : G.vertices()):
3      setPred(v, NULL)
4      setDist(v, -1)
5
6    foreach (Edge e : G.edges()):
7      setLabel(e, UNEXPLORED)
8
9    foreach (Vertex v : G.vertices()):
10     if getDist(v) == -1:
11       DFS(G, v)
```
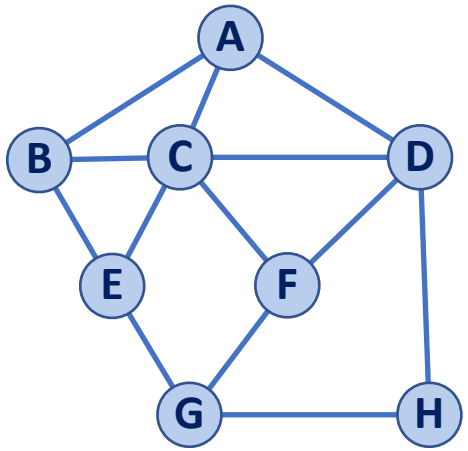
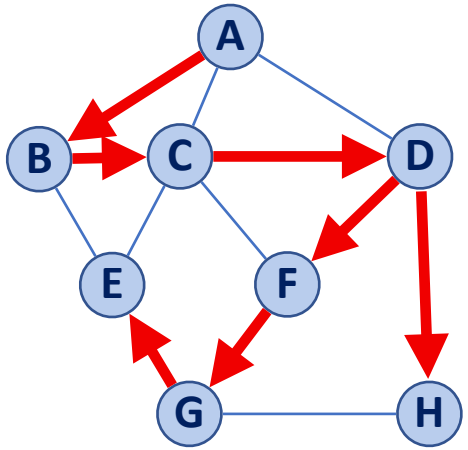

```
12 DFS(G, v):
13
14     foreach (Vertex w : G.adjacent(v)):
15       if( getDist(w) == -1):
16         setLabel((v, w), DISCOVERY)
17         setPred(w, v)
18         setDist(w, v + 1)
19         DFS(G, w)
20       else:
21         setLabel((v, w), BACK)
```



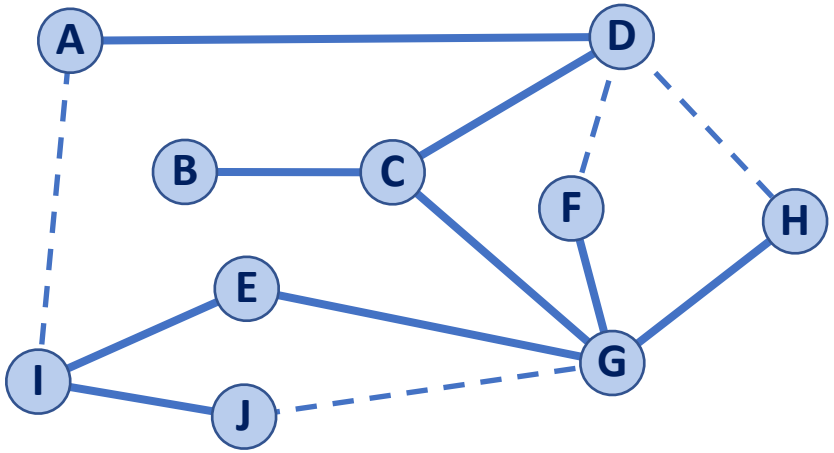
```
12 DFS(G, v):
13
14     foreach (Vertex w : G.adjacent(v)):
15         if( getDist(w) == -1):
16             setLabel((v, w), DISCOVERY)
17             setPred(w, v)
18             setDist(w, v + 1)
19             DFS(G, w)
20         else:
21             setLabel((v, w), BACK)
```



v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	2	B	A B D E F
D	3	C	A C F H
E	6	G	B C G
F	4	D	C D G
G	5	F	E F H
H	4	D	D G

A B C D F G E H

Traversal: DFS



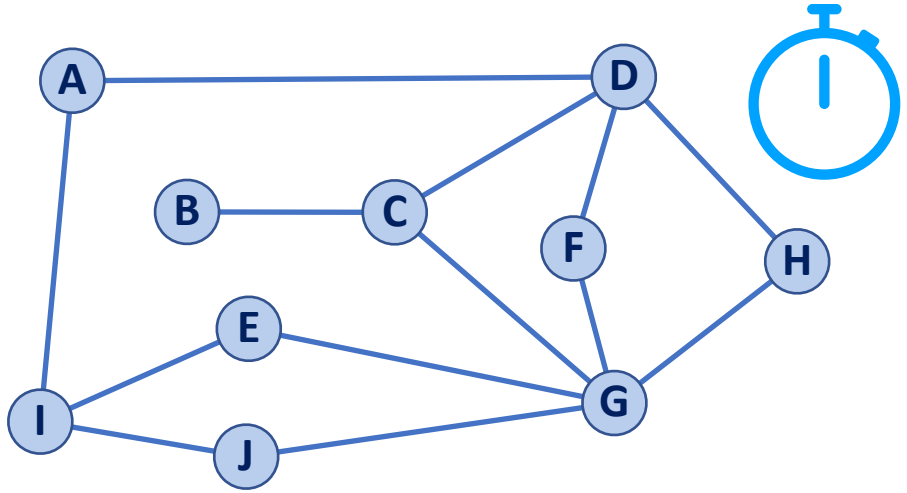
Does distance have meaning here?

