

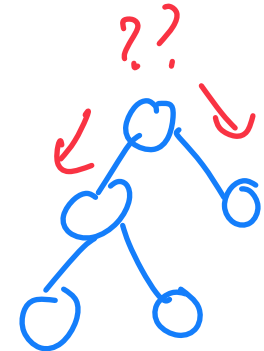
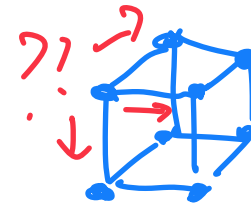
Data Structures

Iterators and Tree Fundamentals

CS 225

Brad Solomon

Friday 😊
September 13, 2024

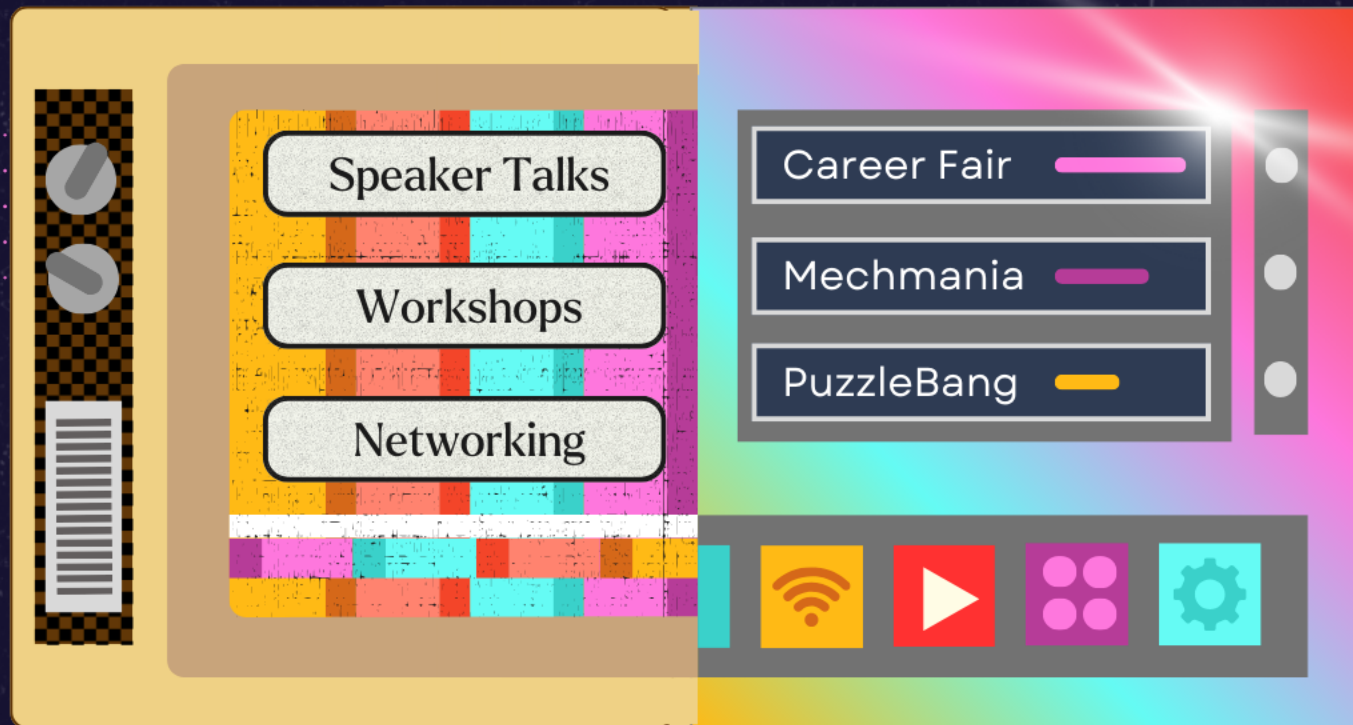


UNIVERSITY OF
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September 18th - 22nd

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Learning Objectives

Queues?

Discuss the importance of iterators

Review trees and binary trees

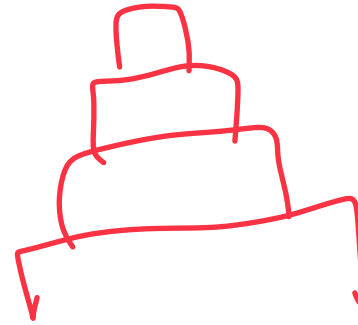
Practice tree theory with recursive definitions and proofs

Discuss the tree ADT

or

Stack ADT

- [Order]: LIFO

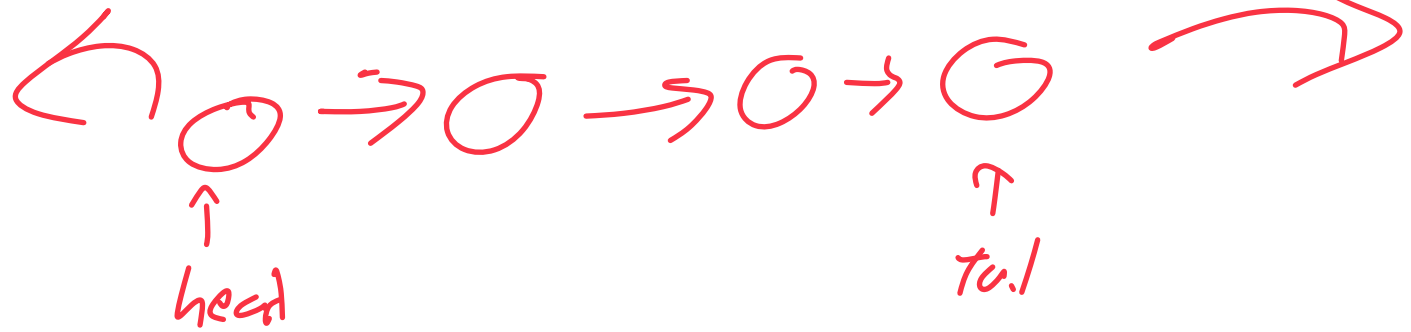


- [Implementation]: Array (such as `std::vector`)

