

Data Structures

Binary Search Trees 2

CS 225

September 23, 2024

Brad Solomon



UNIVERSITY OF
ILLINOIS
URBANA - CHAMPAIGN

Department of Computer Science

Exam 2 (10/02 — 10/04)

Autograded MC and one coding question

Manually graded short answer prompt

Practice exam will be released on PL

Topics covered can be found on website

Registration started September 19

<https://courses.engr.illinois.edu/cs225/fa2024/exams/>

Learning Objectives

Build conceptual and coding understanding of BST

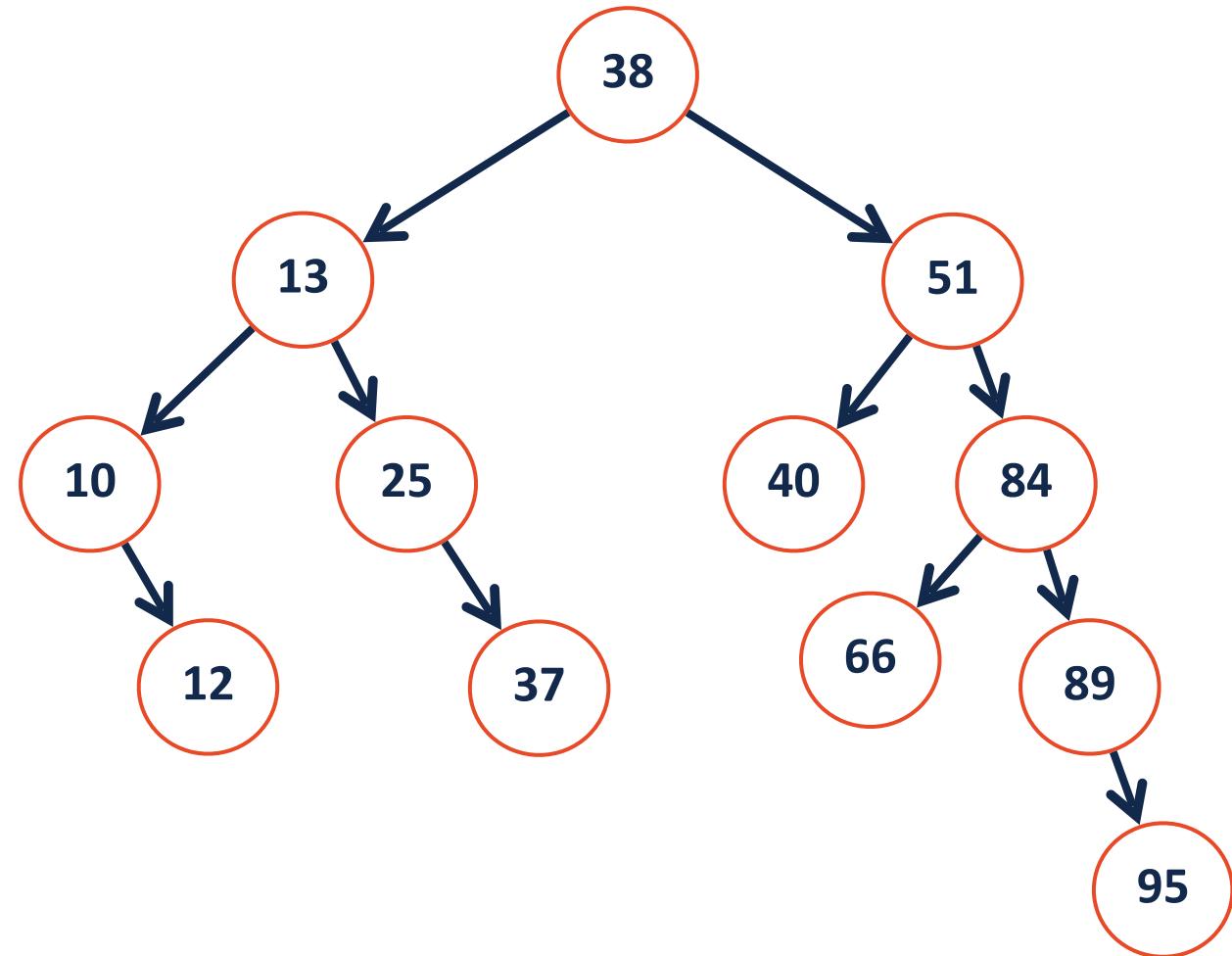
Discuss pros and cons of BST (and possible improvements)

Binary Search Tree (BST)

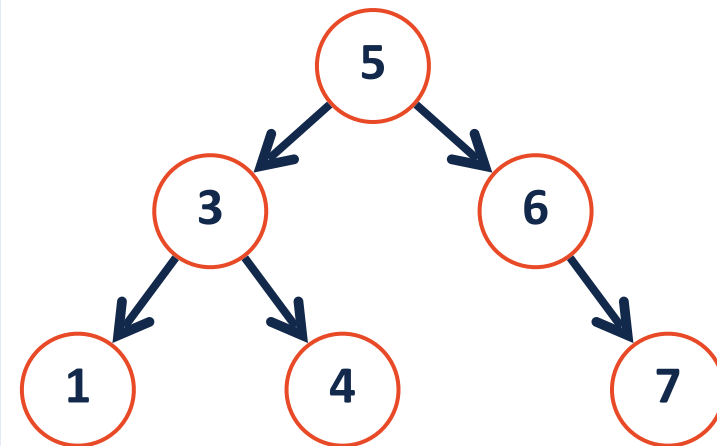
A **BST** is a binary tree $T = \text{TreeNode}(val, T_L, T_r)$ such that:

$\forall n \in T_L, n.val < T.val$

$\forall n \in T_R, n.val > T.val$

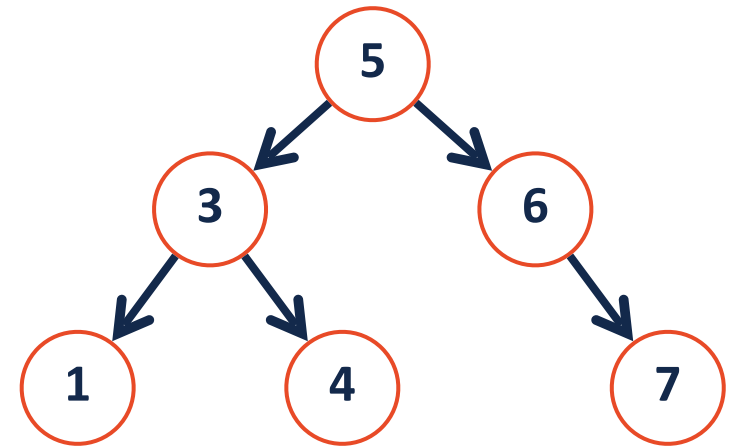


```
1 template<typename K, typename V>
2
3 TreeNode *& _find(TreeNode *& root, const K & key) {
4
5
6 // Base Case
7 if(root == nullptr || root->key == key){
8     return root;
9 }
10
11 // Recursive Step ("Combining step" is 'return')
12 if (root->key > key){
13     return _find(root->left, key);
14 }
15
16 return _find(root->right, key);
17
18
19 }
20
21
22
23
```



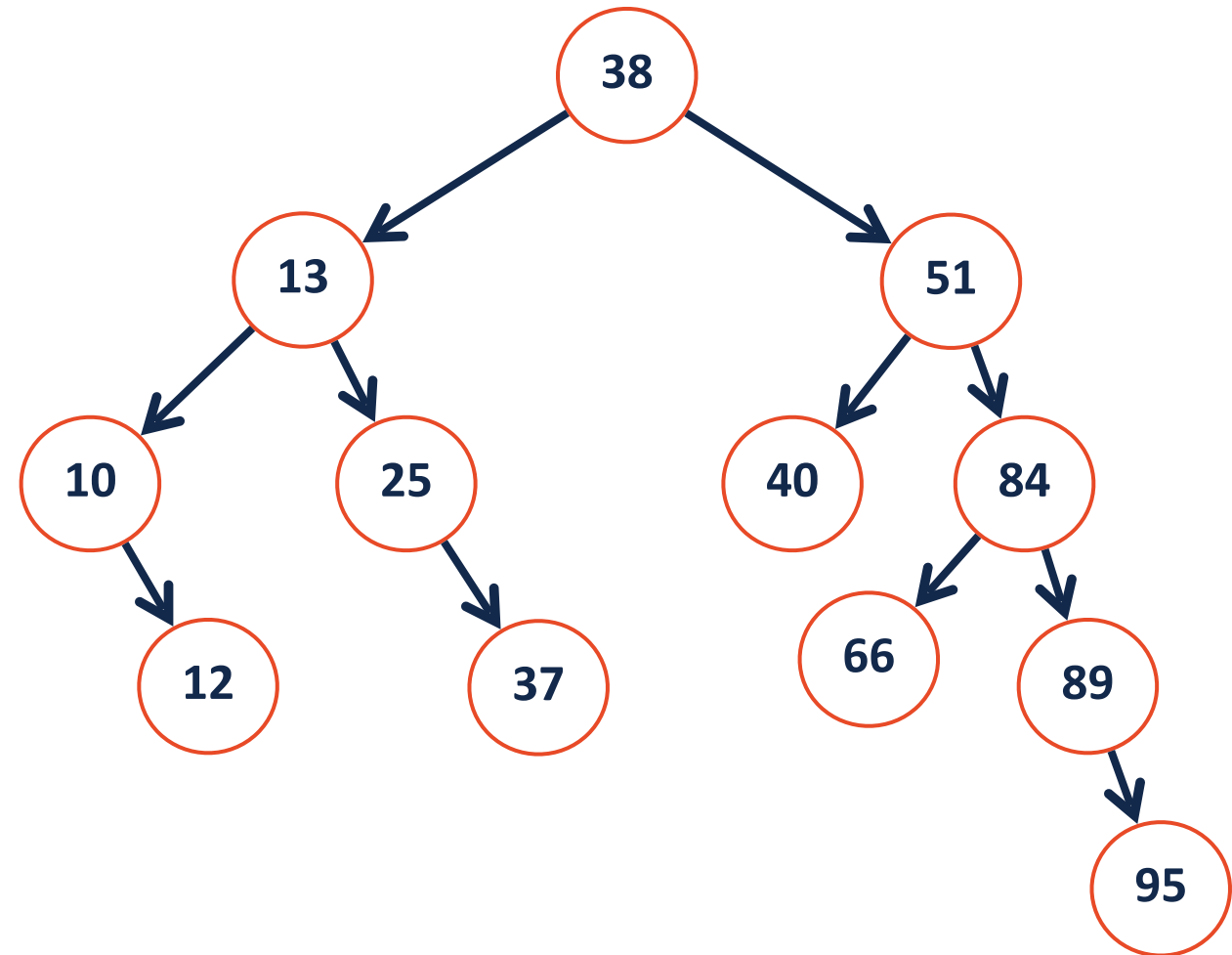


```
1 template<typename K, typename V>
2
3 void _insert(const K & key, const V & val) {
4
5
6     TreeNode *& tmp = _find(root, key);
7
8
9     tmp = new treeNode(key, val);
10
11
12 }
```



