

Data Structures

Array Lists

CS 225

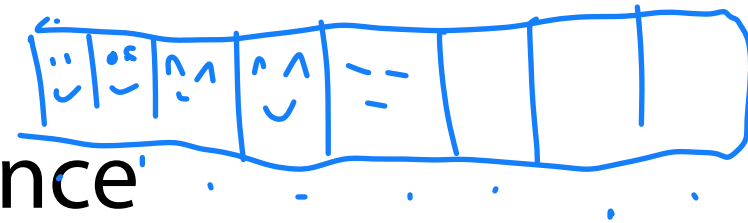
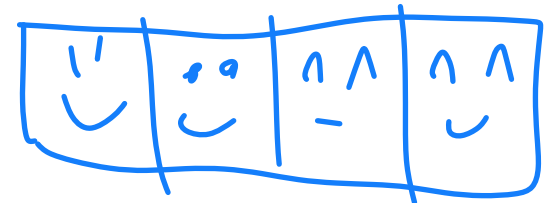
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Post your art on #mp-art!



Exam 1 (9/18 — 9/20) *Cover up to today! 9/9*

Autograded MC and one coding question

Manually graded short answer prompt

Practice exam will be released on PL

Topics covered can be found on website

Registration started August 22

<https://courses.engr.illinois.edu/cs225/fa2024/exams/>

Learning Objectives

Discuss data variables for implementing array lists

review

~~Introduce~~ array list implementations



Discuss amortized analysis

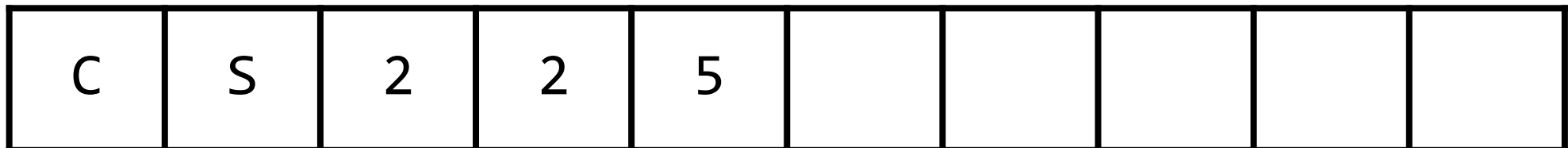
Consider extensions to lists

List Implementations

1. Linked List