- [Runtime]: $O(1)$ ^{*} Push and Pop

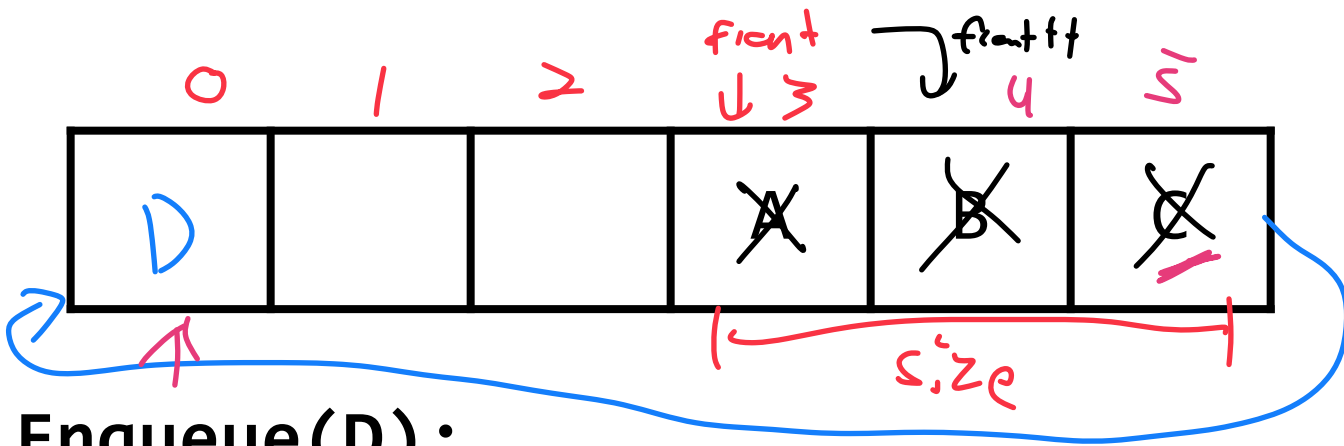
Queue ADT

- [Order]: FIFO



- [Implementation]: Circular Queue as Array

- [Runtime]: $O(1)$



Enqueue(D):

Insert D at index $(\text{size} + \text{front}) \% \text{capacity}$

$$(3 + 3) \% 6 = 0$$

size++

Dequeue(): Remove data at index front

$\text{front} = (\text{front} + 1) \% \text{capacity}$

size--

$$(5 + 1) \% \text{capacity}$$

```

Queue<int> q;
...
q.enqueue(D);
q.dequeue();
q.dequeue();
q.dequeue();
q.dequeue();
q.enqueue(E);

```

size++ until
size == capacity

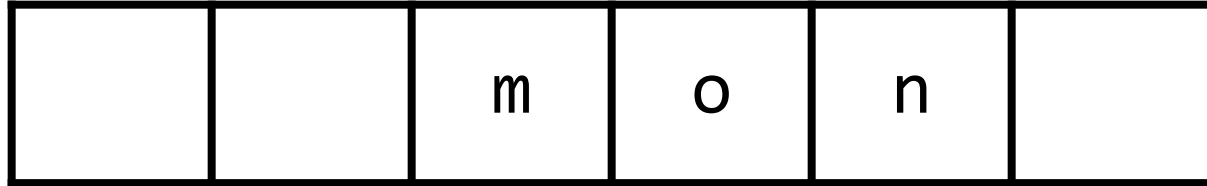
Size: ~~3~~ ~~4~~ ~~5~~ ~~6~~ 1

Front: ~~3~~ ~~4~~ ~~5~~ ~~0~~ 1

unused ints

Capacity: 6

Queue Data Structure: Resizing



```
Queue<char> q;
```

```
...
```

```
q.enqueue(d);
```

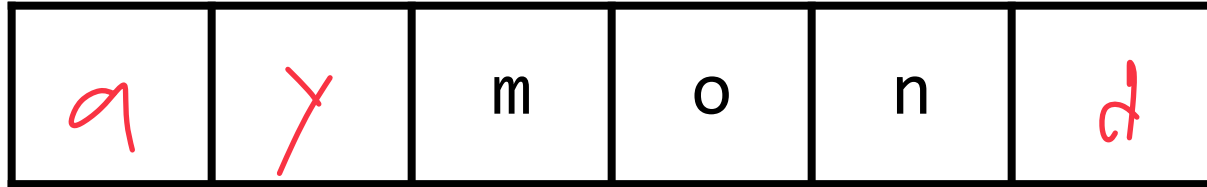
```
q.enqueue(a);
```

```
q.enqueue(y);
```

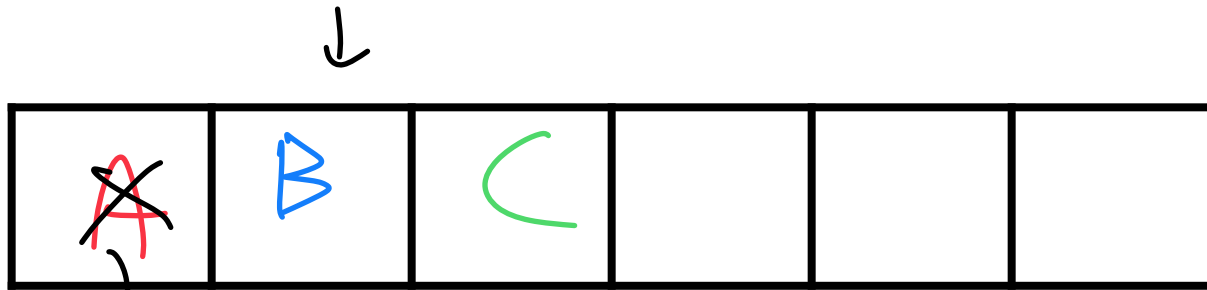
```
q.enqueue(i);
```

```
q.enqueue(s);
```

Queue Data Structure: Resizing



↑
front



↓
A

front: ~~0~~ 1
size: ~~6~~ 1 2 3

```
Queue<char> q;
...
q.enqueue(d);
q.enqueue(a);
q.enqueue(y);
q.enqueue(i);
q.enqueue(s);
```



Queue Data Structure: Resizing

Queue<char> q;

...