BST Remove

remove (40)



BST Remove

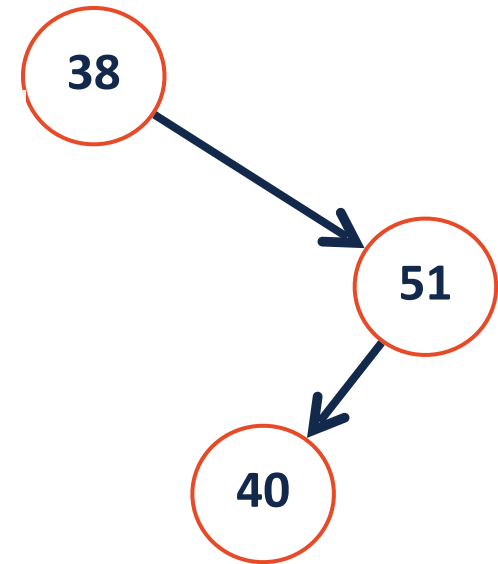
0-Child Case

```
TreeNode *& t = _find(root, 40);
```

```
delete t;
```

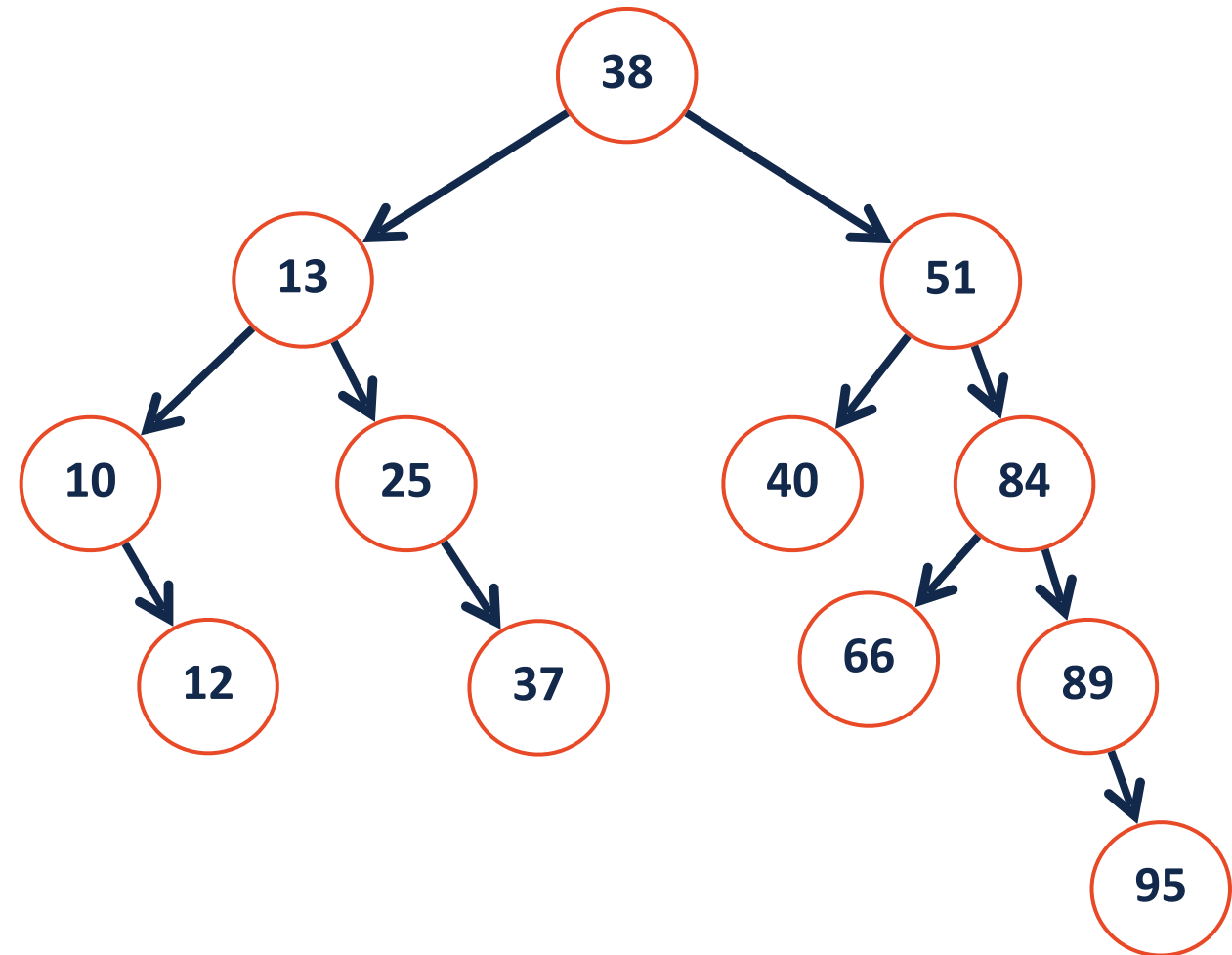
```
t = nullptr;
```

remove(40)



BST Remove

remove (25)