Do our edge labels have meaning here?

Discovery Edge

Back Edge

Running time of DFS



Labeling:

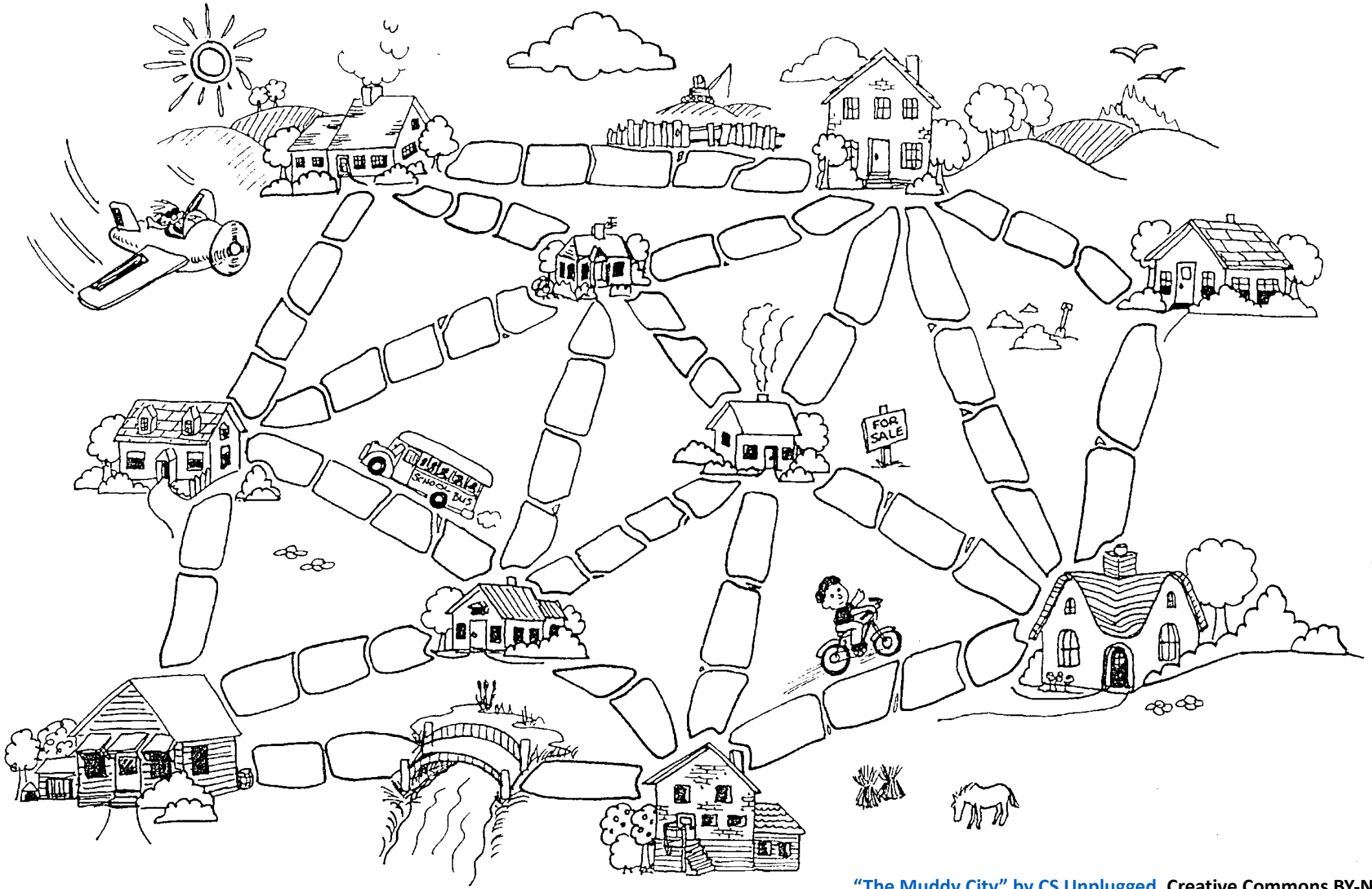
- Vertex:

- Edge:

Queries:

- Vertex:

- Edge:



Minimum Spanning Tree Algorithms

Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

Output: A graph G' with the following properties:

- G' is a spanning graph of G
- G' is a tree (connected, acyclic)
- G' has a minimal total weight among all spanning trees