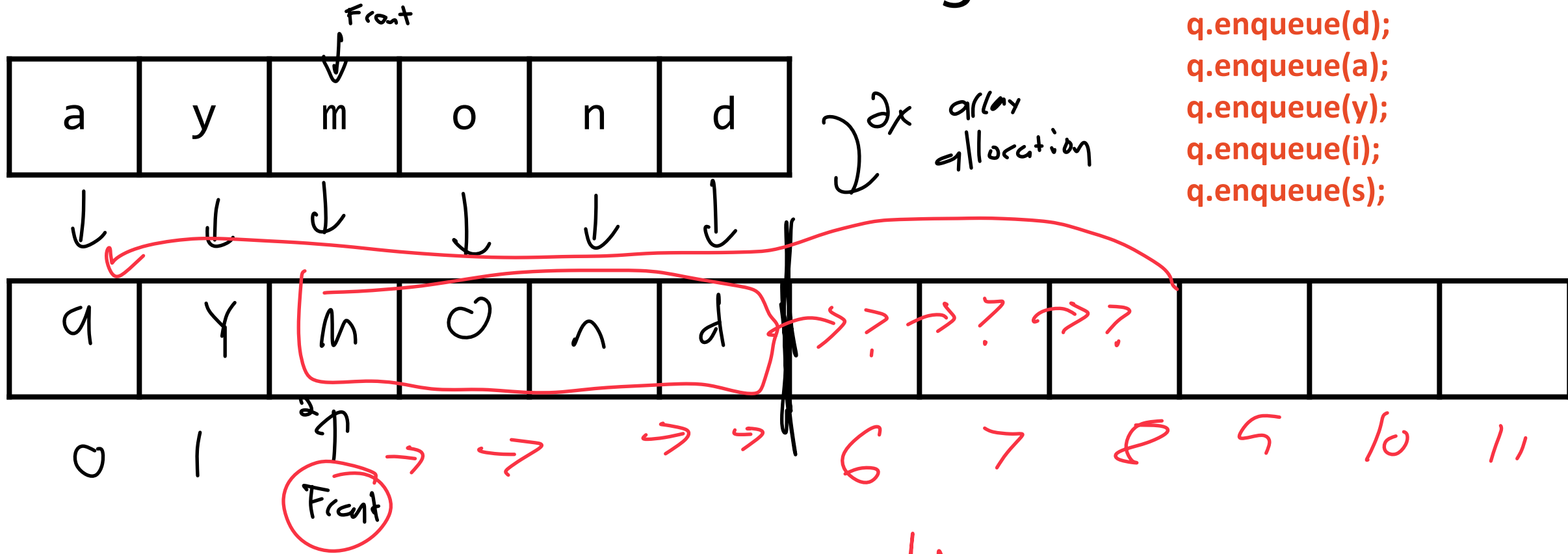
q.enqueue(d);

q.enqueue(a);

q.enqueue(y);

q.enqueue(i);

q.enqueue(s);



Broken circular queue!

Not this!

Queue Data Structure: Resizing

Queue<char> q;

...

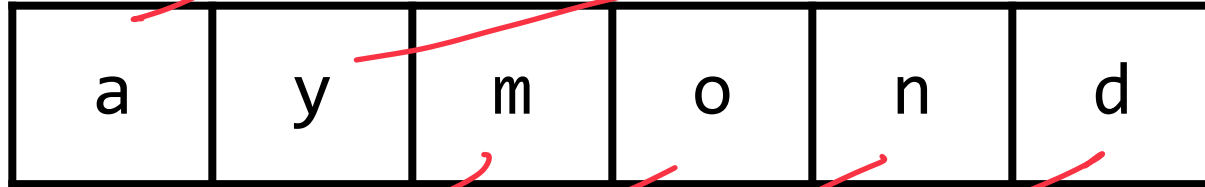
q.enqueue(d);

q.enqueue(a);

q.enqueue(y);

q.enqueue(i);

q.enqueue(s);