BST Remove

1-Child Case

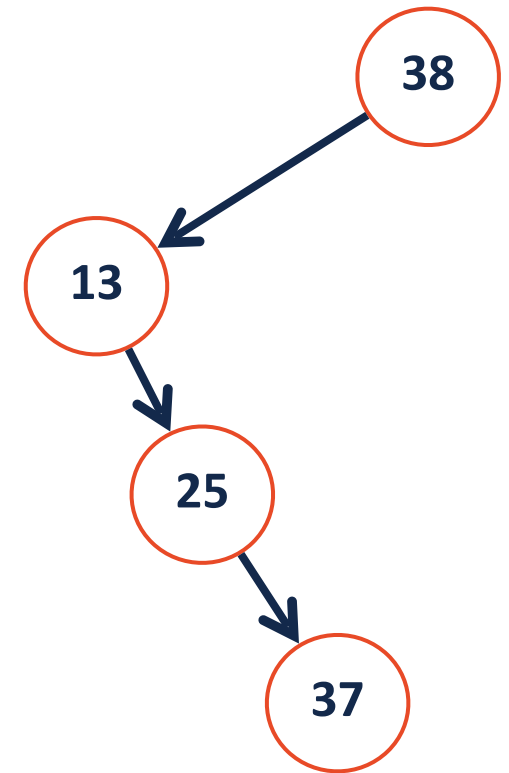
```
TreeNode *& t = _find(root, 25);
```

```
TreeNode * tmp = t;
```

```
t = t->right;
```

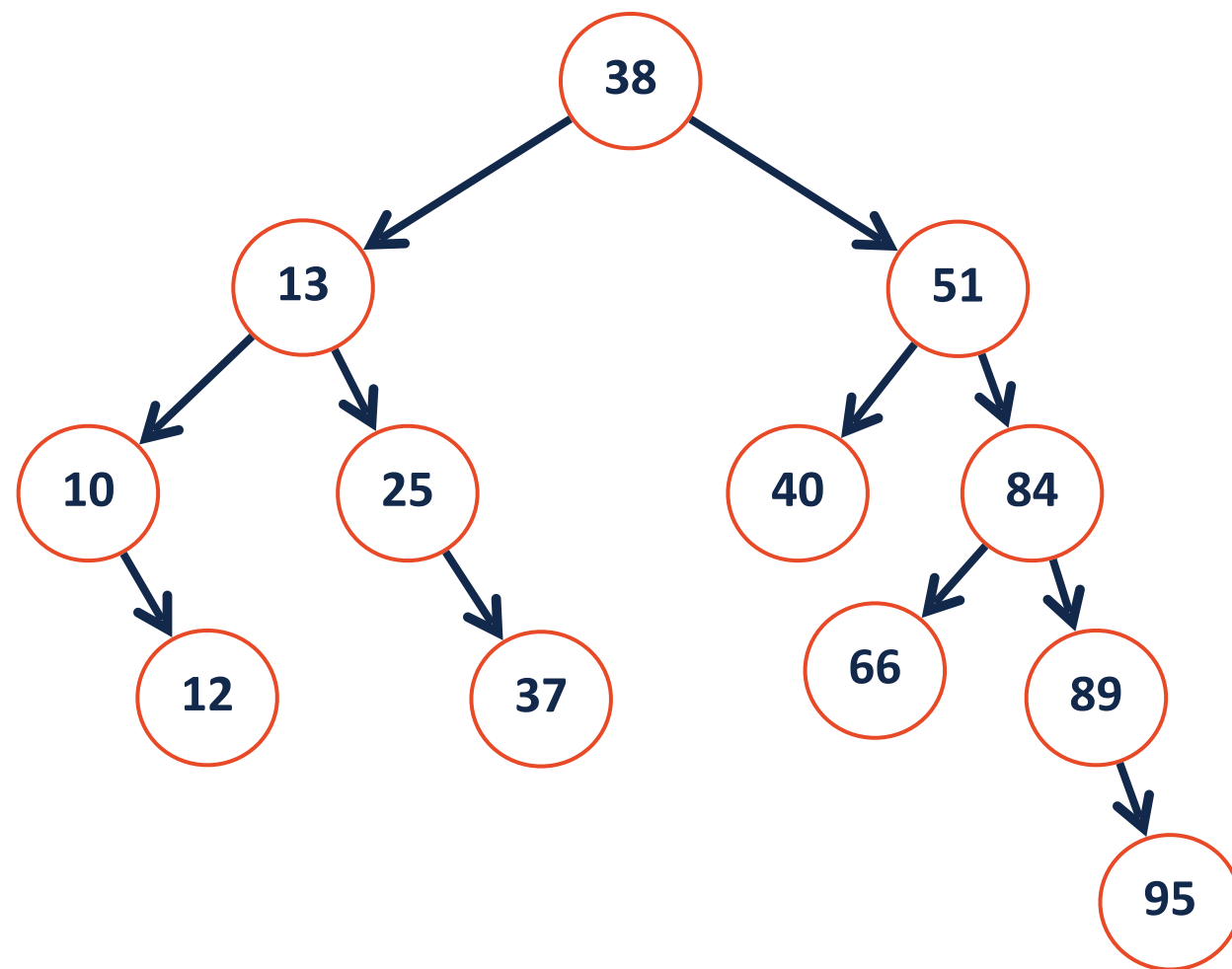
```
delete tmp;
```

remove (25)



BST Remove

remove (13)