2. Array List



Linked List Runtimes



@Front

@RefPointer

@Index

Insert

$O(1)$

$O(1)$

$O(n)$

↳ Find $O(n)$

↳ Modification $O(1)$

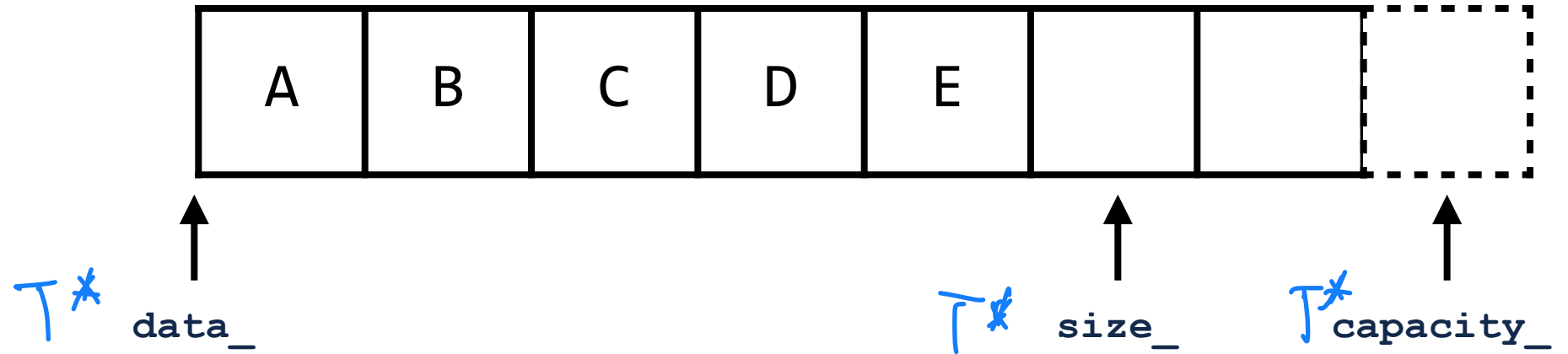
Delete

$O(1)$

$O(1)$

$O(n)$

Array List



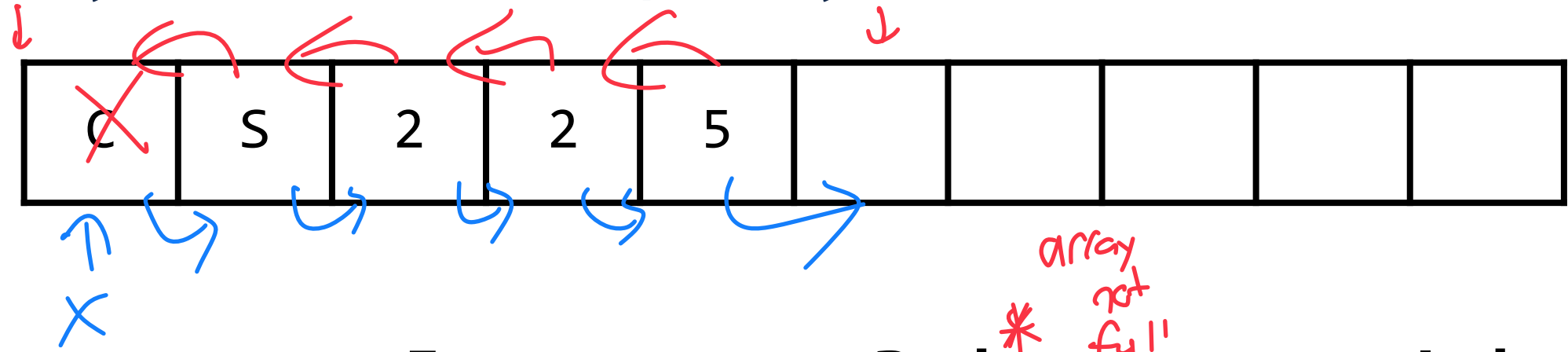
In C++, vector is implemented as:

- 1) **Data:** Stored as a pointer to array start
- 2) **Size:** Stored as a pointer to the next available space
- 3) **Capacity:** Stored as a pointer past the end of the array

Array List: Not at capacity

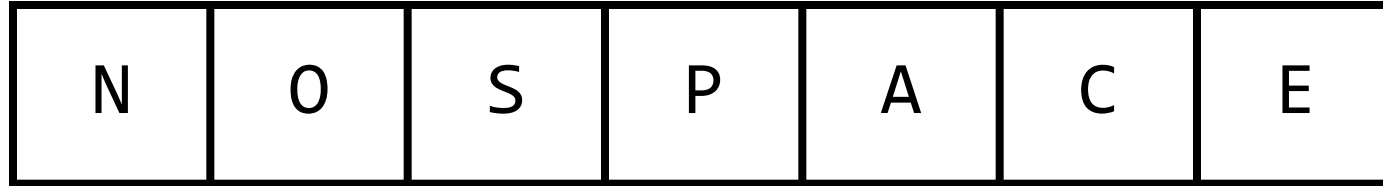
insert(x)

Capacity
↓

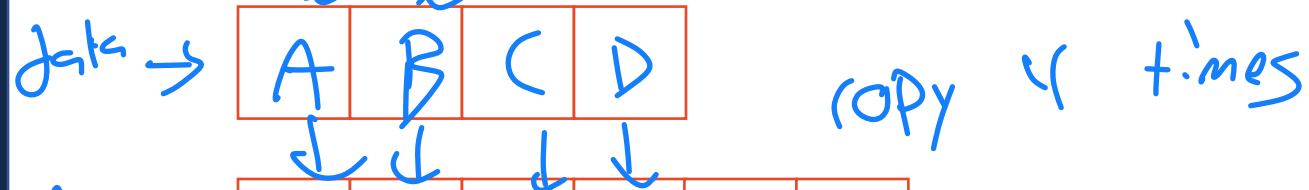


	@Front	@Back * array not full	@Index
Insert	$O(n)$	$O(1)$	$O(n)$
Delete	$O(n)$	$O(1)$	$O(n)$

Array List: addspace(data)