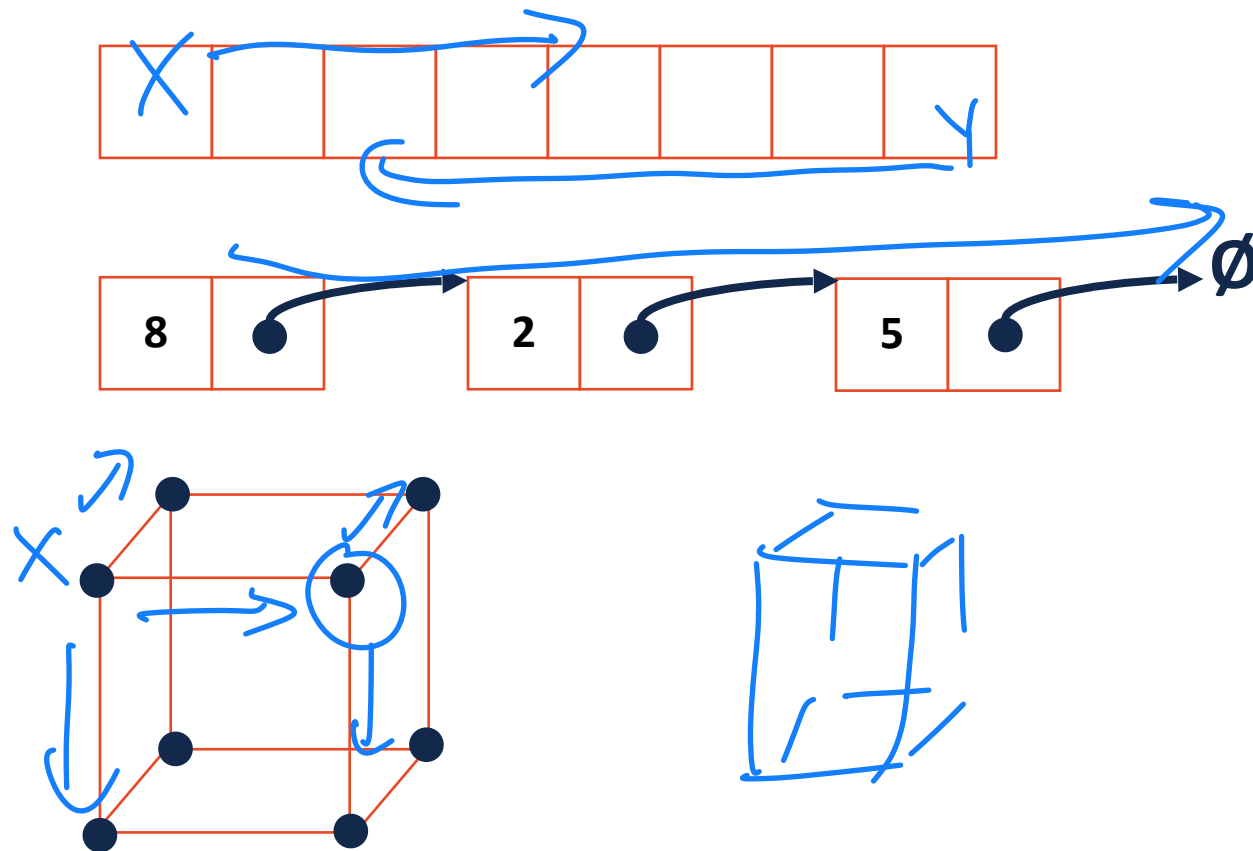


Front = 0

$O(1)^*$ amortized time

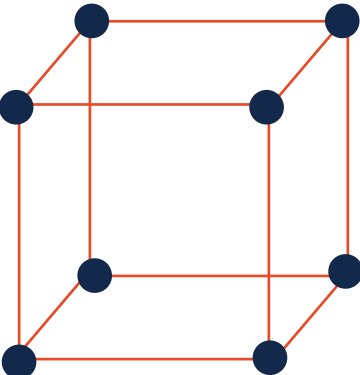
Iterators \rightarrow mp_lists

We want to be able to loop through all elements for any underlying implementation in a systematic way



Iterators

We want to be able to loop through all elements for any underlying implementation in a systematic way

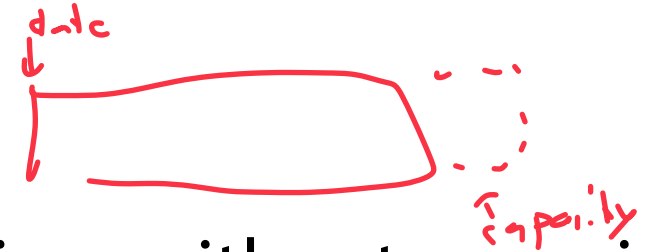


$*i++$
↓

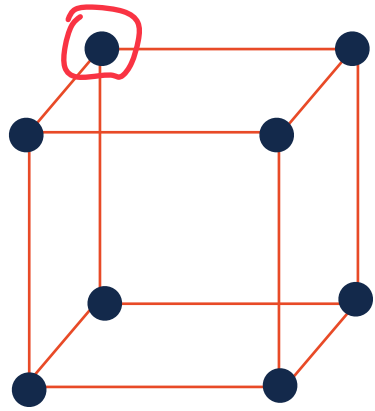
Cur. Location	Cur. Data	Next
ListNode * <u>curr</u>	$curr \rightarrow data$	$curr \rightarrow next$
unsigned index	$A[i]$ $getData(i)$	$index++$
Some form of (x, y, z)	?? ..	?? ..

Iterators

Cube myCube



Iterators provide a way to access items in a container without exposing the underlying structure of the container



```
1 Cube::Iterator it = myCube.begin();  
2  
3 while (it != myCube.end()) {  
4     std::cout << *it << " ";  
5     it++;  
6 }  
7
```

(iterator) != (iterator)

*it - de reference

Iterators

For a class to implement an iterator, it needs two functions:

Iterator begin() - return type iterator
↳ points to start position

Iterator end() - return type iterator
↳ points to one mem address past
the end of class

Iterators

The actual iterator is defined as a class **inside** the outer class:

1. It must be of base class **std::iterator**

2. It must implement at least the following operations:

Iterator& operator ++() *- move to next item*

const T & operator *() *- return the data/value at current pos*

bool operator ==(const Iterator &) *- check if iterators are equal*

Iterators

i++
↳ use it at call & then increment



Here is a (truncated) example of an iterator:

```
1 template <class T>
2 class List {
3     class ListIterator : public
4     std::iterator<std::bidirectional_iterator_tag, T> {
5     public:
6
7     * ListIterator& operator++();
8
9     ListIterator& operator--();
10
11    * bool operator!=(const ListIterator& rhs);
12
13    * const T& operator*();
14    };
15
16    ListIterator begin() const;
17
18    ListIterator end() const;
19 };
```

++it
↳ increment first and then use new value

Pre increment

Implementing this gives iterator access


```
1 #include <list>
2 #include <string>
3 #include <iostream>
4
5 struct Animal {
6     std::string name, food;
7     bool big;
8     Animal(std::string name = "blob", std::string food = "you", bool big = true) :
9         name(name), food(food), big(big) { /* nothing */ }
10 };
11
12 int main() {
13     Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
14     std::vector<Animal> zoo;
15
16     zoo.push_back(g);
17     zoo.push_back(p); // std::vector's insertAtEnd
18     zoo.push_back(b);
19
20     for ( std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); ++it ) {
21         std::cout << (*it).name << " " << (*it).food << std::endl;
22     }
23
24     return 0;
25 }
```

Handwritten annotations in red:

- Arrows pointing to `std::string name, food;` and `bool big;` in line 5-7.
- A large arrow pointing to the constructor in line 8.
- An arrow pointing to `std::vector<Animal> zoo;` in line 14.
- Arrows pointing to `zoo.push_back(g);`, `zoo.push_back(p);`, and `zoo.push_back(b);` in lines 16-18.
- Arrows pointing to `std::vector<Animal>::iterator` in line 20.
- Arrows pointing to `zoo.begin()`, `zoo.end()`, and `++it` in line 20.
- Arrows pointing to `(*it).name` and `(*it).food` in line 21.
- The word `Animal` written below line 21, with an arrow pointing to `(*it).name`.

