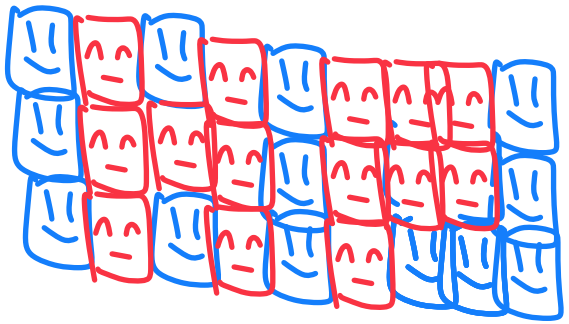
# Data Structures

# KD Tree

CS 225

Brad Solomon

October 2, 2024

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Department of Computer Science

# Informal Early Feedback Released!

A larger anonymous survey designed to give feedback to staff

Collective extra credit opportunity!

Studying what aspects of class are most / least helpful

# Learning Objectives

Past AVL trees

Explore the need and use of range search

Introduce the KD Tree

Go over C++ concepts for mp_mosaics

# Summary of Balanced BST

**AVL Trees**

- Max height: 1.44 * lg(n)
- Rotations:

# Summary of Balanced BST

**AVL Trees**

- Max height: 1.44 * lg(n)
- Rotations:
  - Zero rotations on find
  - One rotation on insert
  - O(**h**) == O(**lg(n)**) rotations on remove

**Red-Black Trees**

- Max height: 2 * lg(n)
- Constant number of rotations on insert (max 2), remove (max 3).

# Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```
std::map<K, V> map;
```

myDict[K] = V;

```
V & std::map<K, V>::operator[]( const K & )
```

↳ find & insert

```
std::map<K, V>::erase( const K & )
```

# Red-Black Trees in C++


C++ provides us a balanced BST as part of the standard library:

```
iterator std::map<K, V>::lower_bound( const K & );
```

↳ first item equal to k or larger than k

```
iterator std::map<K, V>::upper_bound( const K & );
```

↳ smallest value greater than k

# Summary of Balanced BST

**Pros:**

O(log N) for insert, find, remove

**Optimal range queries in 1D**

**Cons:**

O(log N) isn't that great

Large in-memory requirement

# Range-based Searches

Consider a collection of points on a 1D line: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

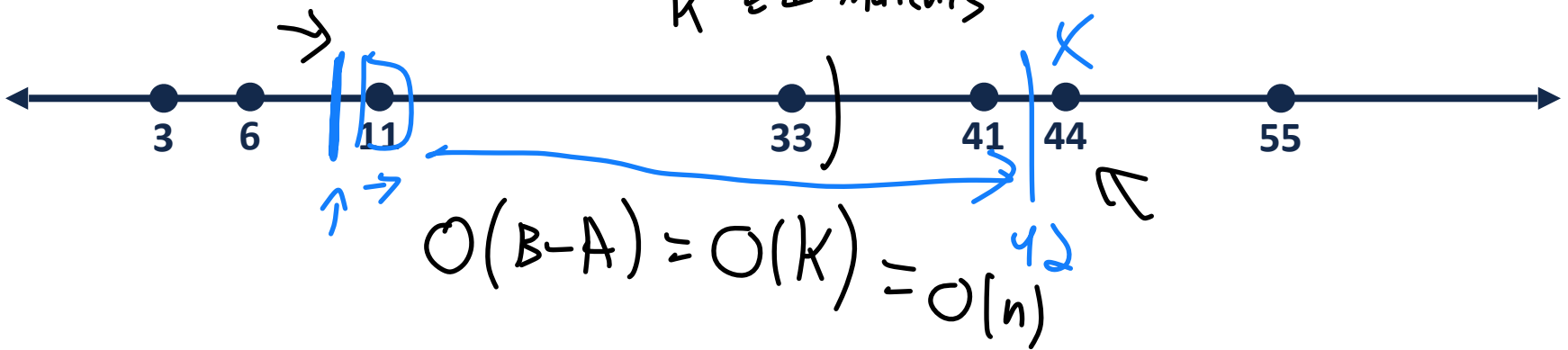
If I want to find all values between [A, B], how could I implement this?

A = 10
B = 42

1) Start with naive
↳ Loop through every item, check if in range

2) Sort list & find first match (then walk)
↳ O(n log n)
↳ Binary search O(log n)
O(n)

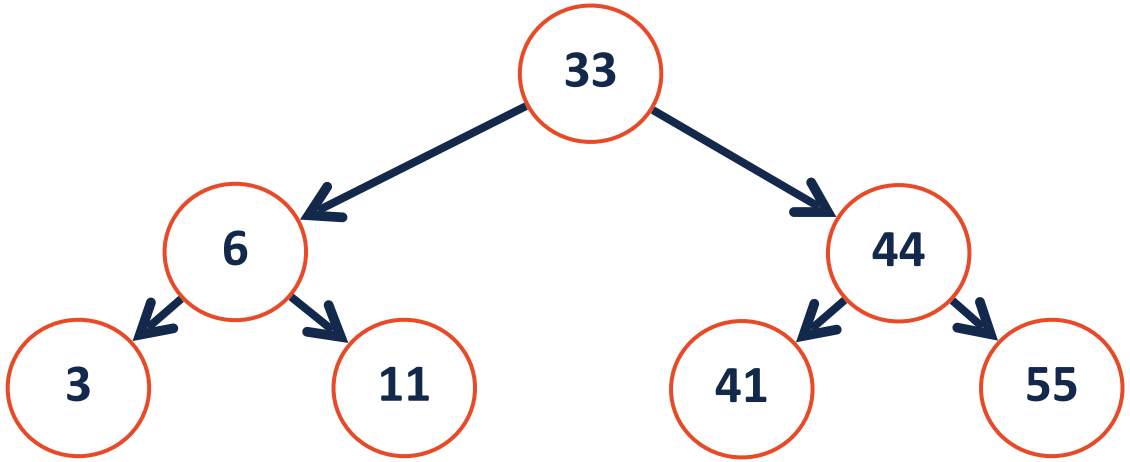
A = -∞
B = ∞



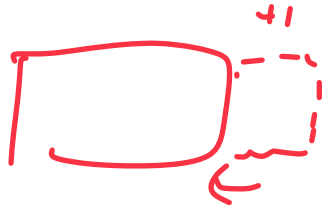
# Range-based Searches

Consider a collection of points on a 1D line: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

If I want to find all values between [A, B], how could I implement this?



# Range-based Searches

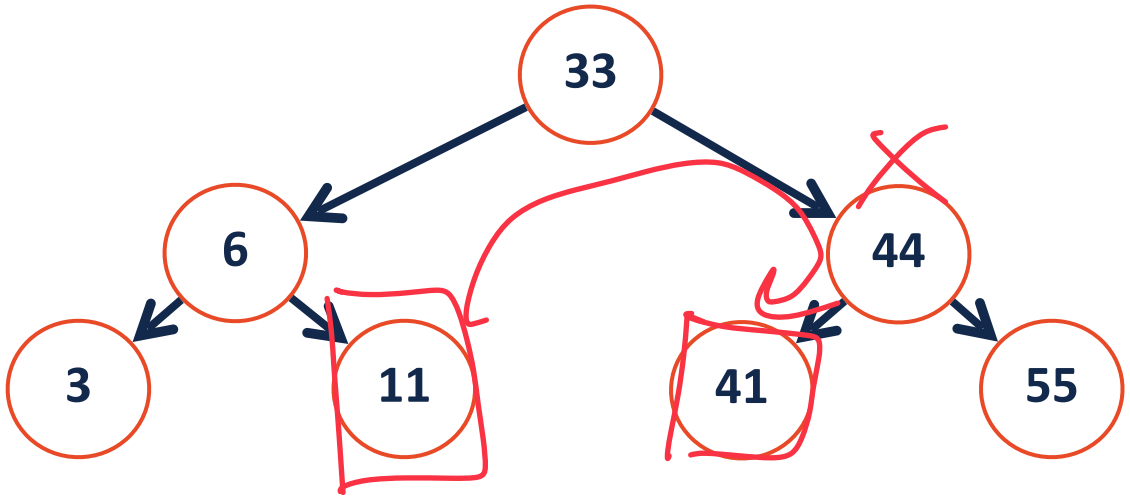


Consider a collection of points on a 1D line: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

If I want to find all values between [A, B], how could I implement this?

A = 10

B = 42




Begin()

end()