BST In-Order _____

In-Order Predecessor

Rightmost left child

IOP(38) =

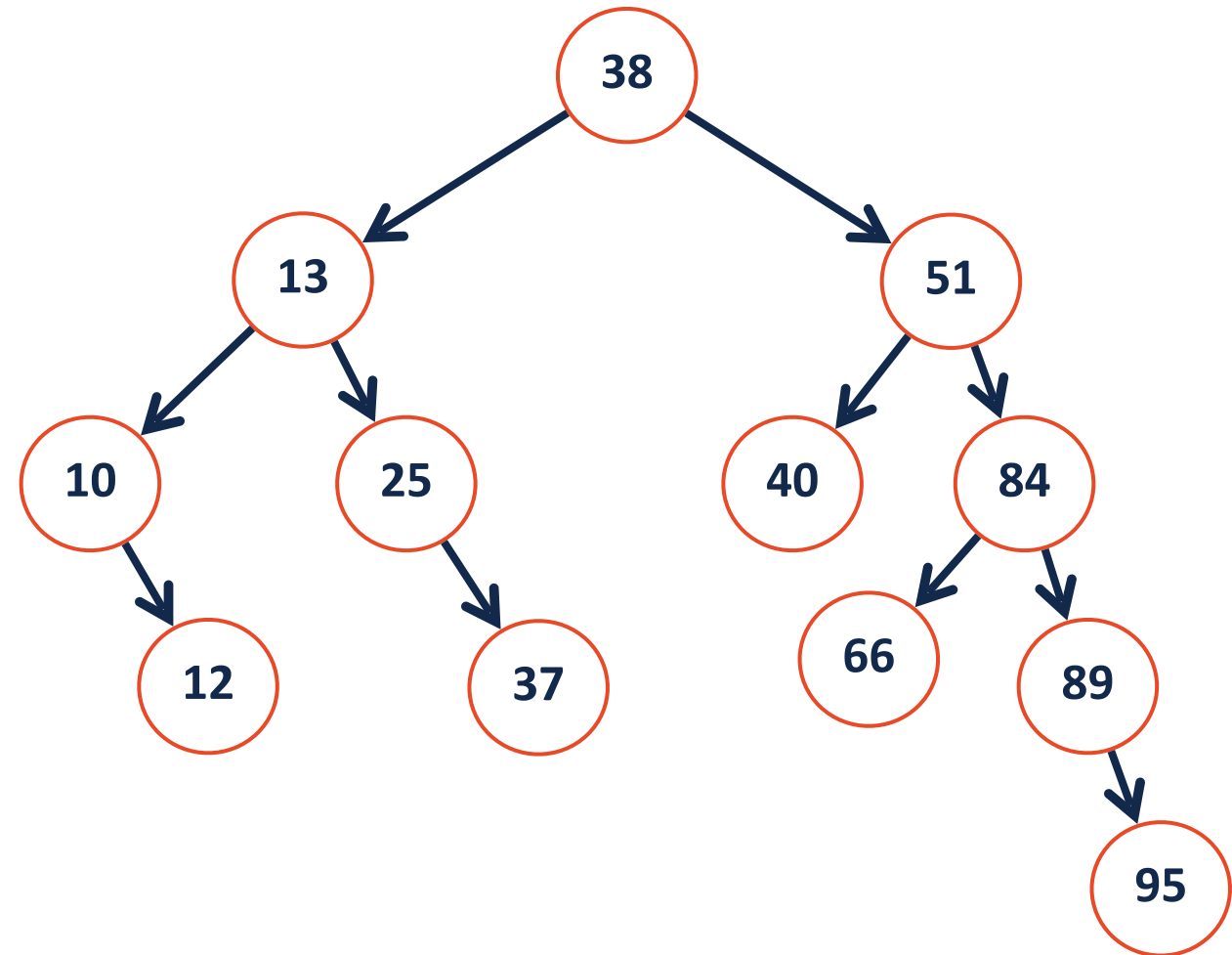
IOP(84) =

In-Order Successor

Leftmost right child

IOS(38) =

IOS(84) =



BST Remove

2-Child Case

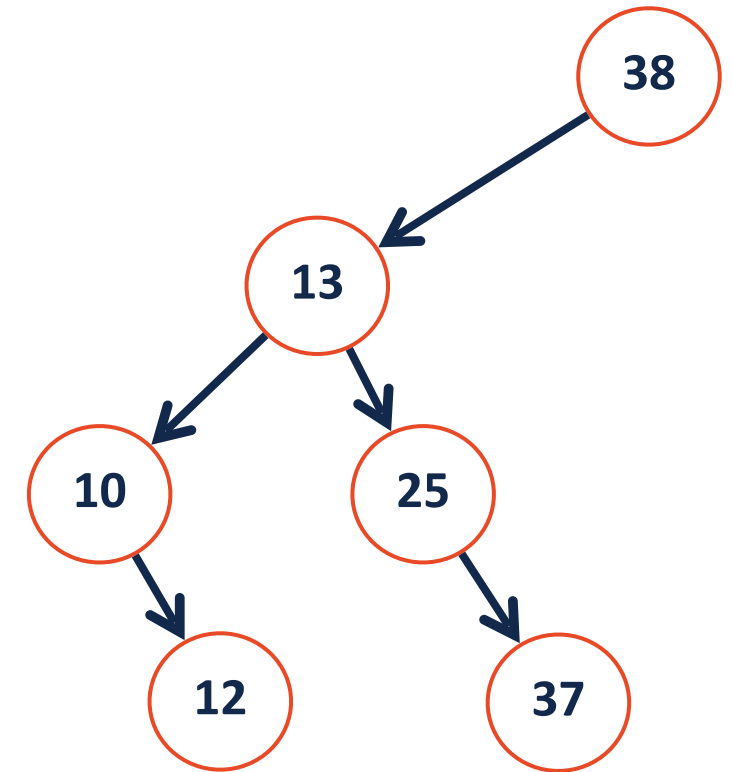
```
TreeNode *& t = _find(root, 13);
```

```
TreeNode * IOP = getIOP(t);
```

```
swap(t, iop);
```