1
2
3
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7
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9
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16
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18
19
20
21
22
23
24
25

$$A = []$$
$$A[i++] \quad \text{vs} \quad A[++i]$$



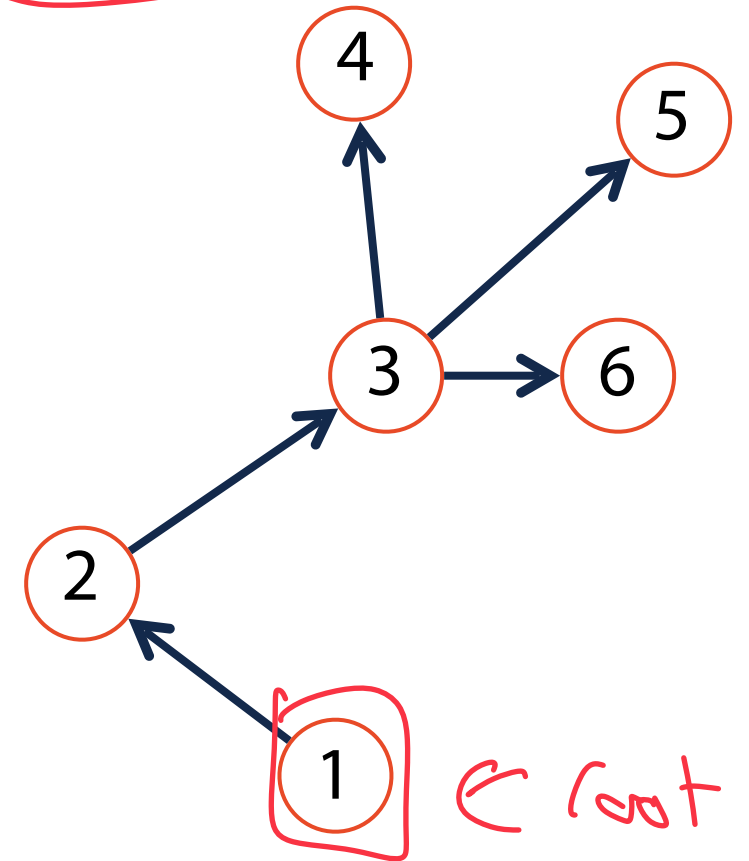
```
1
2 std::vector<Animal> zoo;
3
4
5 /* Full text snippet */
6
7 for (std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); ++it ) {
8     std::cout << (*it).name << " " << (*it).food << std::endl;
9 }
10
11
12 /* Auto Snippet */
13
14 for (auto it = zoo.begin(); it != zoo.end(); ++it ) {
15     std::cout << (*it).name << " " << (*it).food << std::endl;
16 }
17
18 /* For Each Snippet */
19
20 for ( const Animal & animal : zoo ) {
21     std::cout << animal.name << " " << animal.food << std::endl;
22 }
23
24
25
```

Trees

A non-linear data structure defined recursively as a collection of nodes where each node contains a value and zero or more connected nodes.

[In CS 225] a tree is also:

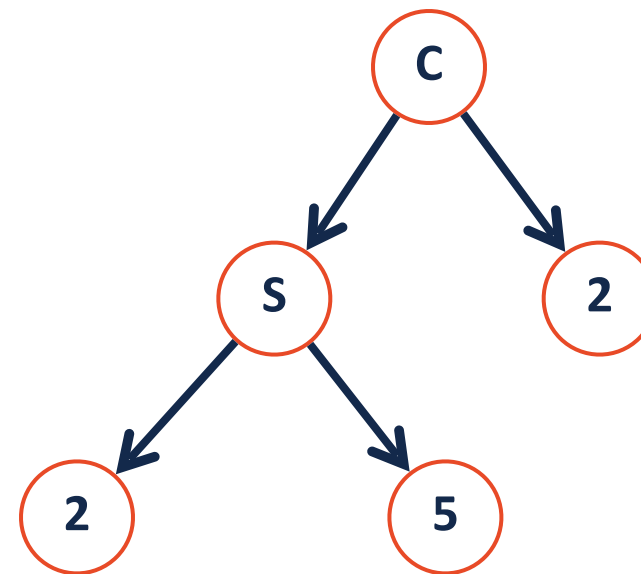
- 1) Acyclic — No path from node to itself
- 2) Rooted — A specific node is labeled root



Binary Tree

A **binary tree** is a tree T such that:

1. $T = \emptyset$



2. $T = (\overset{\text{root}}{\cancel{\text{data}}}, T_L, T_R)$

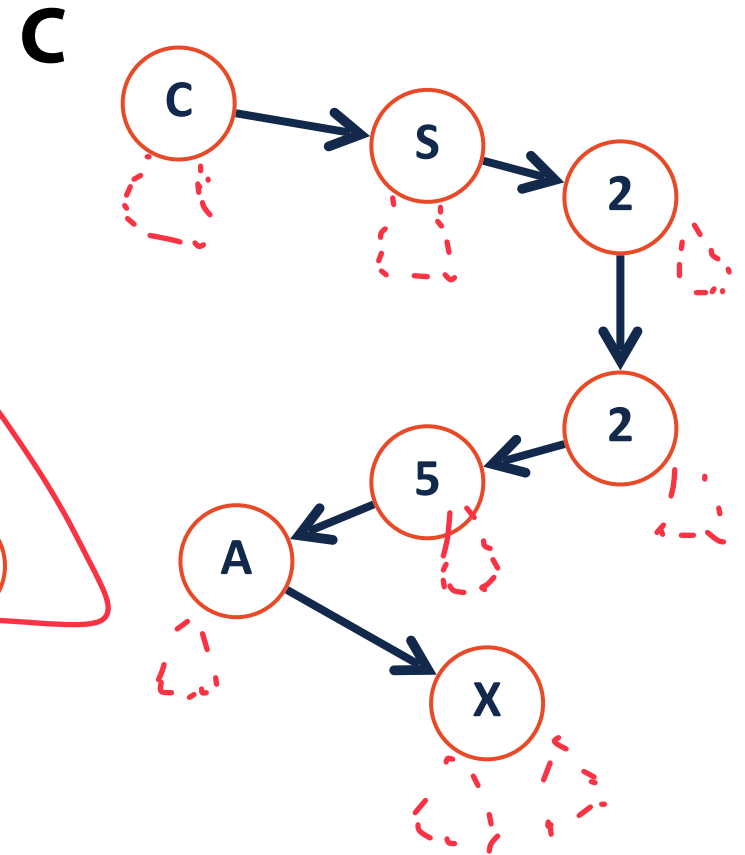
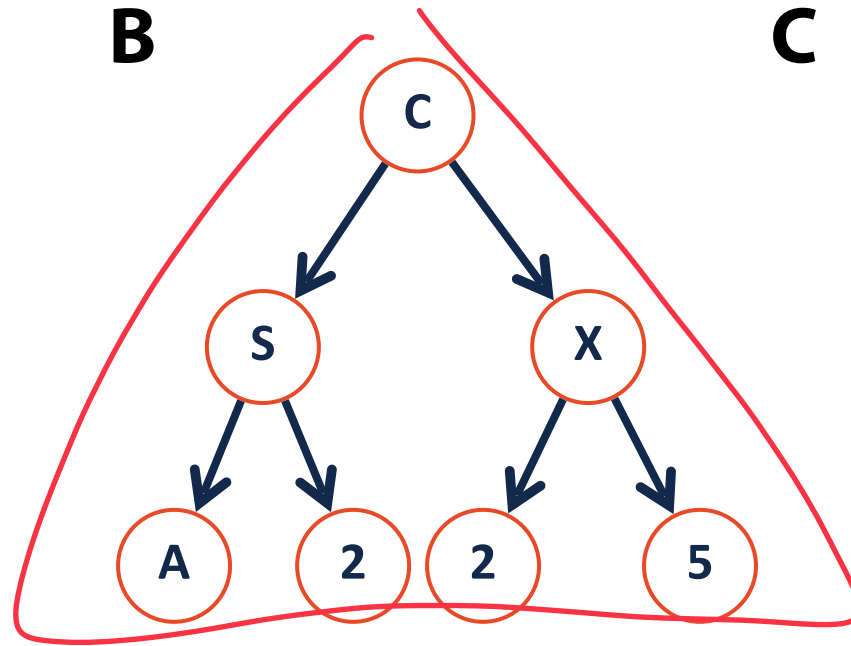
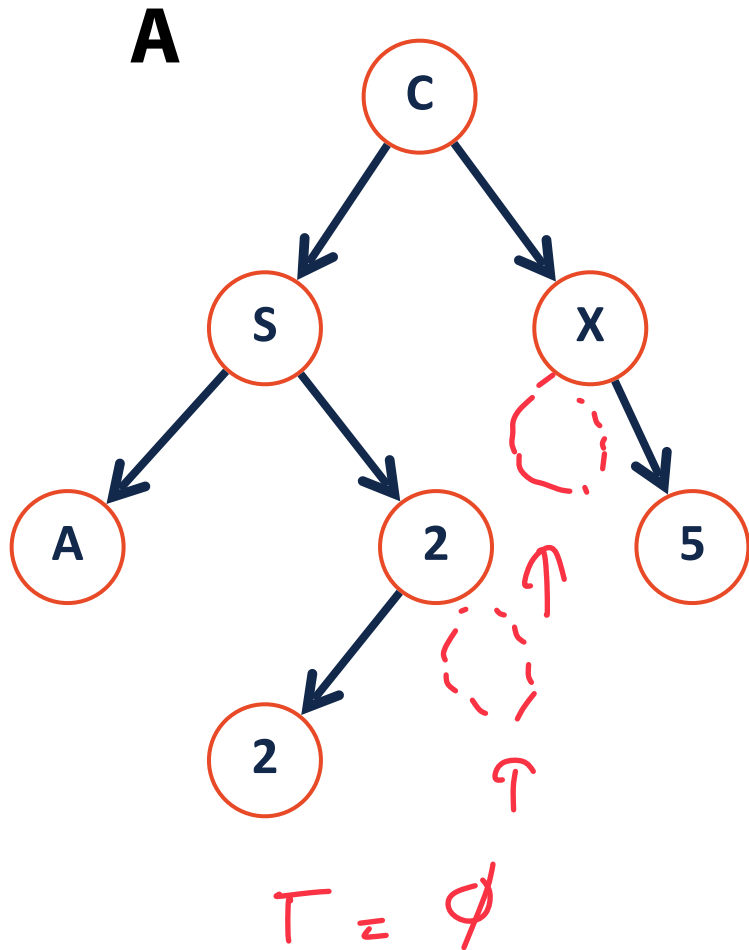


Which of the following are binary trees?

75% say "All"
20% say "A+B"



Join Code: 225



Binary Tree

Lets define additional terminology for different **types** of binary trees!

1.

On Monday!

2.

↳ Started just last slide

3.

(Tree ADT brainstorm)

Binary Tree: full

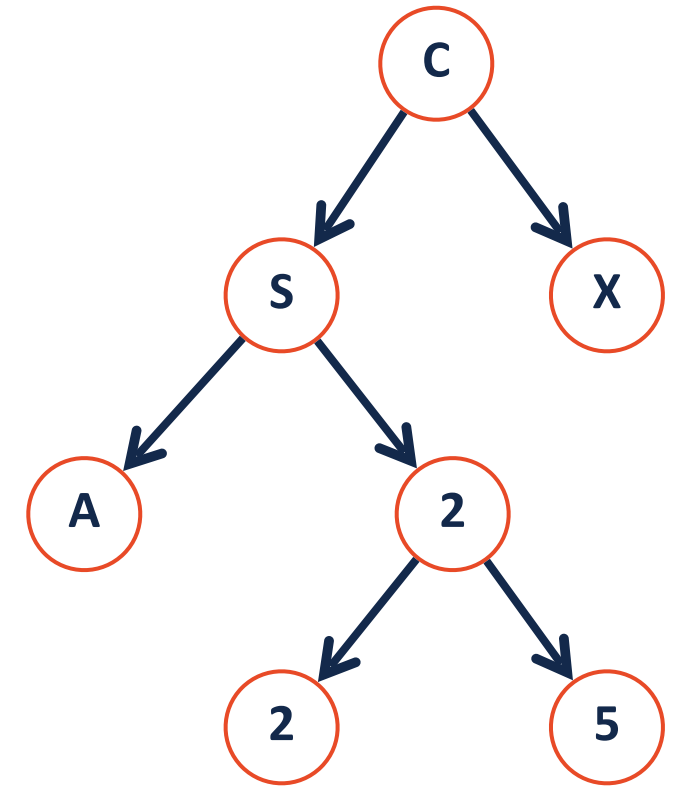
A **full tree** is a binary tree where every node has either 0 or 2 children

A tree **F** is **full** if and only if:

1.