```
1
2 for(auto it = myMap.lower_bound(A); it != myMap.upper_bound(B); ++it){
3
4 // Do Stuff
5 }
```

# Range-based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

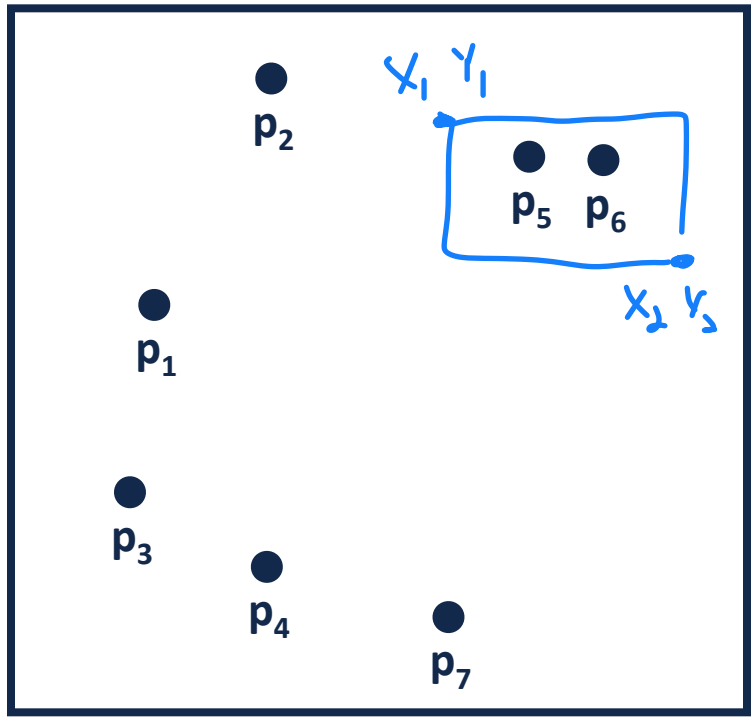
What points in rectangle [ $\mathbf{(x_1, y_1)}$, $\mathbf{(x_2, y_2)}$ ]?

Naive: Loop through all points, check if in range

Brainstorm:

↳ Split into two ranges (1D) X & Y ↔ ↕

↳ Must be in X range & Y range



# Range-based Searches

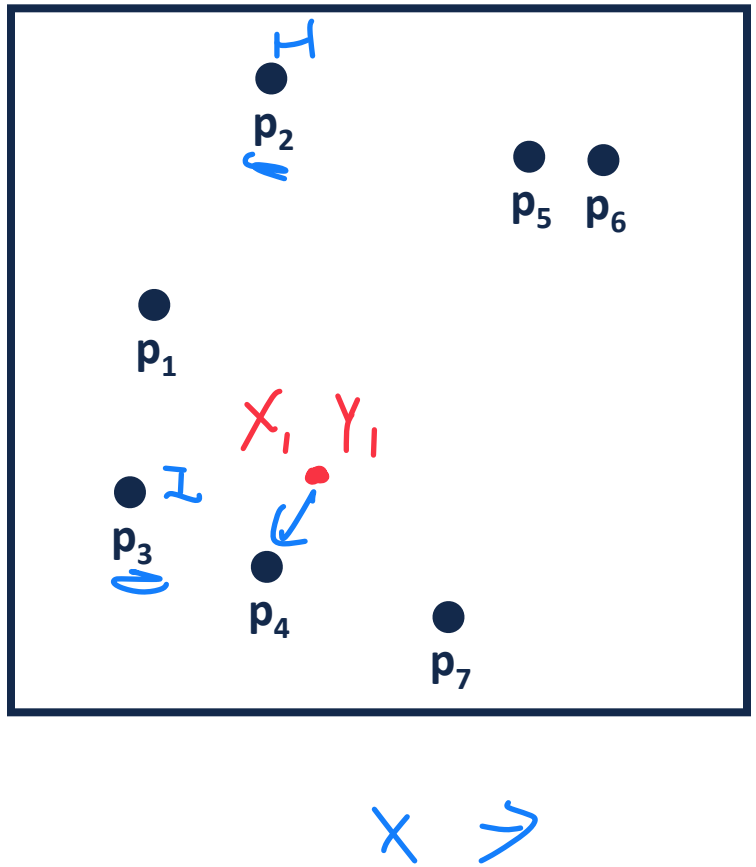
Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

What is nearest point to $\mathbf{(x_1, y_1)}$?