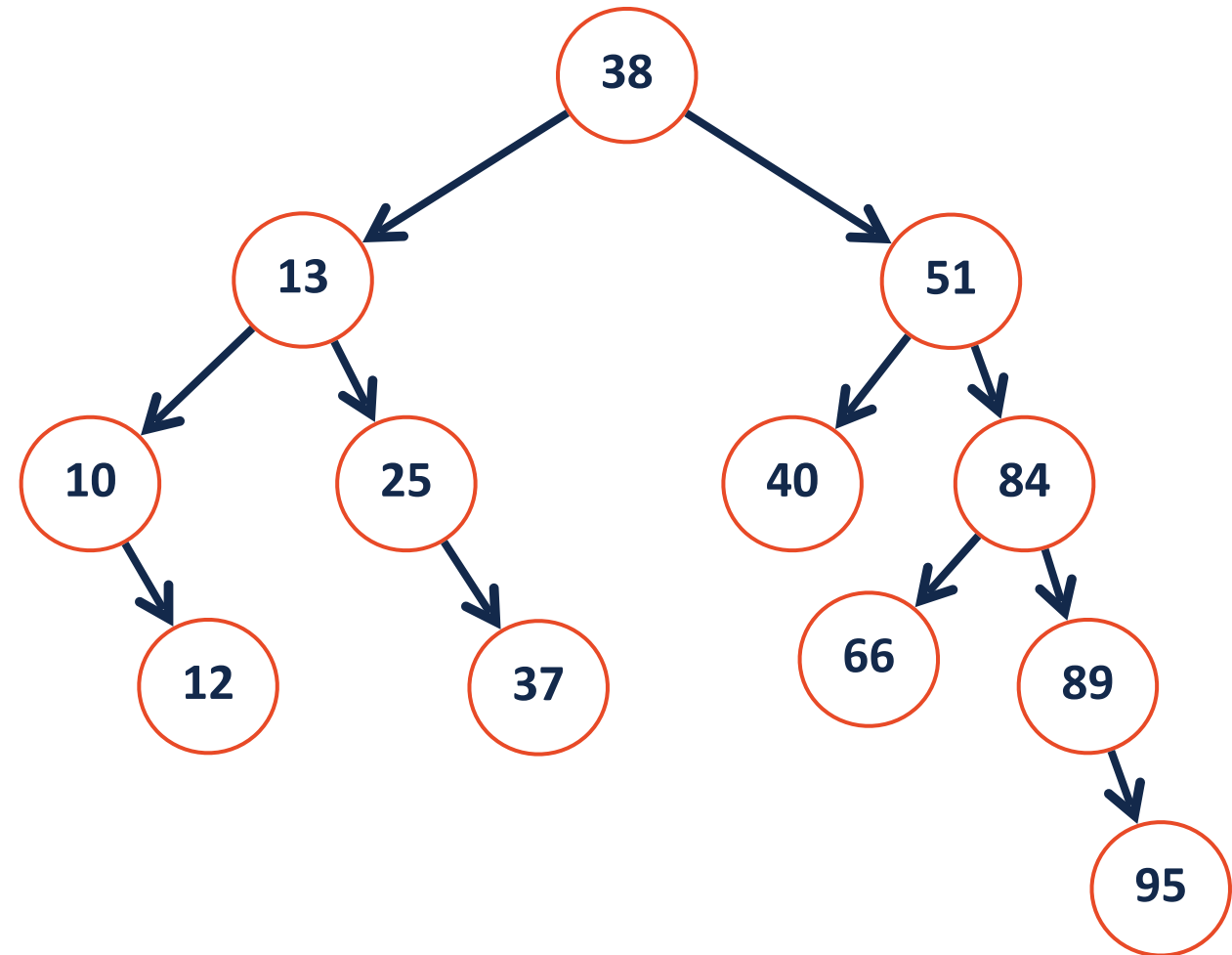
```
remove(13); //starting from t
```

remove (13) 