2.

3.



Binary Tree: full

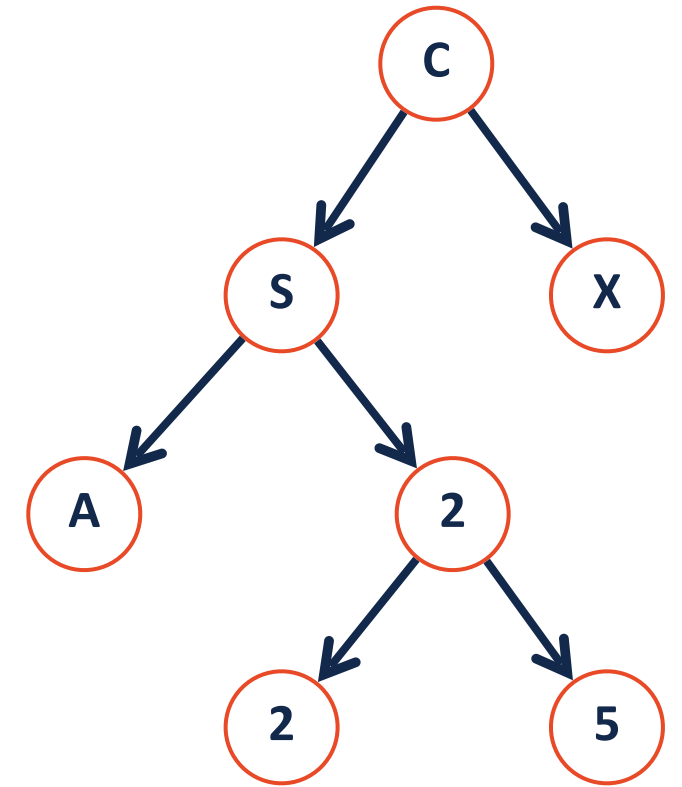
A **full tree** is a binary tree where every node has either 0 or 2 children

A tree **F** is **full** if and only if:

1. $F = \emptyset$

2. $F = (data, \emptyset, \emptyset)$

3. $F = (data, F_l \neq \emptyset, F_r \neq \emptyset)$



Binary Tree: perfect

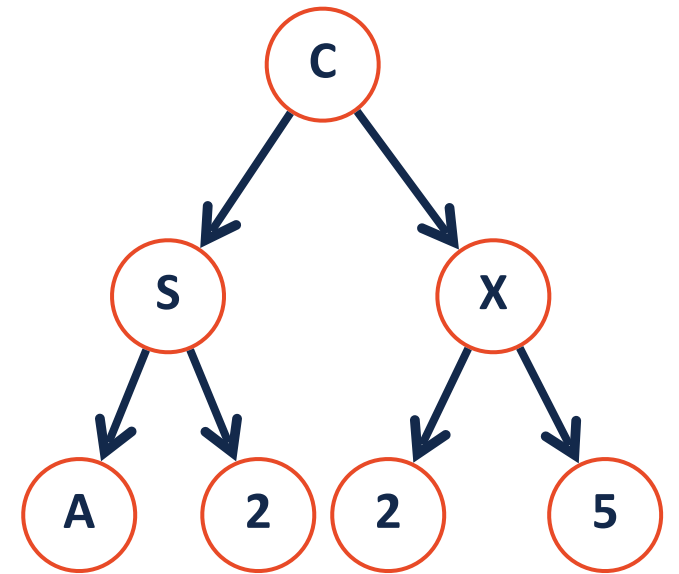
A **perfect tree** is a binary tree where...

Every internal node has 2 children and all leaves are at the same level.

A tree **P** is **perfect** if and only if:

1.

2.



Binary Tree: perfect

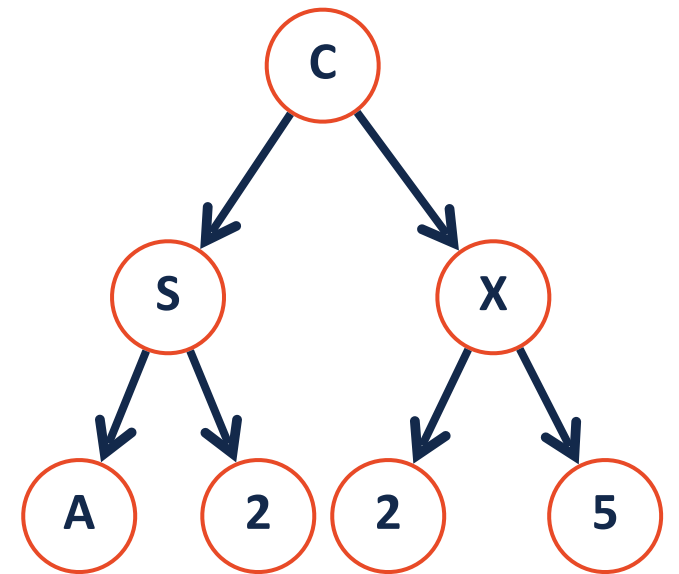
A **perfect tree** is a binary tree where...

Every internal node has 2 children and all leaves are at the same level.

A tree **P** is **perfect** if and only if:

$$1. P_h = (data, P_{h-1}, P_{h-1})$$

$$2. P_0 = (data, \emptyset, \emptyset) \equiv P_{-1} = \emptyset$$



Binary Tree: complete

A **complete tree** is a B.T. where...

All levels except the last are completely filled.