Naive: Look at every point, calc dist, store min

Split into x & y tests

↳ Doesn't work!



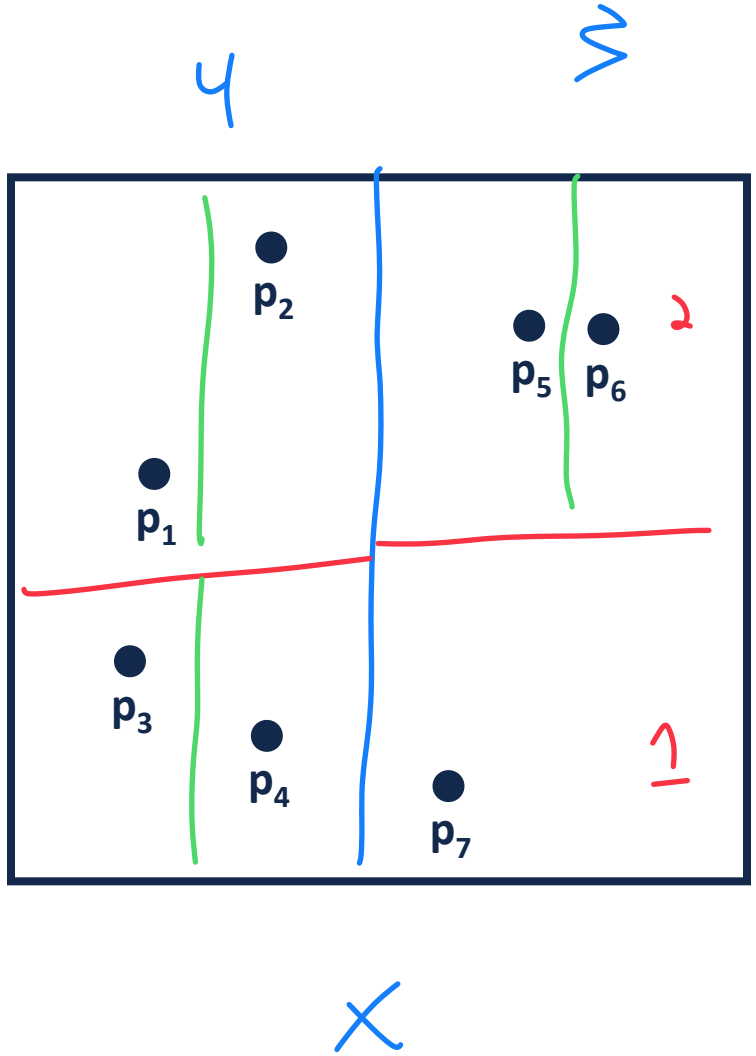
# Range-based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$

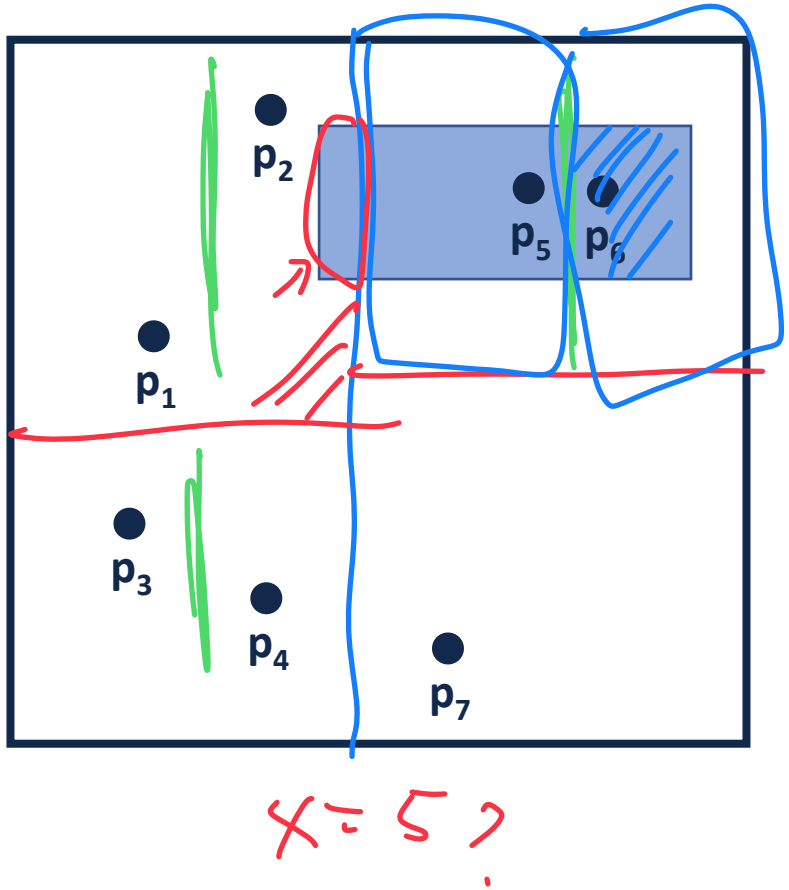
**Tree Construction:**

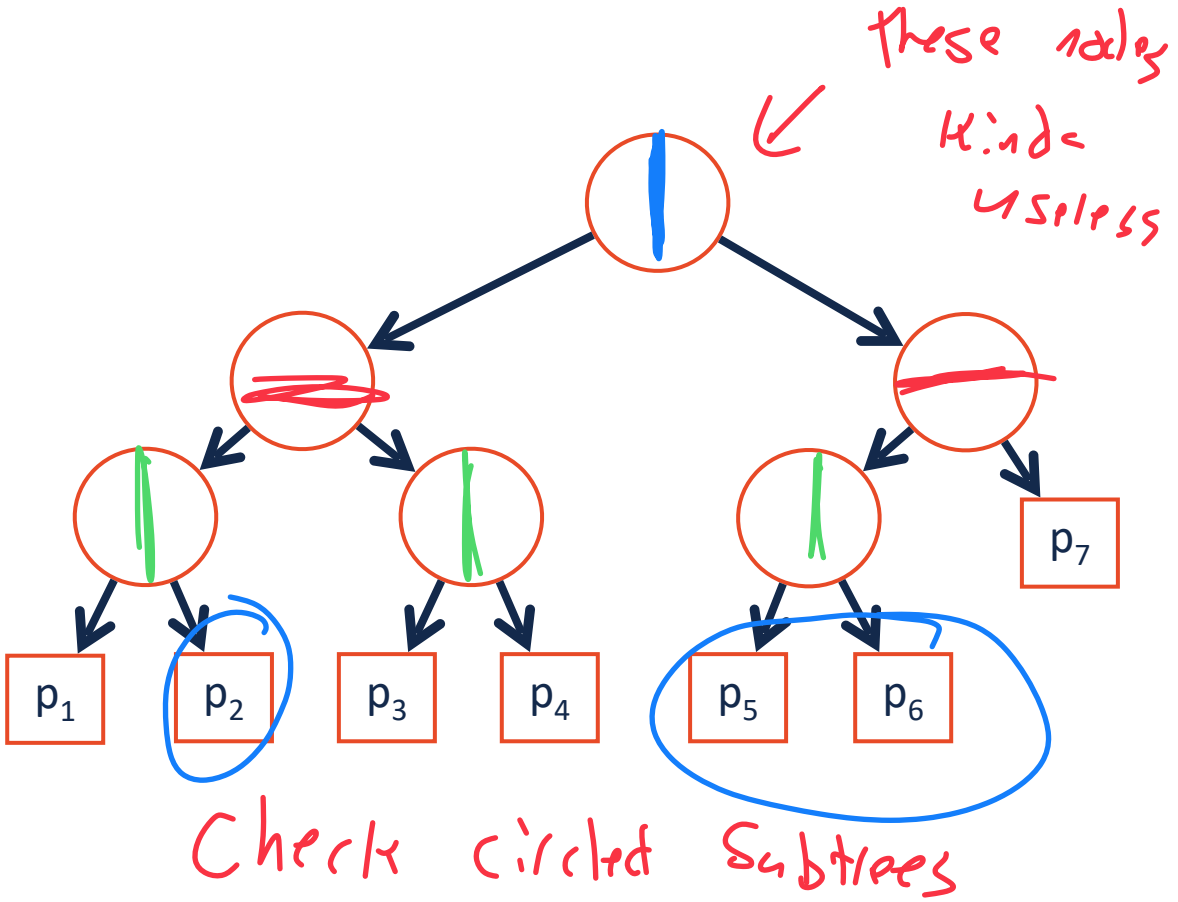
Split space down middle (# of nodes)

↳ Alternate dim

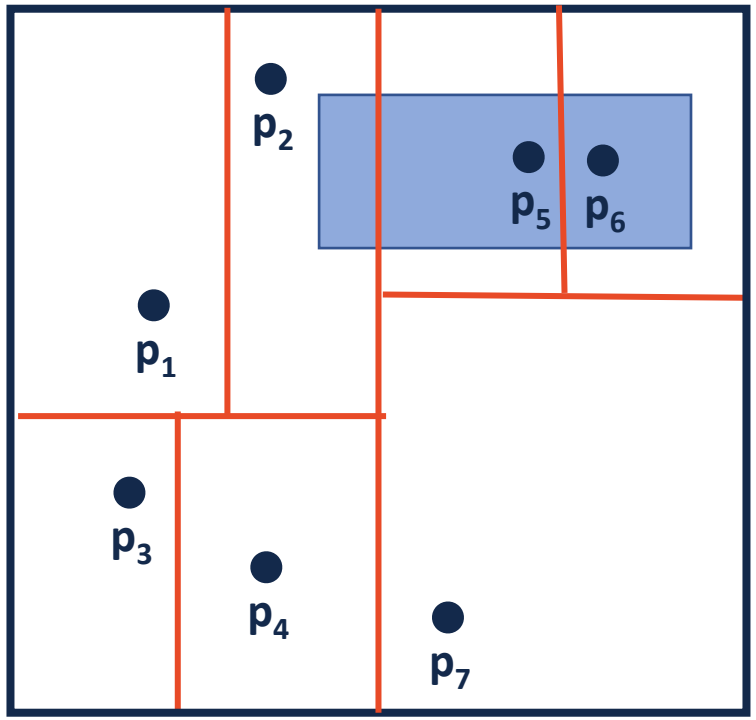


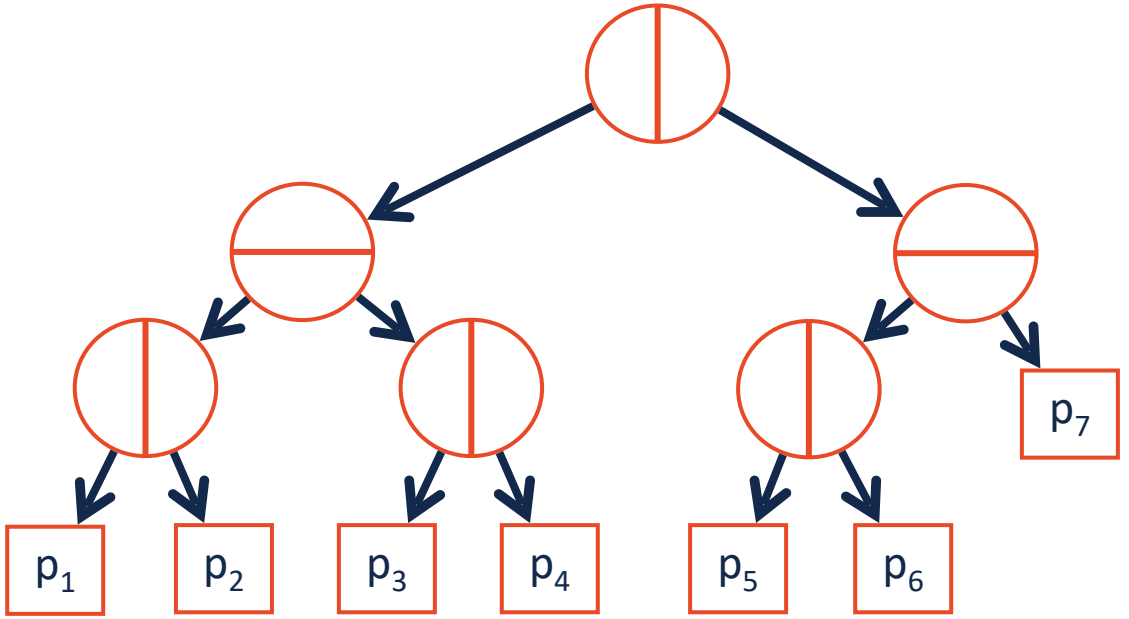
# Range-based Searches



# Range-based Searches



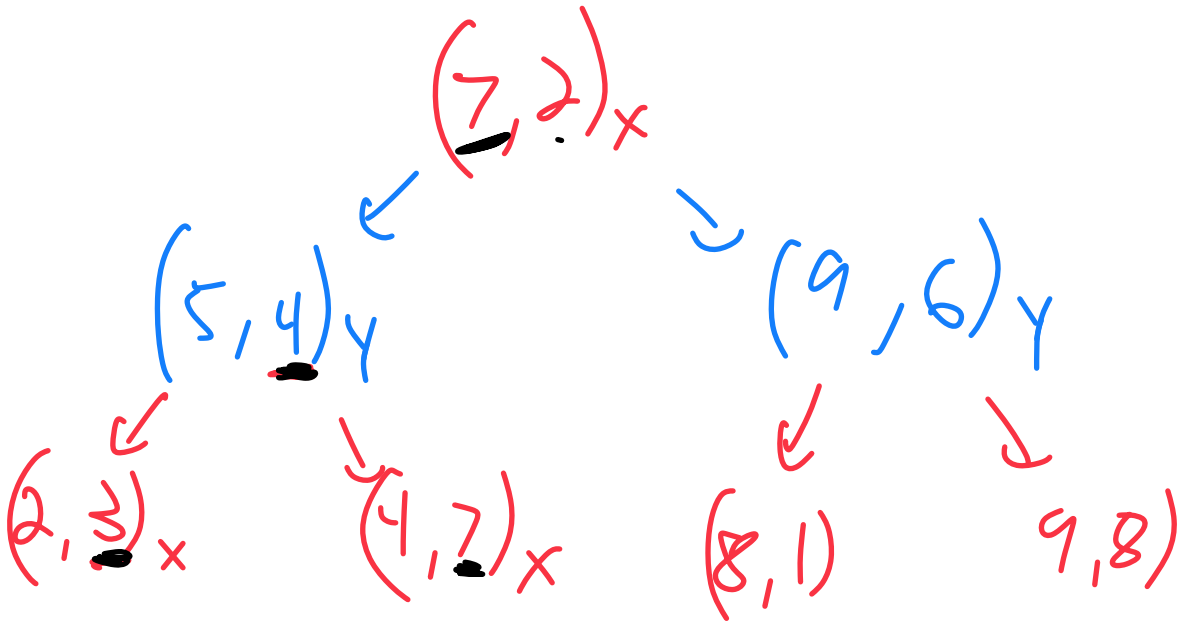



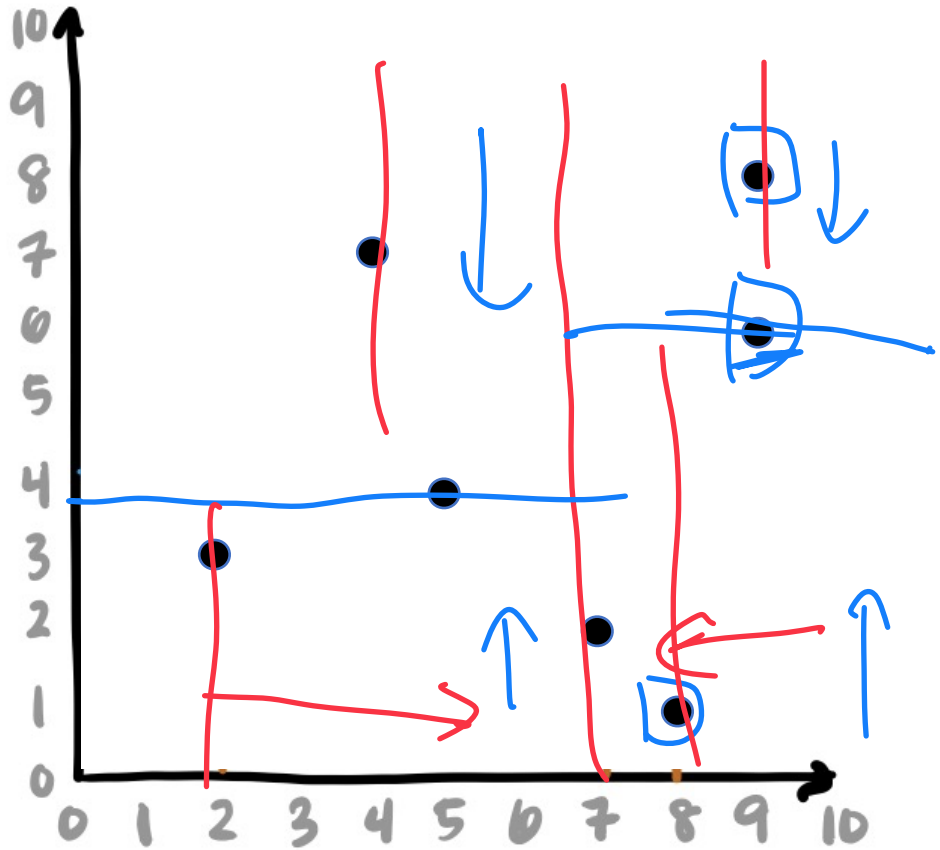