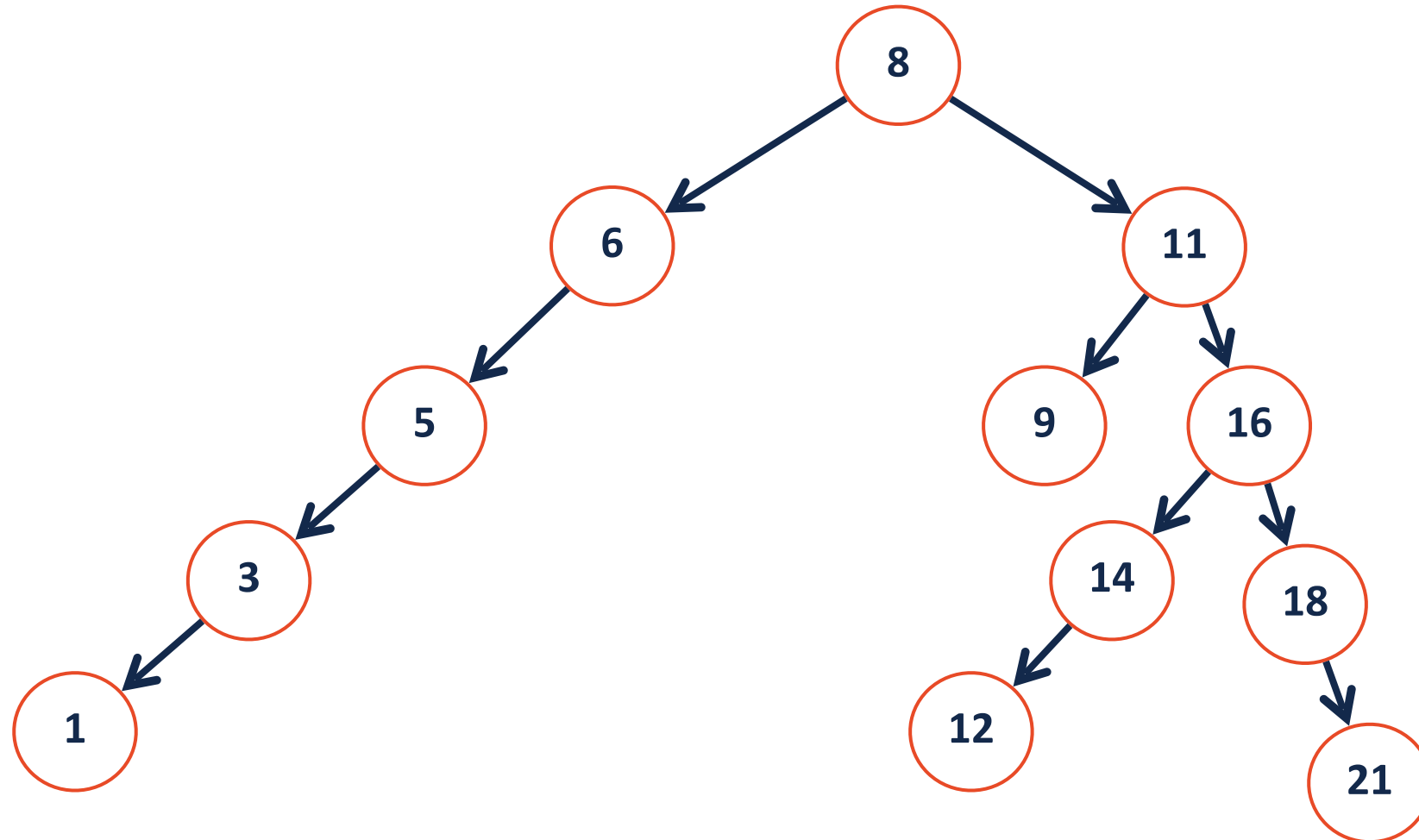
BST Remove

remove (51)

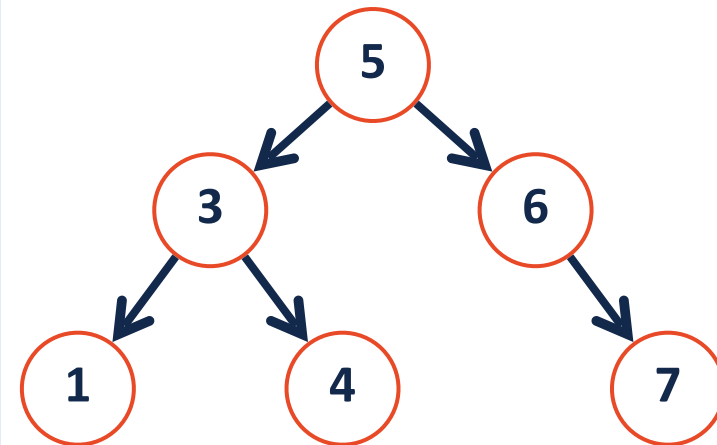


BST Remove

What will the tree structure look like if we remove node 16 using IOS?



```
1 template<typename K, typename V>
2
3 void _remove(TreeNode *& root, const K & key) {
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23 }
```



BST Analysis – Running Time



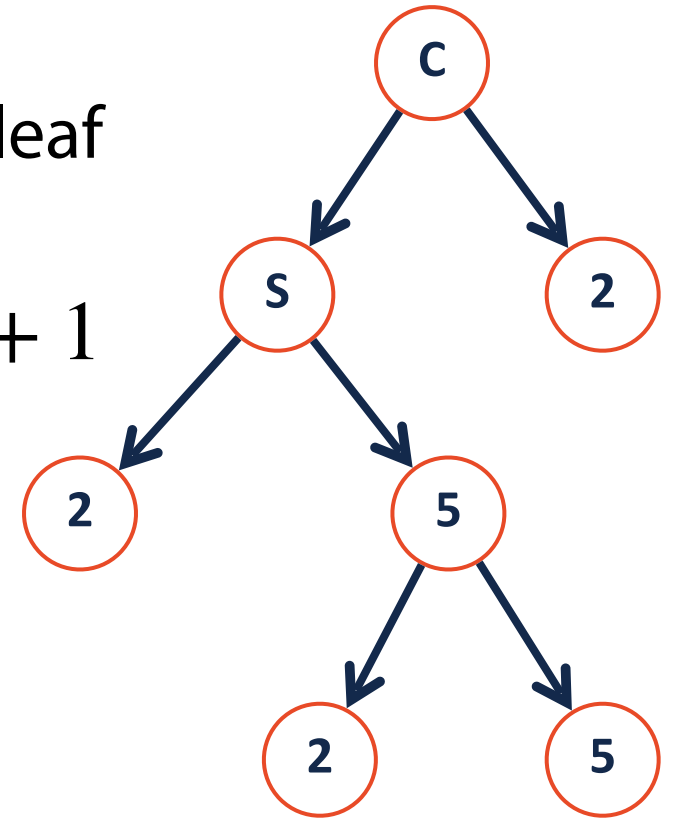
| Operation | BST Worst Case |
|-----------|----------------|
| find | |
| insert | |
| remove | |
| traverse | |

Binary Tree Height

Height: The length of the longest path from root to leaf

$$\text{Height}(\text{root}) = \max (\text{Height}(T_L), \text{Height}(T_R)) + 1$$

Given this recursion, what is base case?