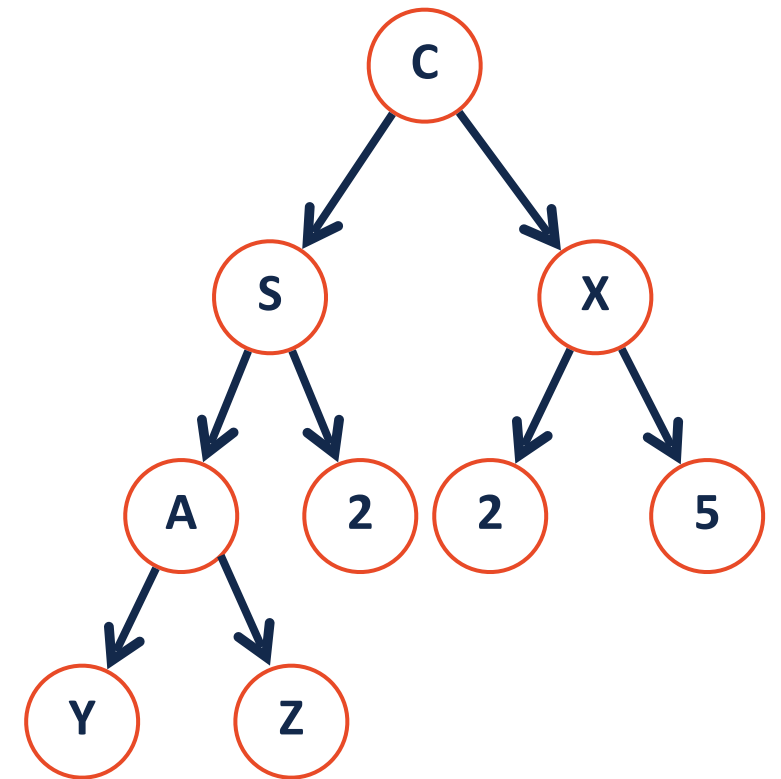
The last level contains at least one node (and is pushed to left)

A tree **C** is **complete** if and only if:

1.

2.

3.



Binary Tree: complete

A **complete tree** is a B.T. where...

All levels except the last are completely filled.

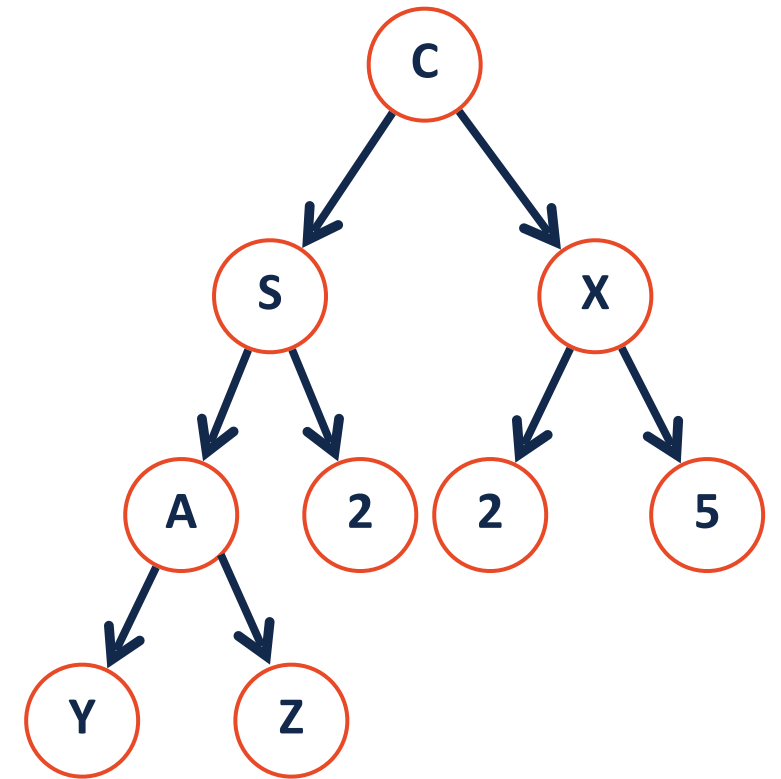
The last level contains at least one node (and is pushed to left)

A tree **C** is **complete** if and only if:

1. $C_h = (data, C_{h-1}, P_{h-2})$

2. $C_h = (data, P_{h-1}, C_{h-1})$

3. $C_{-1} = \emptyset$



Binary Tree



Why do we care?

1. Terminology instantly defines a particular tree structure
2. Understanding how to think 'recursively' is very important.

Binary Tree: Thinking with Types

Is every **full** tree **complete**?

Is every **complete** tree **full**?

Binary Tree: Practicing Proofs

Theorem: If there are n objects in our representation of a binary tree, then there are _____ NULL pointers.

Binary Tree: Practicing Proofs

Theorem: If there are n objects in our representation of a binary tree, then there are $n+1$ NULL pointers.

Base Case:

Binary Tree: Practicing Proofs

Theorem: If there are n objects in our representation of a binary tree, then there are $n+1$ NULL pointers.

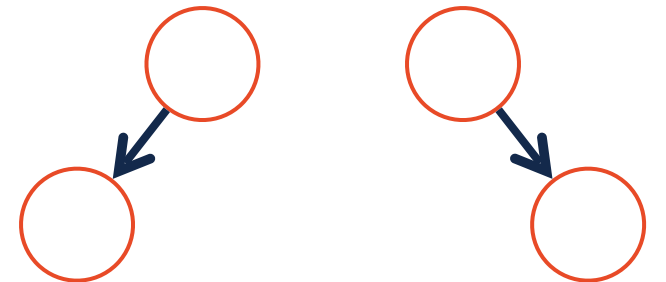
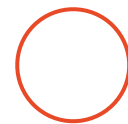
Base Case:

Let $F(n)$ be the max number of NULL pointers in a tree of n nodes

$N=0$ has one NULL

$N=1$ has two NULL

$N=2$ has three NULL



Theorem: If there are n objects in our representation of a binary tree, then there are $n+1$ NULL pointers.

Induction Step:

Theorem: If there are n objects in our representation of a binary tree, then there are $n+1$ NULL pointers.



IS: Assume claim is true for $|T| \leq k - 1$, prove true for $|T| = k$

By def, $T = r, T_L, T_R$. Let q be the # of nodes in T_L

Since r exists, $0 \leq q \leq k - 1$. By IH, T_L has $q + 1$ NULL

All nodes not in r or T_L exist in T_R . So T_R has $k - q - 1$ nodes

$k - q - 1$ is also smaller than k so by IH, T_R has $k - q$ NULL

Total number of NULL is the sum of T_L and T_R : $q + 1 + k - q = k + 1$

Tree ADT

Tree Nodes

↳ get child(i)

↳ get Data()

↳ ?? insert Left / Right / i ?

Tree

↳ height

↳ root

↳ insert (index)