Find median point in x

# Nearest Neighbor: k-d tree

A **k-d tree** is similar but splits on points:

(7,2), (5,4), (9,6), (4,7), (2,3), (8,1), (9,8)




why is (4,7) to the right of (5,4)?

# Nearest Neighbor: k-d tree

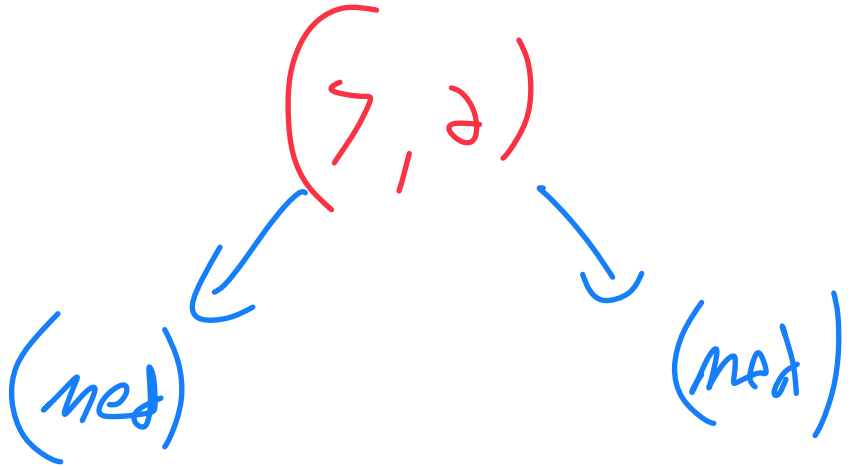
k-dimensions

x, y, z, q, ...

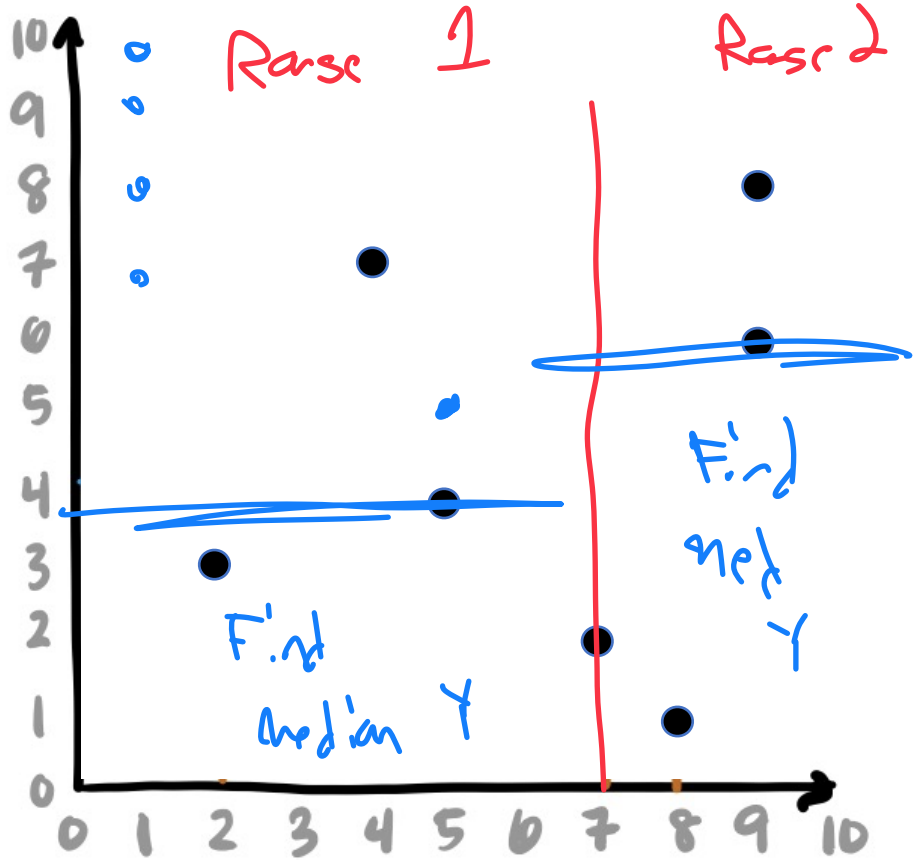
Alt splitting dimensions

A **k-d tree** is similar but splits on points:

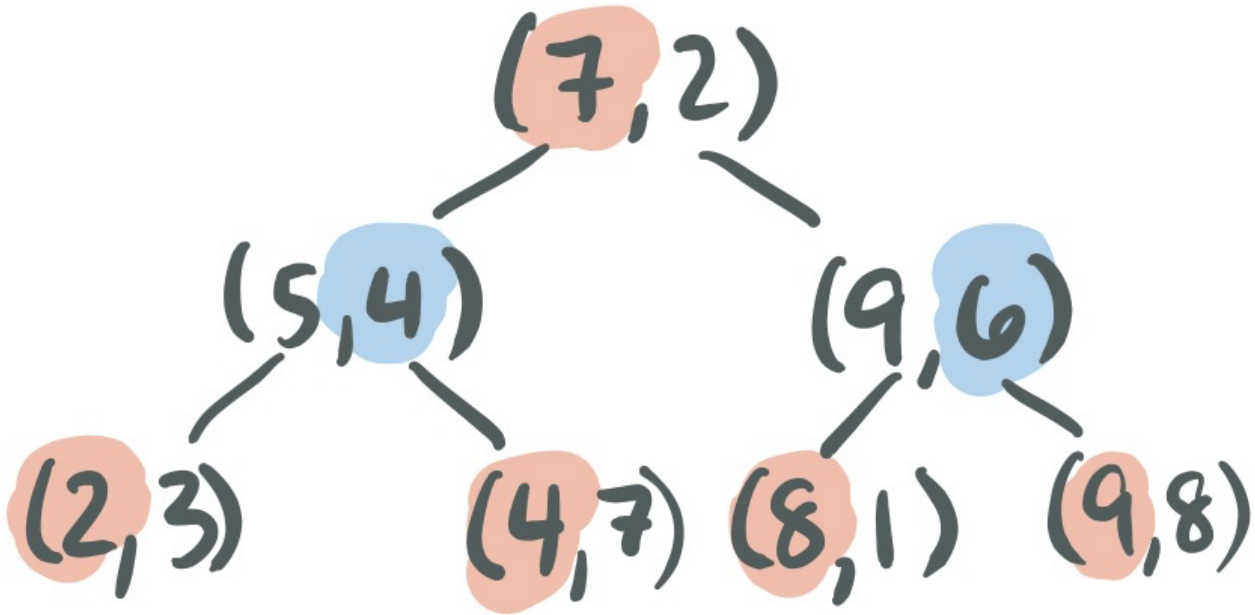
(7,2), (5,4), (9,6), (4,7), (2,3), (8,1), (9,8)

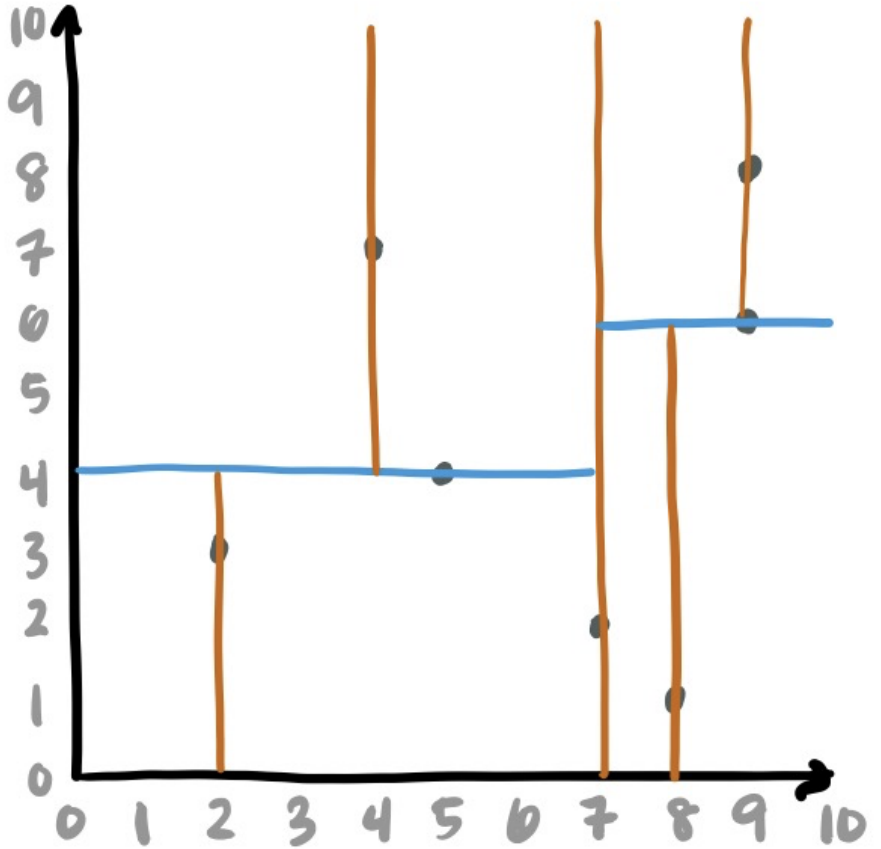



This is good for height!