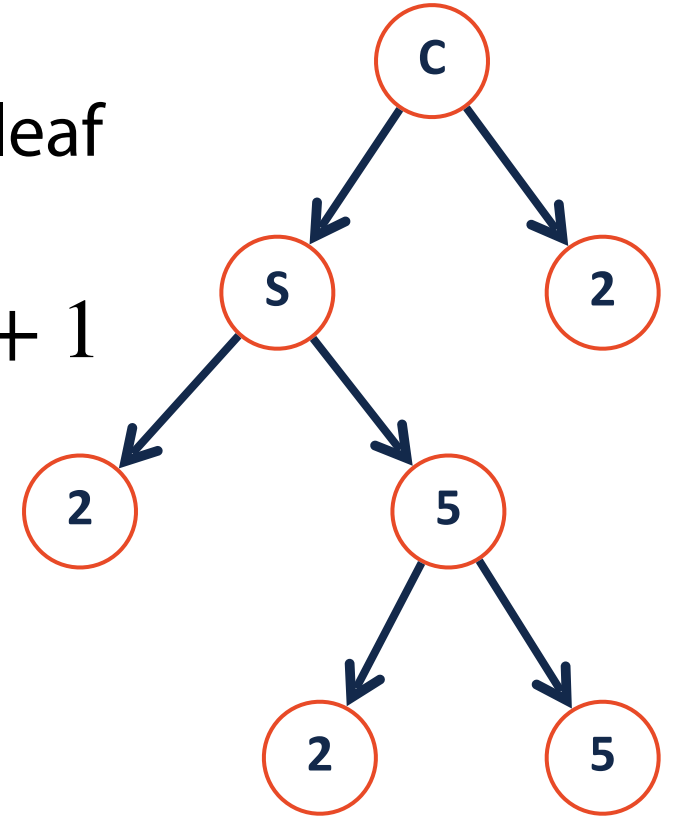
Binary Tree Height

Height: The length of the longest path from root to leaf

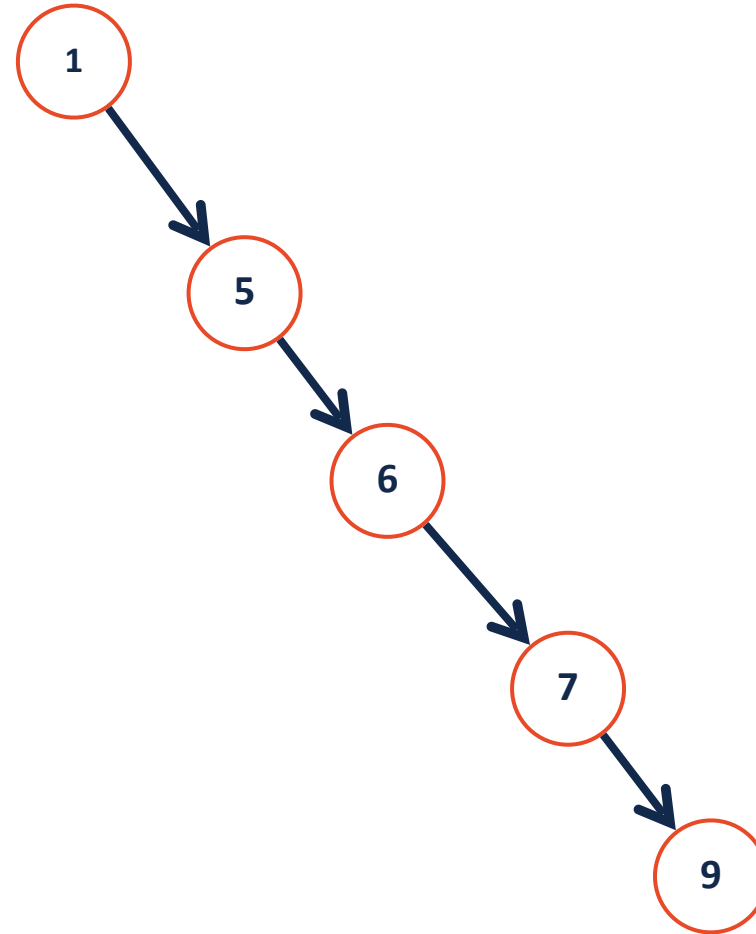
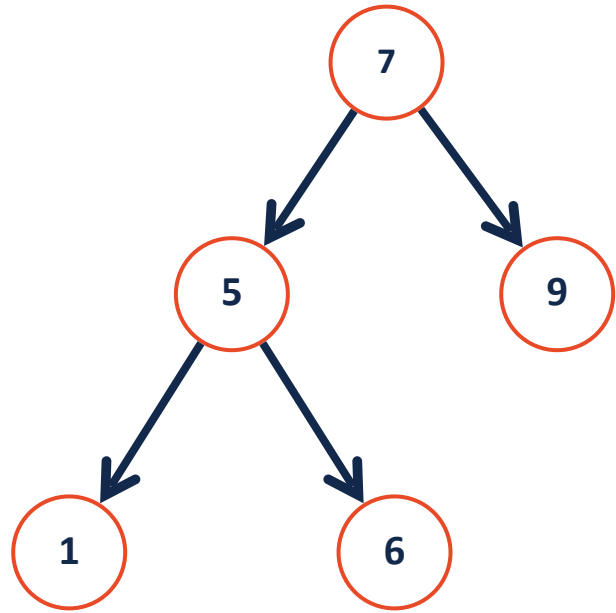
$$\text{Height}(\text{root}) = \max (\text{Height}(T_L), \text{Height}(T_R)) + 1$$

Given this recursion, what is base case?

$$\text{Height}(\emptyset) = - 1$$



Limiting the height of a tree



Option A: Correcting bad insert order

The height of a BST depends on the order in which the data was inserted

Insert Order: [1, 3, 2, 4, 5, 6, 7]

Insert Order: [4, 2, 3, 6, 7, 1, 5]

AVL-Tree: A self-balancing binary search tree

Rather than fixing an insertion order, just correct the tree as needed!