# Nearest Neighbor: k-d tree

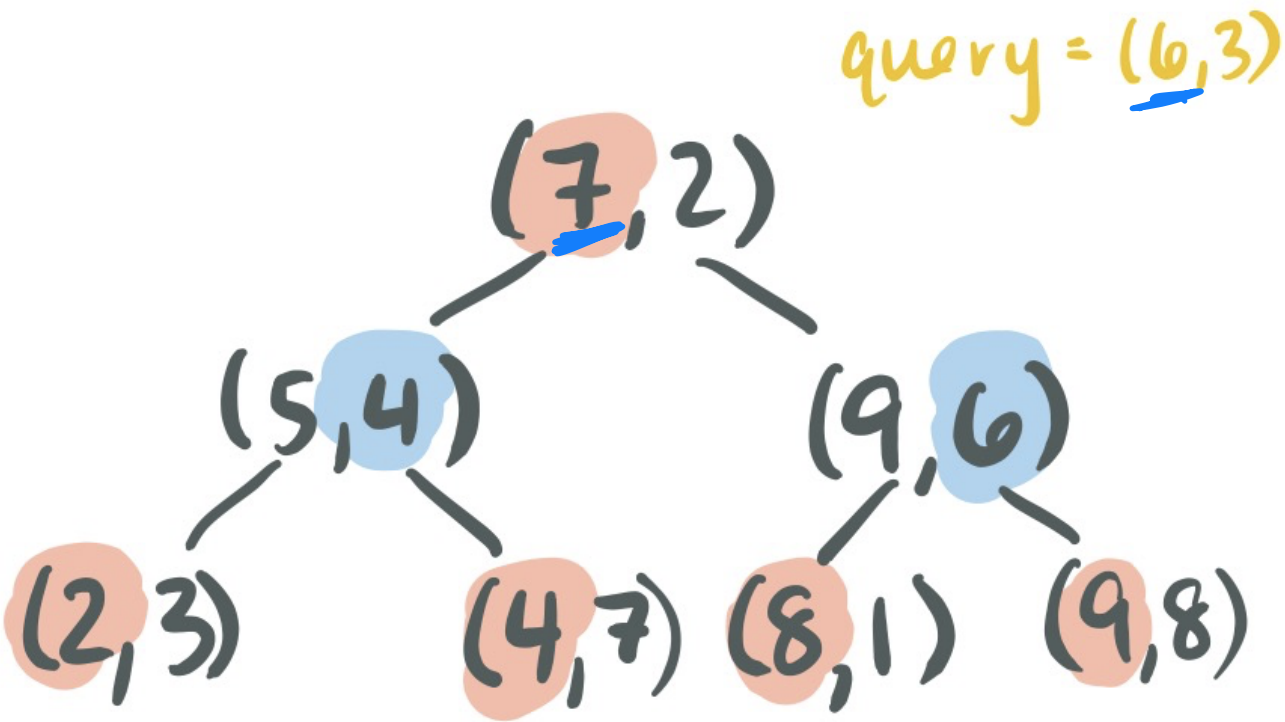


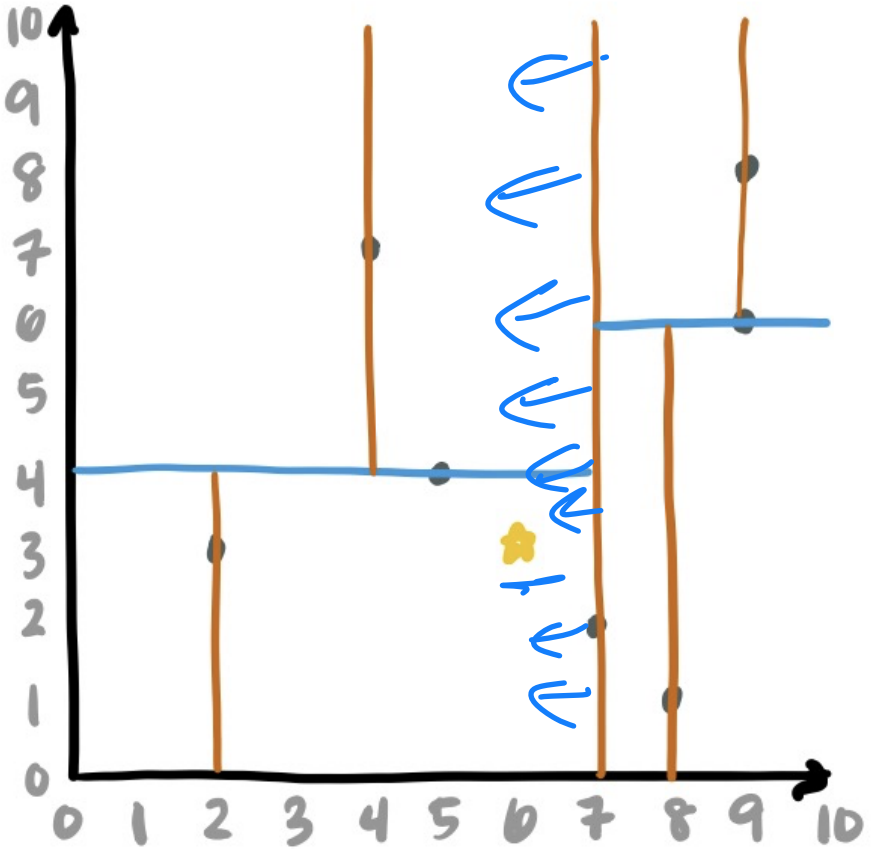



# Nearest Neighbor: k-d tree

check alt dimensions

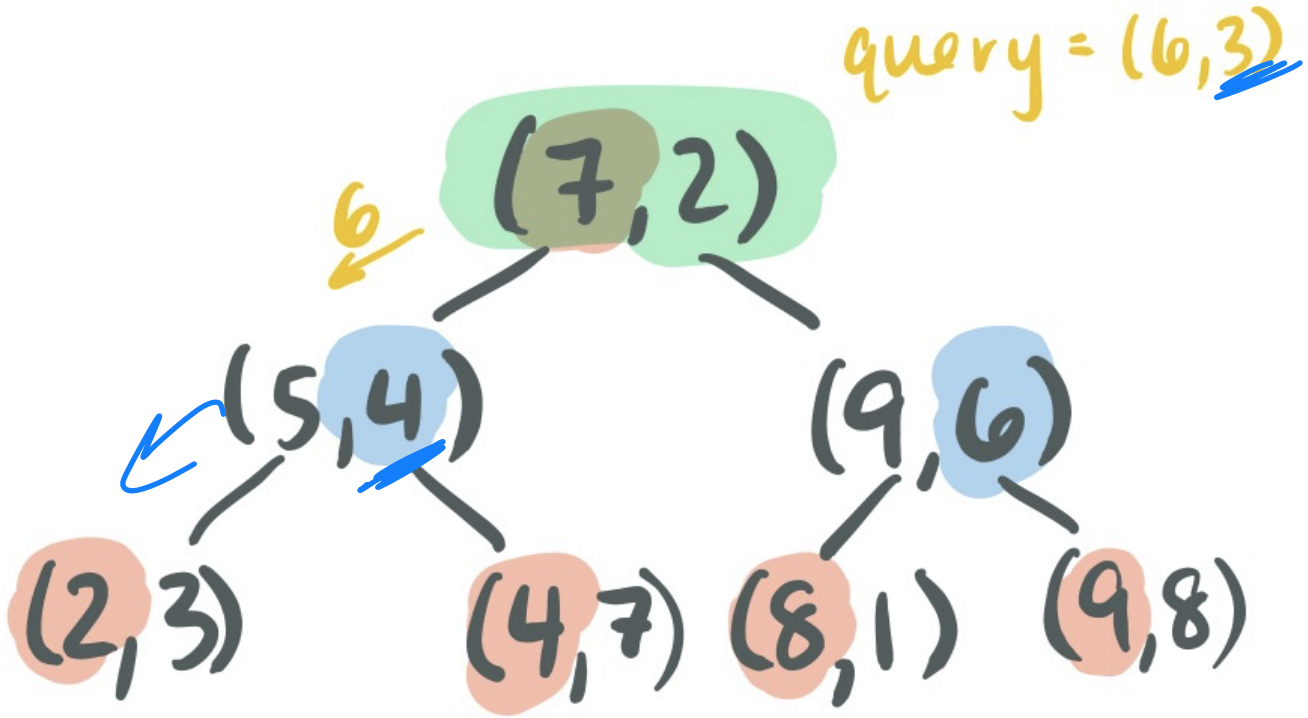
When querying a k-d tree, it acts like a BST* at first…

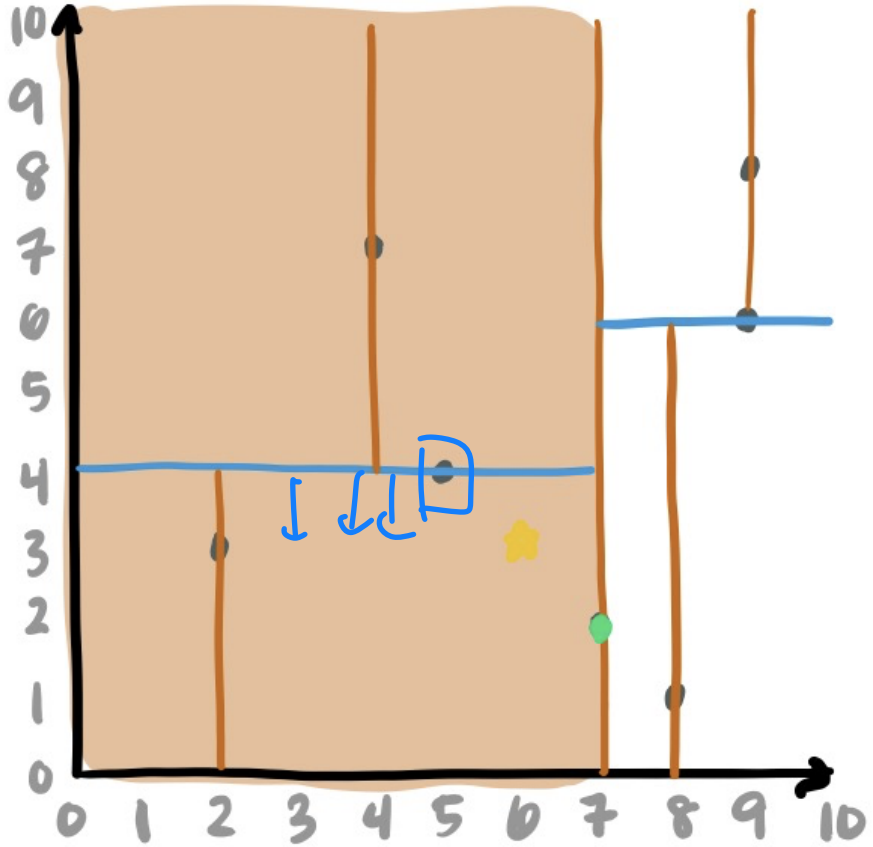


# Nearest Neighbor: k-d tree

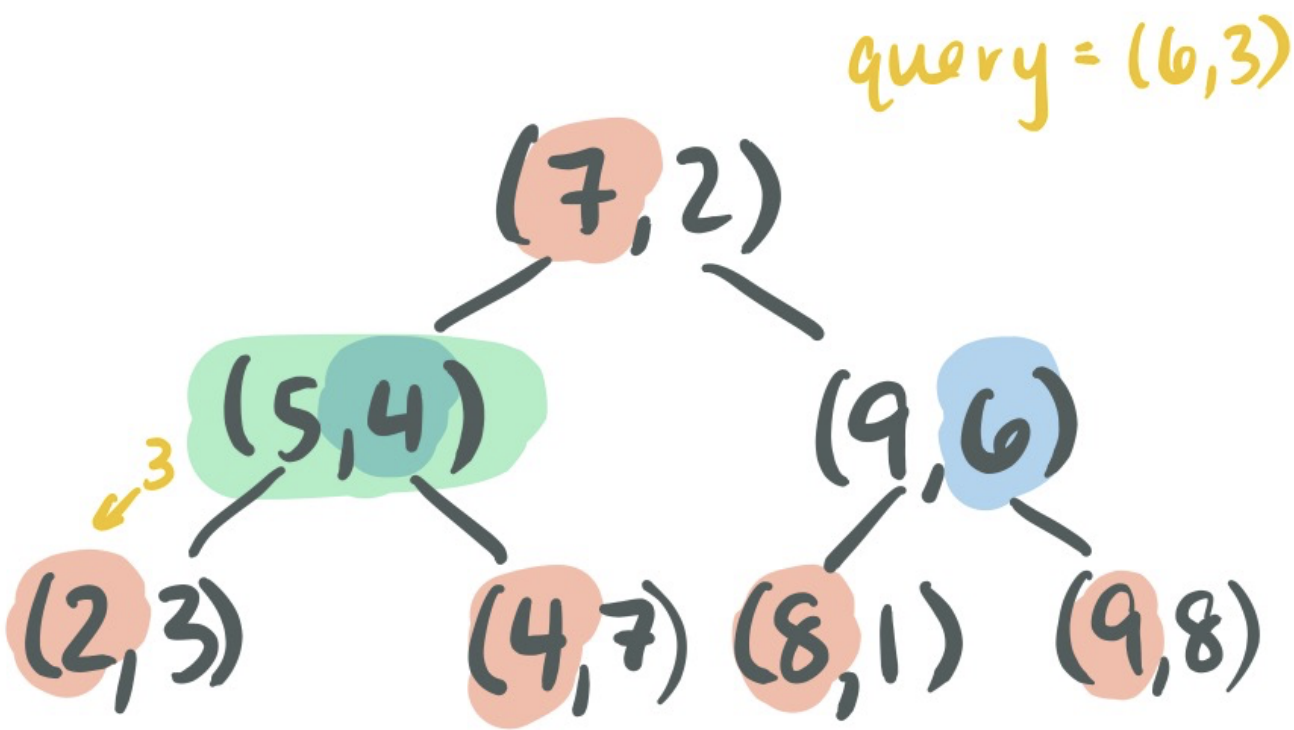
When querying a k-d tree, it acts like a BST* at first…

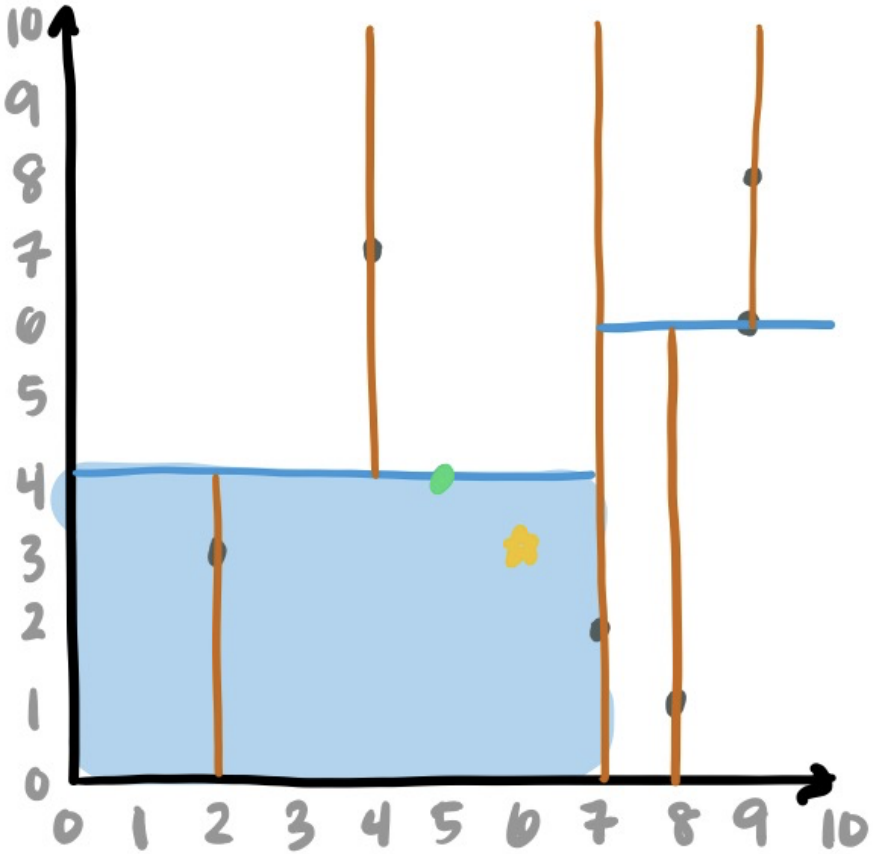


# Nearest Neighbor: k-d tree

When querying a k-d tree, it acts like a BST* at first…

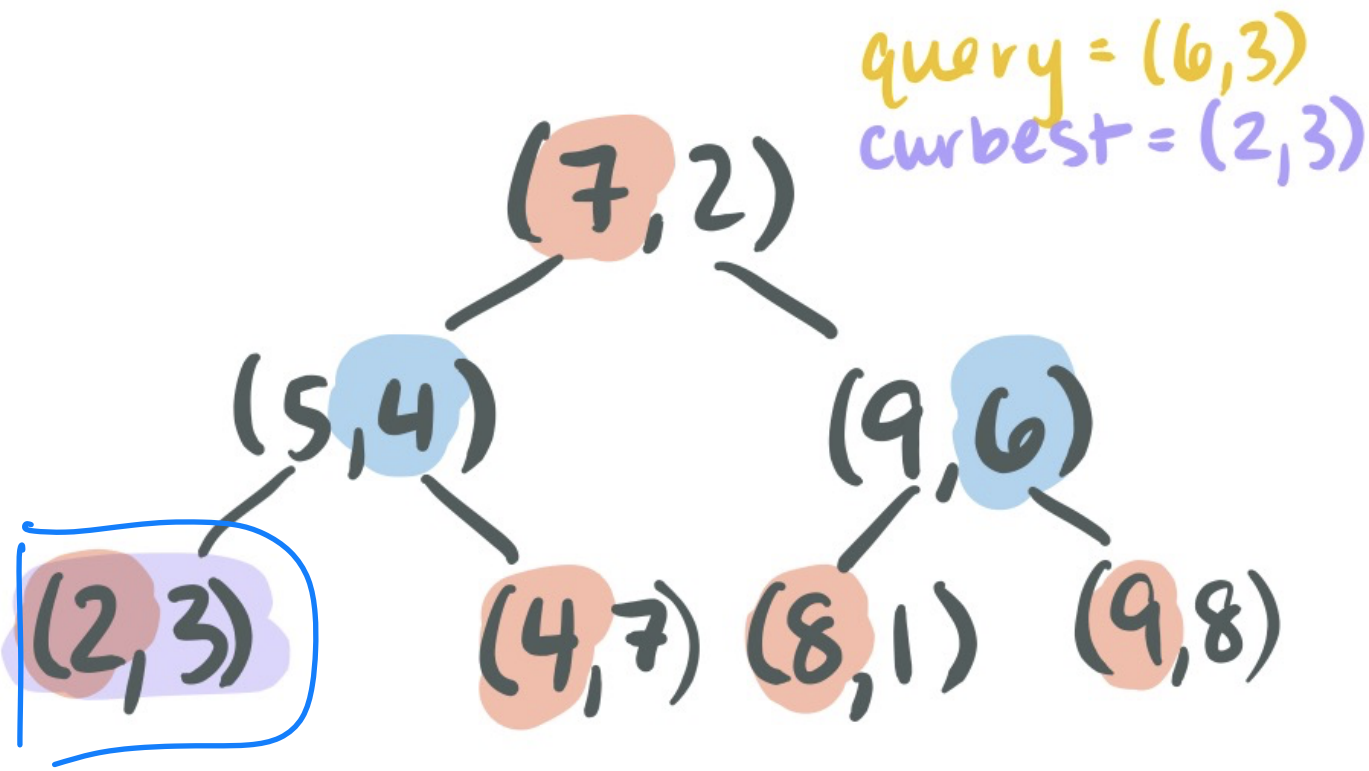


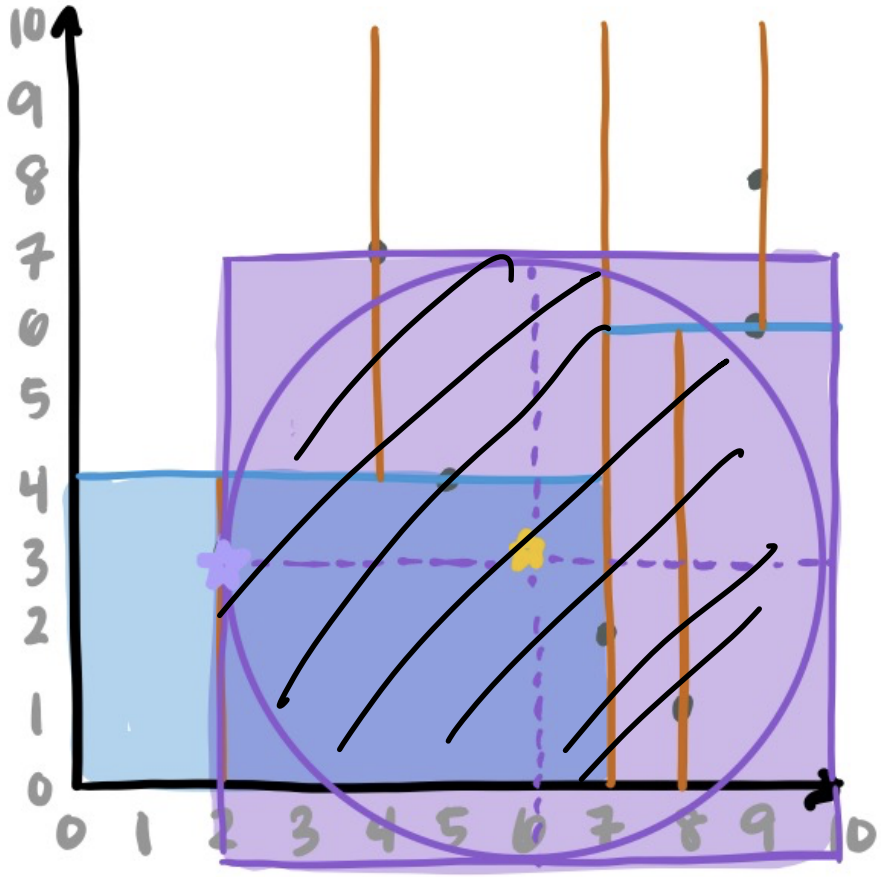



# Nearest Neighbor: k-d tree

When querying a k-d tree, it acts like a BST* at first…

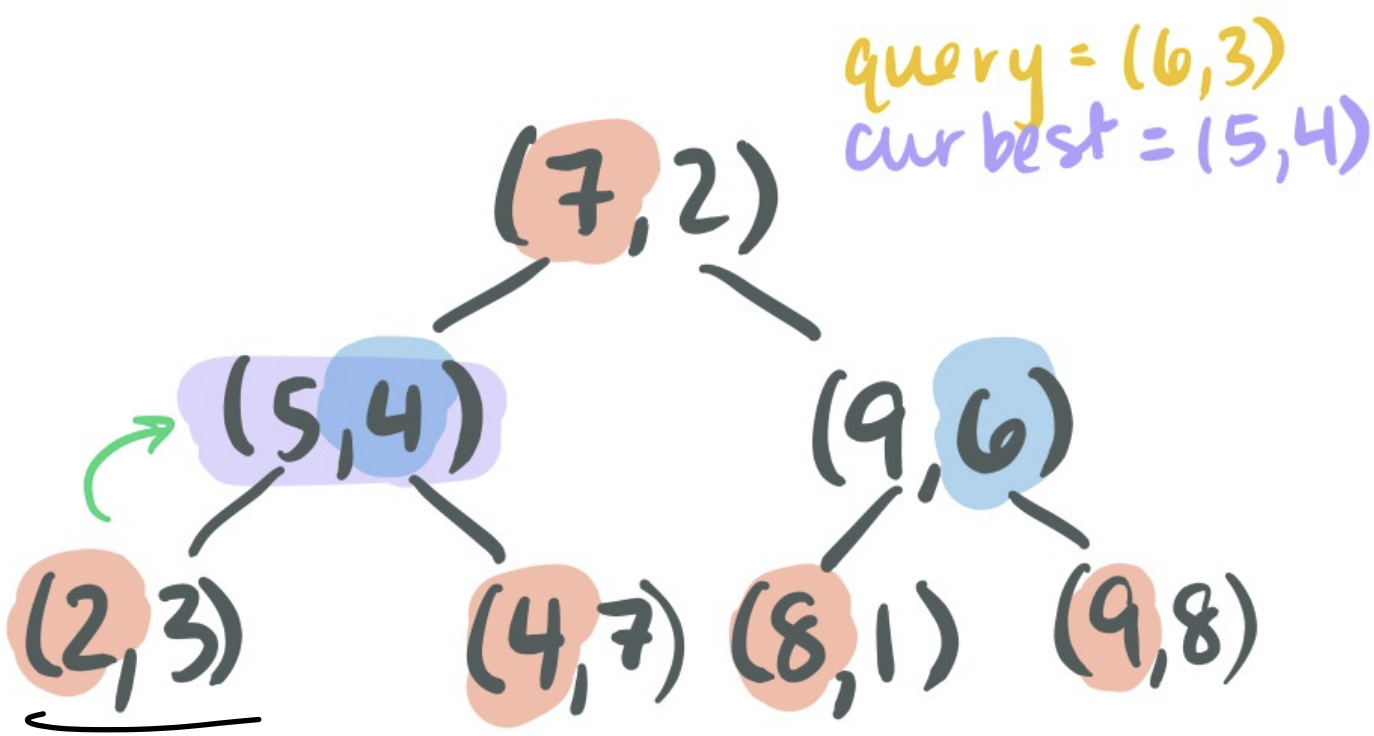
… But if we dont find exact match, have to find nearest neighbor

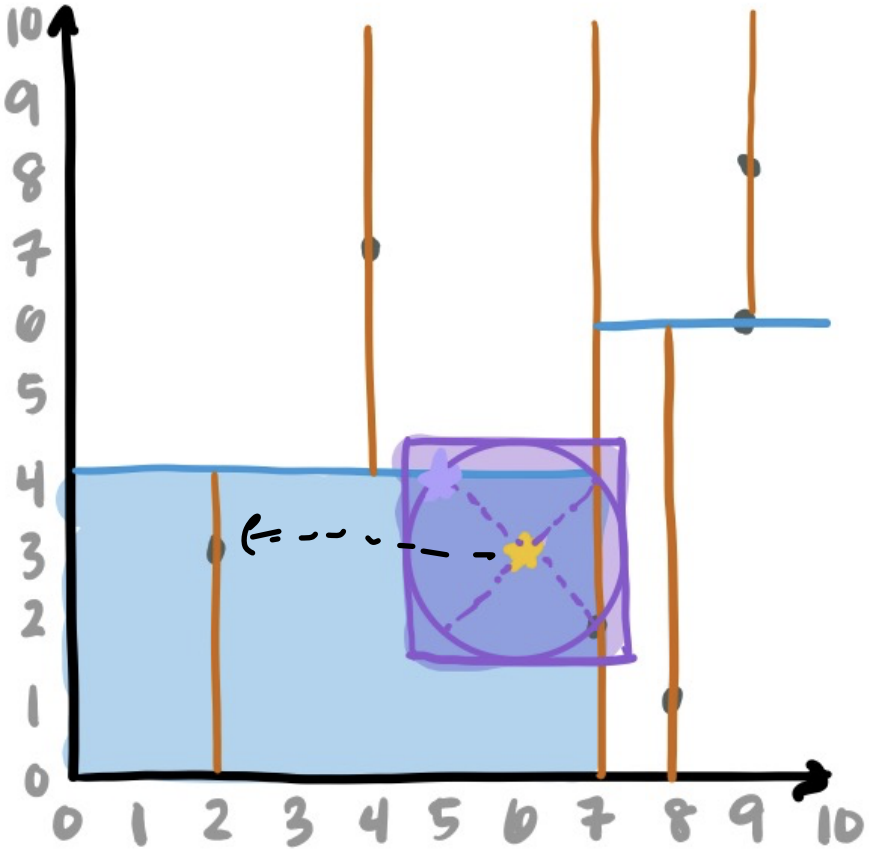


# Nearest Neighbor: k-d tree

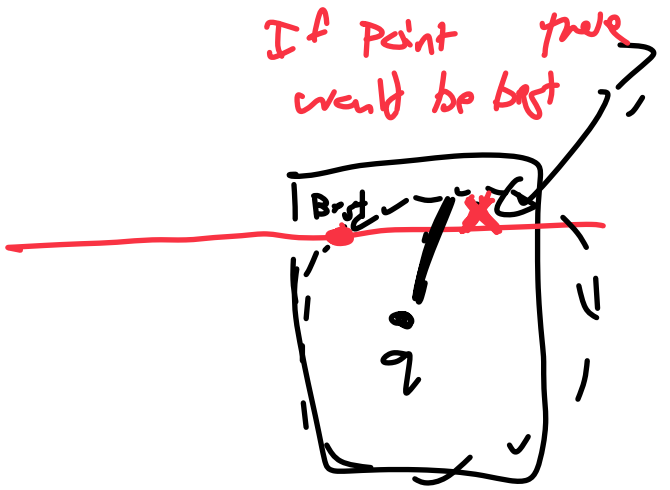
**Backtracking:** start recursing backwards -- store "best" possibility as you trace back



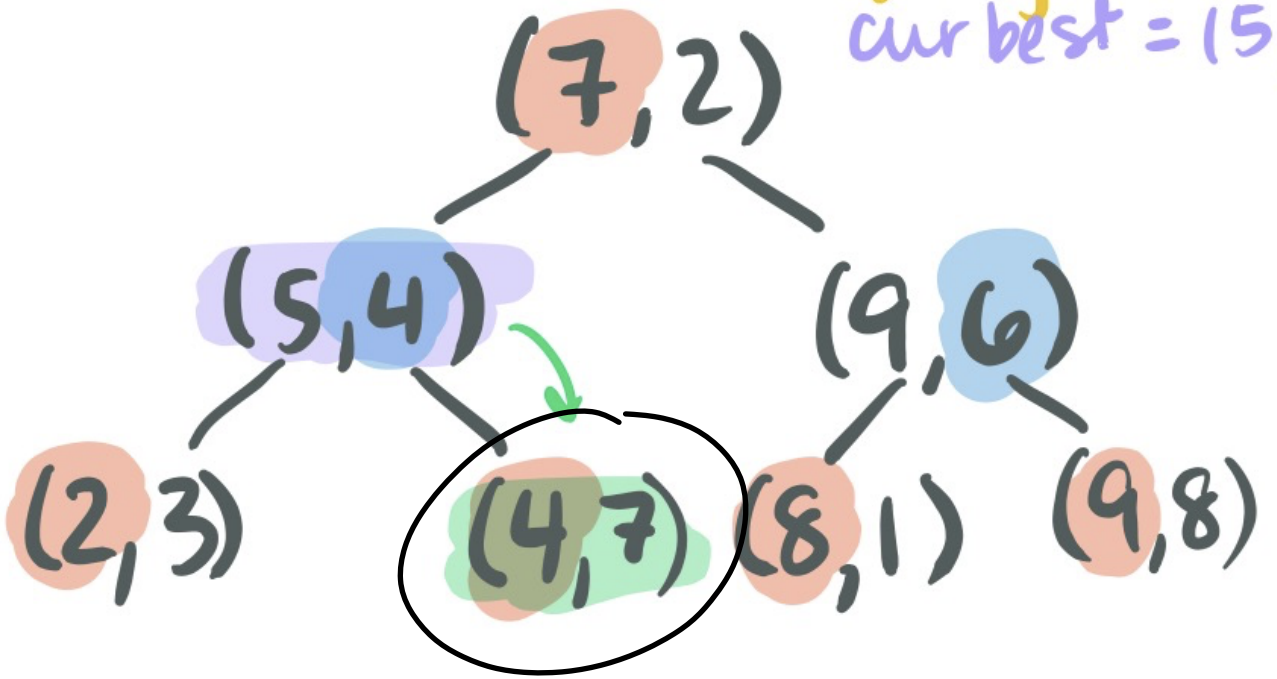



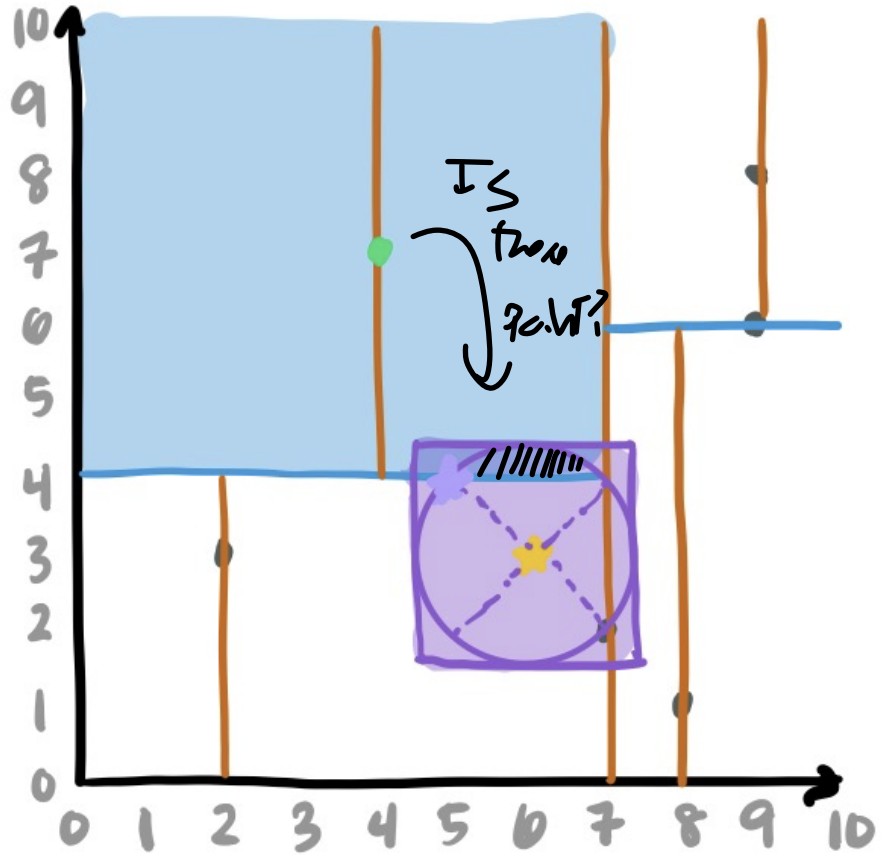
# Nearest Neighbor: k-d tree

Stopped here
will redo Friday!



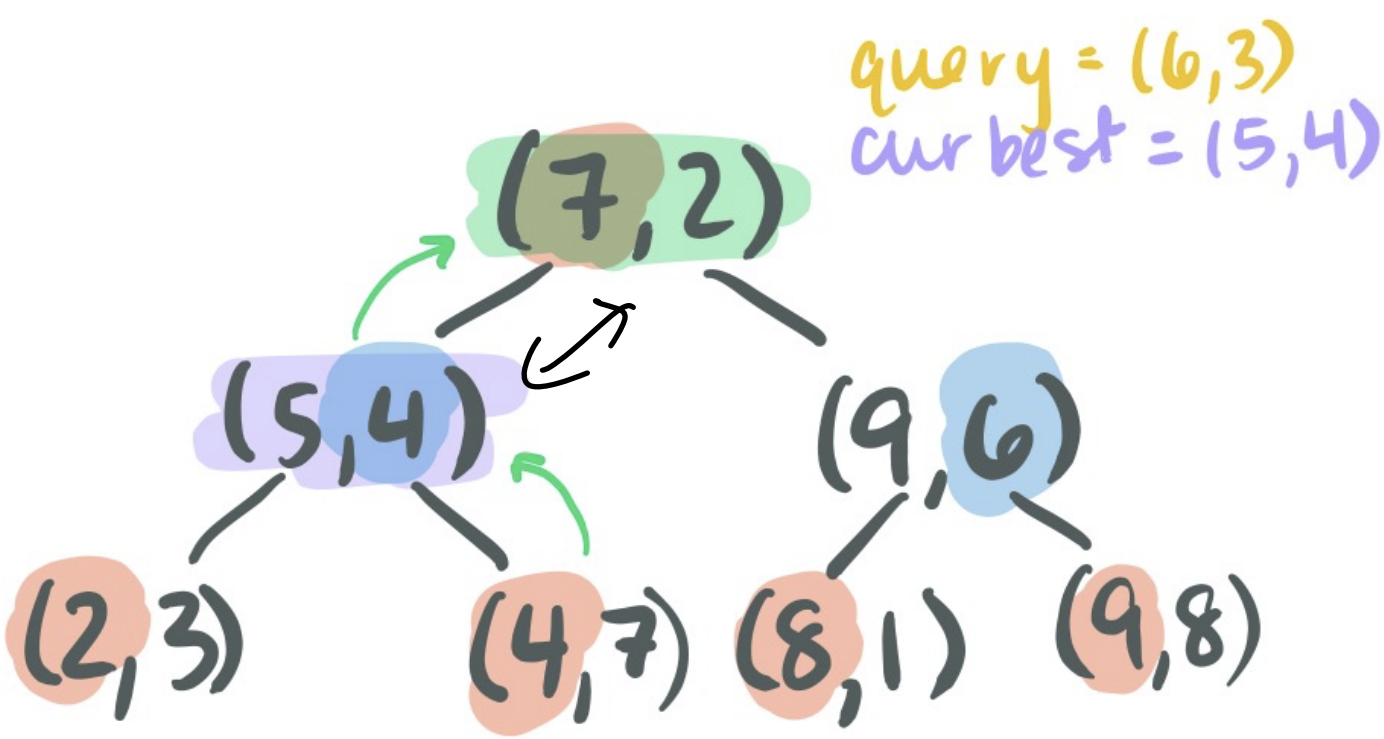

query = (6,3)
cur best = (5,4)

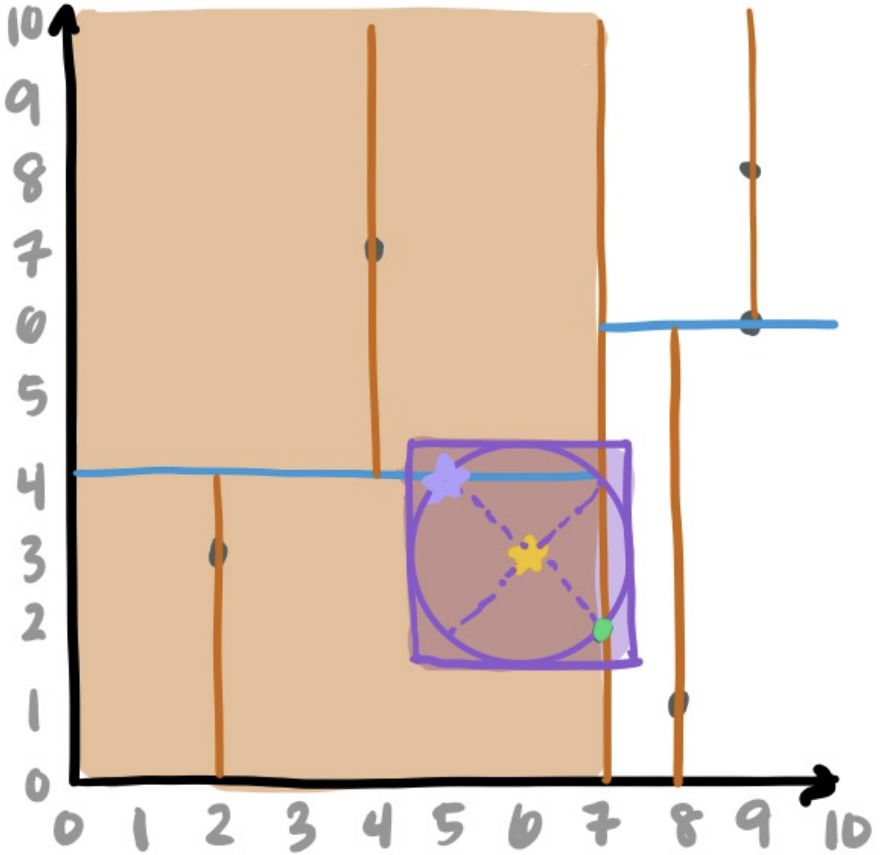


# Nearest Neighbor: k-d tree

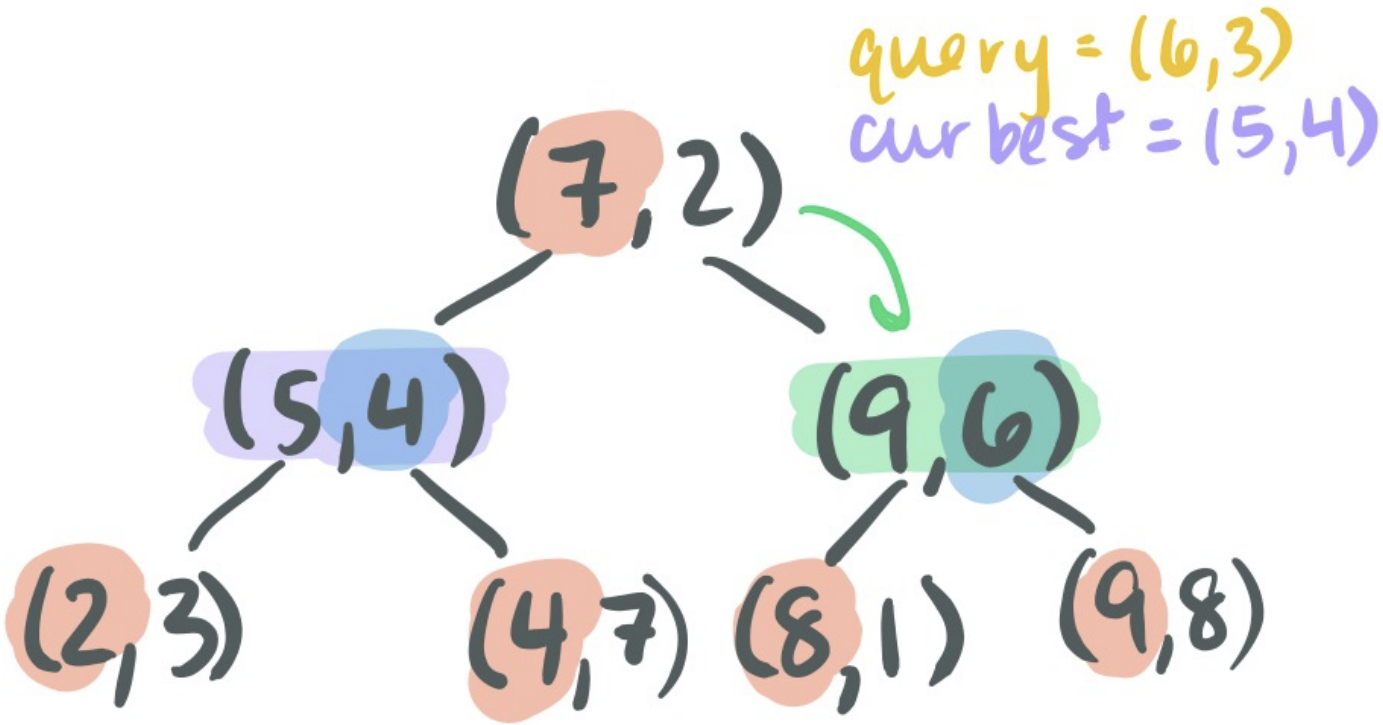
On ties, use smallerDimVal to determine which point remains curBest

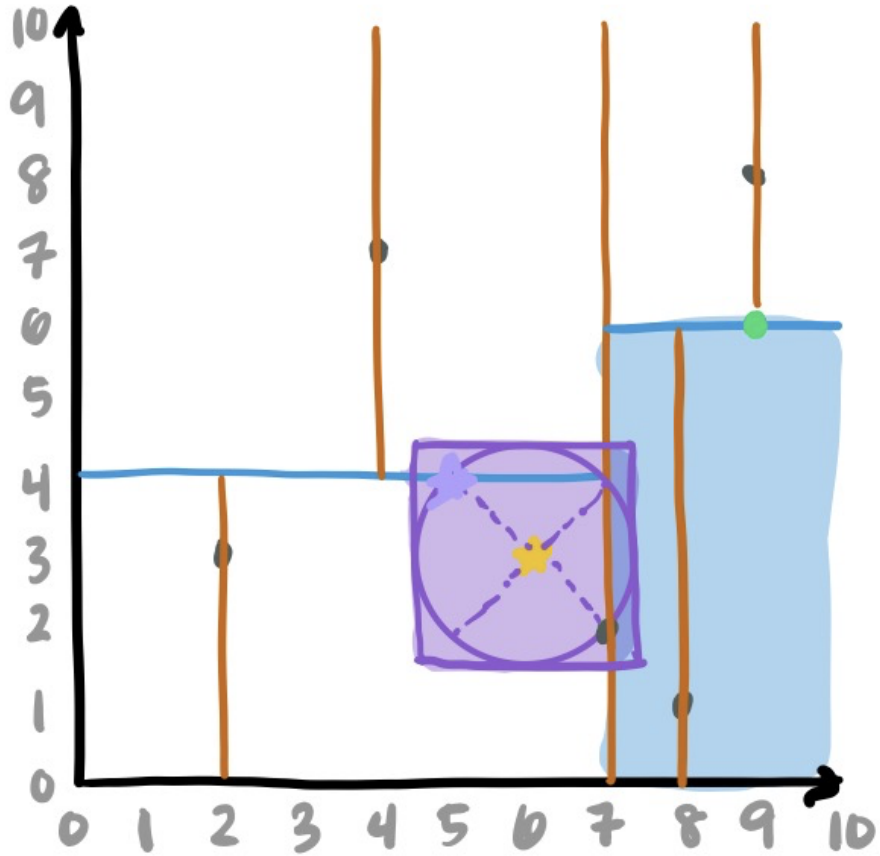


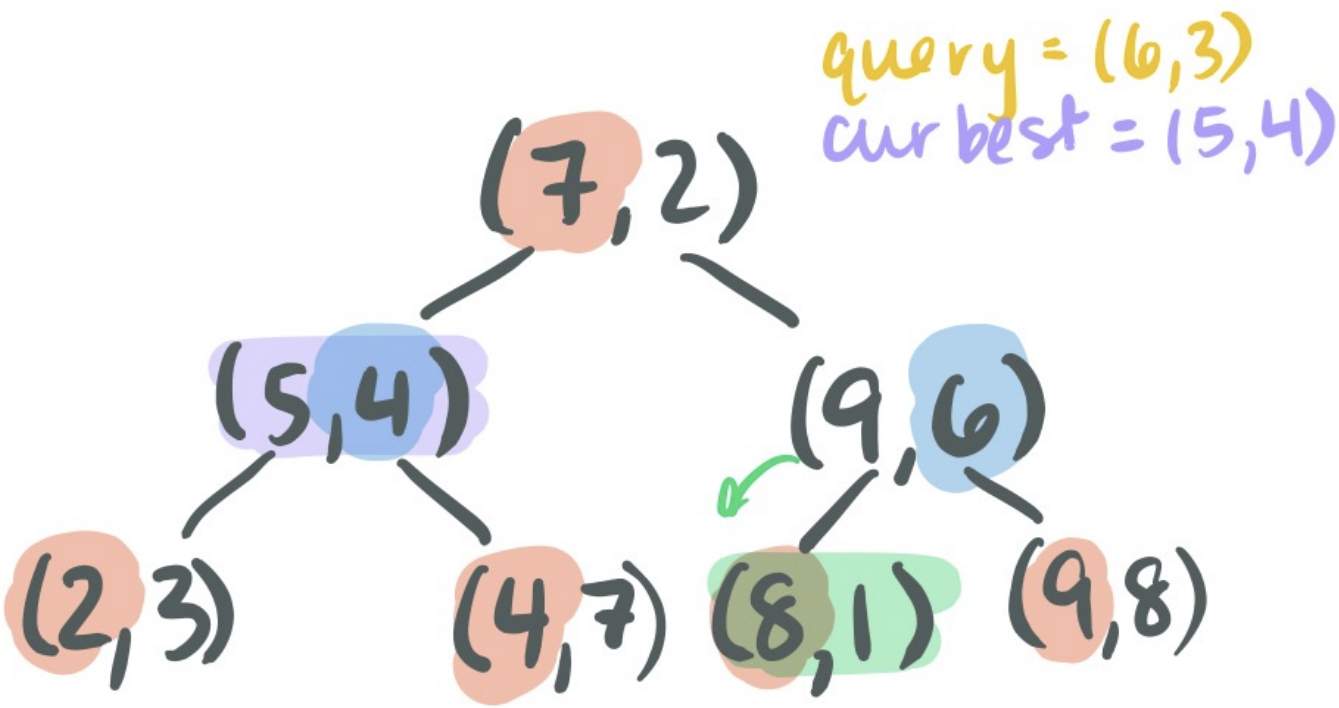
# Nearest Neighbor: k-d tree

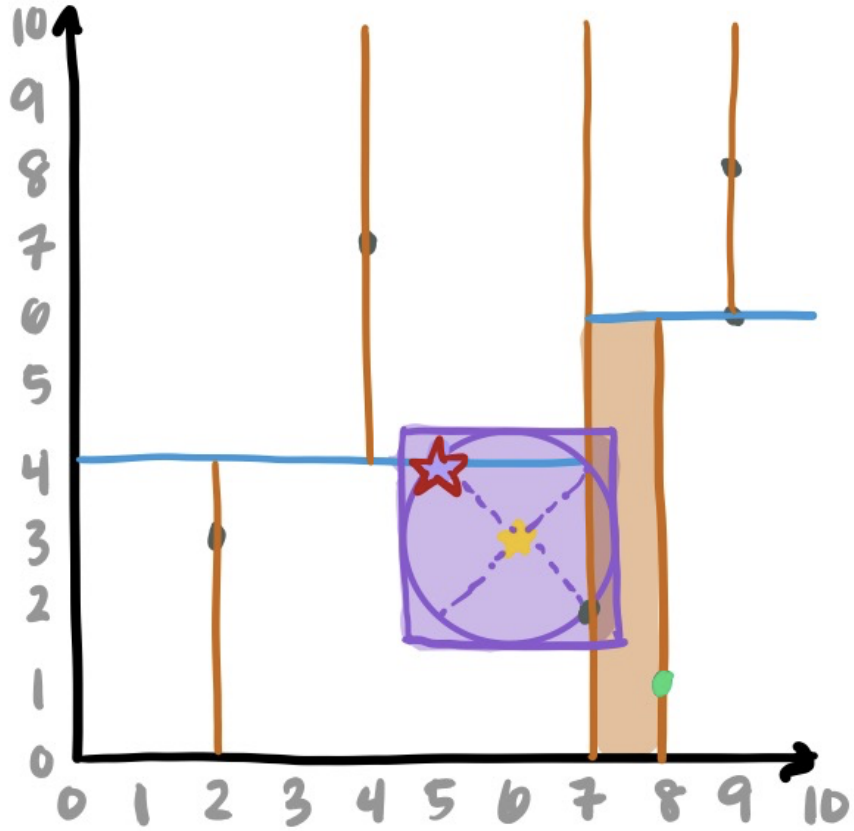
**Why do we need to explore this subtree?**



# Nearest Neighbor: k-d tree




BEST: (5,4)

# Tips and Tricks for MP_Mosaics

1. Review, understand, and use **quickselect**

```
1  template <typename RandIter, typename Comparator>
2  void select(RandIter start, RandIter end, RandIter k, Comparator cmp)
3  {
4      /**
5       * @todo Implement this function!
6       */
7
8    }
9
```

2. Review, understand, and use **lambda functions**

# Understanding ‘randIter’

An iterator is a container giving access in different ways:

Forward

Bidirectional

Random Access

# Implementing quickselect with RandIter

Random Access Iterator lets you:

**Swap items using std::swap()**

```
1  template <typename RandIter, typename Comparator>
2  void BlackBox(RandIter A, RandIter B)
3  {
4
5     std::swap(*A, *B);
6
7
8  }
9
```

**Hint:** Look at pseudo-code for quickselect!

# Implementing quickselect with RandIter

Random Access Iterator lets you:

**Access container indices using math operations**

```
randIter A;

auto nth = *(A + n);
```

**Get distance between two iterators**

```
randIter A, B;

 A < B;          // True if A is earlier in container than B

 A - B;          // The distance between A and B
```

# Implementing quickselect with RandIter

Random Access Iterator lets you:

Do most things you'd expect an array to be able to do!

The power of the **Interface!**

https://en.cppreference.com/w/cpp/iterator/random_access_iterator

# Tips and Tricks for MP_Mosaics

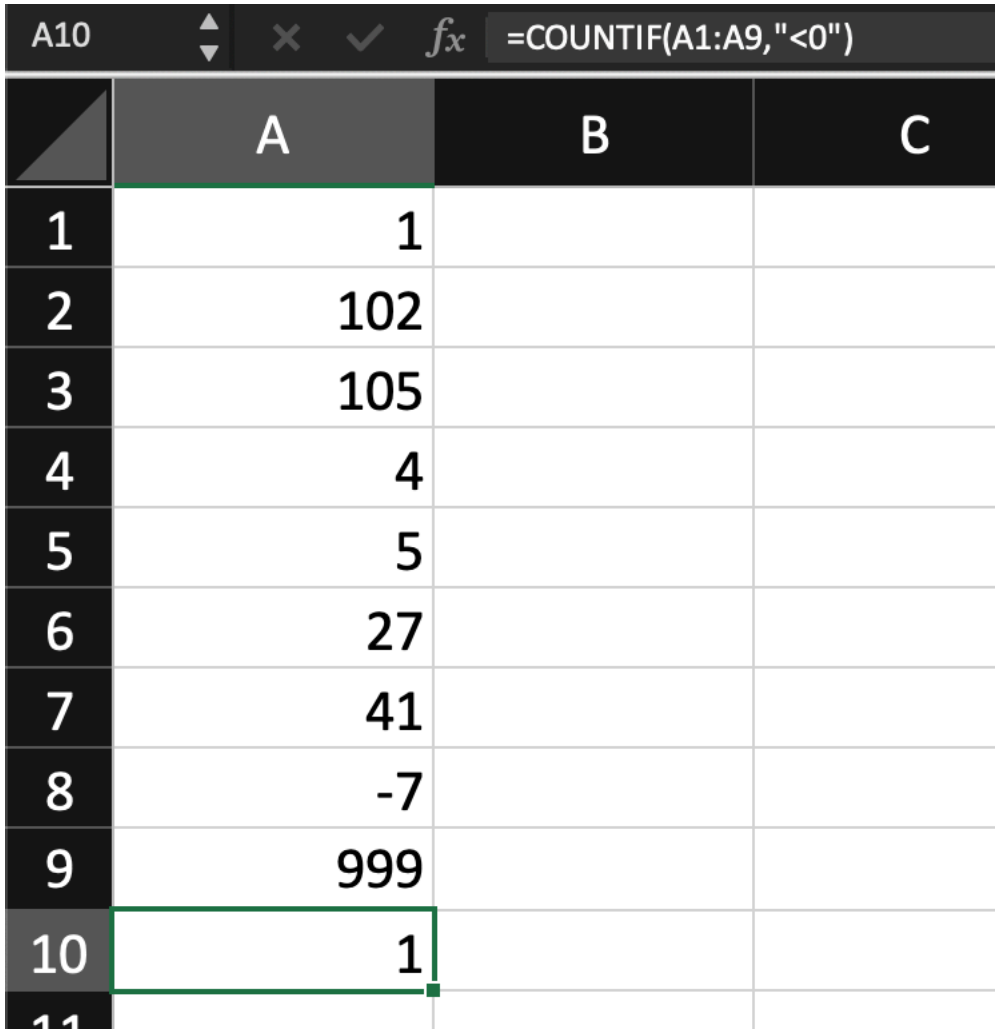
1. Review, understand, and use **quickselect**

```
1  template <typename RandIter, typename Comparator>
2  void select(RandIter start, RandIter end, RandIter k, Comparator cmp)
3  {
4      /**
5       * @todo Implement this function!
6       */
7
8    }
9
```

2. Review, understand, and use **lambda functions**

# Functions as arguments

Consider the function from Excel
COUNTIF(*range*, *criteria*)



# Functions as arguments

**Countif.hpp**

```
10 template <typename Iter, typename Pred>
11 int Countif(Iter begin, Iter end, Pred pred) {
12   int count = 0;
13   auto cur = begin;
14
15   while(cur != end) {
16     if(pred(*cur))
17       ++count;
18     ++cur;
19   }
20
21   return count;
22 }
```

# Lambda Functions in C++

Here are several ways to write a function as an object

**main.cpp**

```
1  bool isNegative(int num) { return (num < 0); }
2
3  class IsNegative {
4  public:
5      bool operator() (int num) { return (num < 0); }
6  };
7
8  int main() {
9    std::vector<int> numbers = {1, 102, 105, 4, 5, 27, 41, -7, 999};
10
11   auto isnegl = [](int num) { return (num < 0); };
12   auto isnegfp = isNegative;
13   auto isnegfunctor = IsNegative();
14
15   cout << "There are " << Countif(numbers.begin(), numbers.end(), _______)
16     << " negative numbers" << std::endl;
17
```

# Lambda Functions in C++

[Capture](Arg List){ Function Body}

# Lambda Functions in C++

## [Capture](Arg List){ Function Body}

**Capture:** Takes the value of object based on when the lambda was defined, NOT the current value of the object!

**Arg List:** Standard way of inputing into a function

**Function Body:** Code can use both capture vars and arg vars

# Lambda Functions in C++

main.cpp

```
29    int big;
30    std::cout << "How big is big? ";
31    std::cin >> big;
32
33    auto isbig = [big](int num) { return (num >= big); };
34
35
36
37    std::cout << "There are " << Countif(numbers.begin(), numbers.end(), isbig)
38      << " big numbers" << std::endl;
}
```

# Lambda Functions in C++

main.cpp

```
29    int big;
30    std::cout << "How big is big? ";
31    std::cin >> big;
32
33    auto isbig = [big](int num) { return (num >= big); };
34
35
36
37    std::cout << "There are " << Countif(numbers.begin(), numbers.end(), isbig)
38      << " big numbers" << std::endl;
  }
```

**Useful for mp_mosaics!**

KD-Tree will split points in one dimension

When comparing, we need to remember what dimension we are in!

# Tips and Tricks for MP_Mosaics

**Final tips:**

The mp_mosaic writeup is long. **READ IT**

The suggestions in the writeup should be followed carefully