Resize Strategy: +2 elements every time



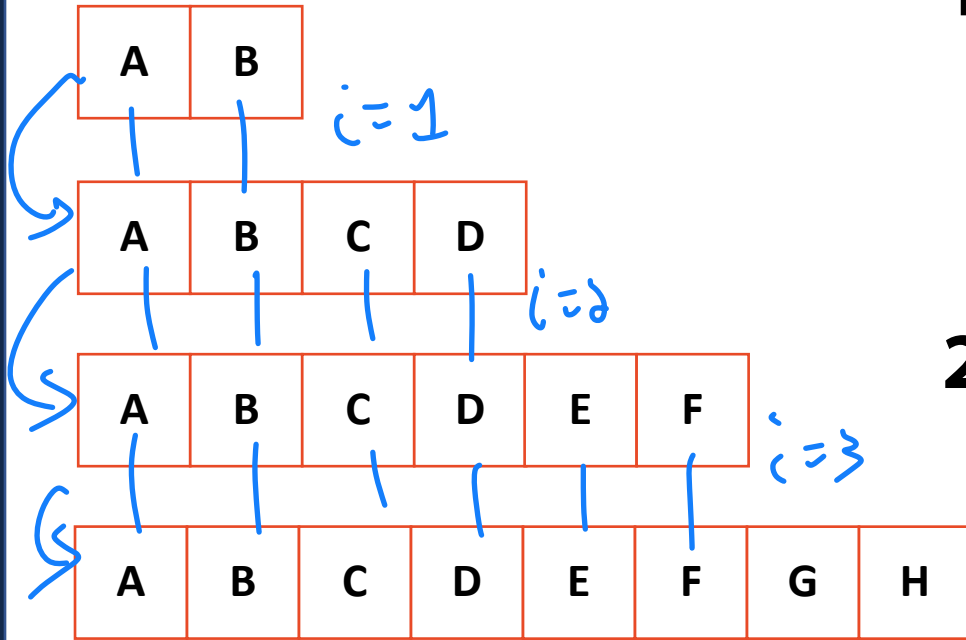
Resize Strategy: +2 elements every time

1) How many copy calls per reallocation?

At iteration i , 2^i copies

2) Total reallocations for N objects?

$$K = N/2$$



Resize Strategy: +2 elements every time

1) How many copy calls per reallocation?

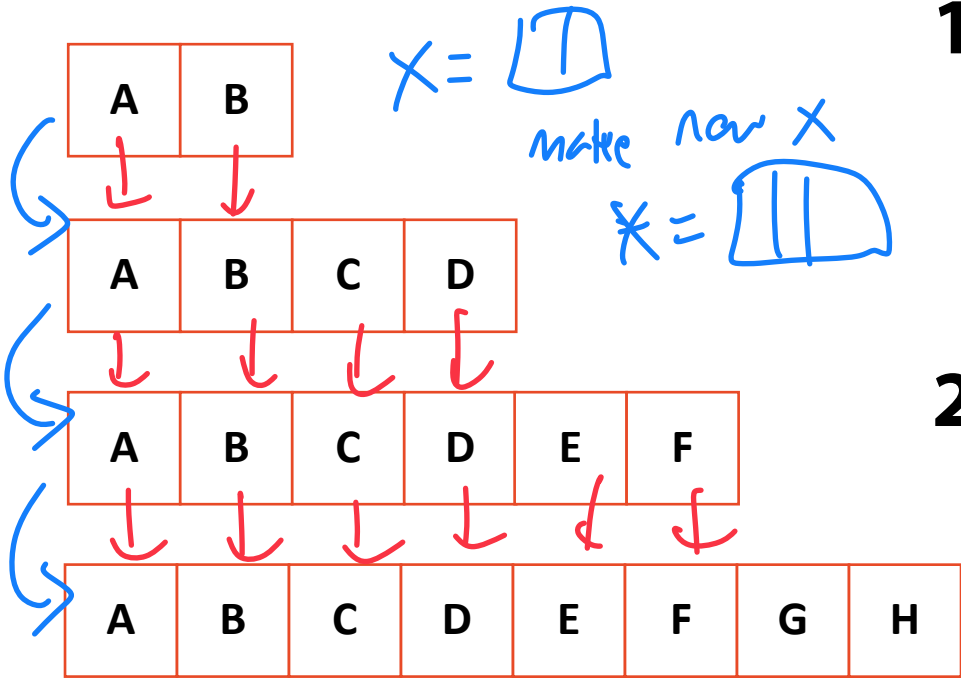
For reallocation i , $2i$ copy calls are made

2) Total reallocations for N objects?

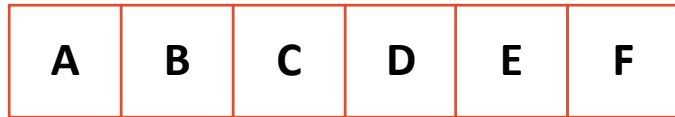
Let k be the number of reallocs, $k = \frac{N}{2}$

$$\sum_{i=1}^k 2i = k(k+1) = k^2 + k$$

Total number of copy calls:



Resize Strategy: +2 elements every time



1) How many copy calls per reallocation?

For reallocation i , $2i$ copy calls are made

2) Total reallocations for N objects?

Let k be the number of reallocs, $k = \frac{N}{2}$

Total number of copy calls:

$$\sum_{i=1}^k 2i = k(k+1) = k^2 + k$$

Handwritten notes: $k = N/2$ $= \frac{N^2}{4} + \frac{N}{2}$

... For N objects: $\frac{N^2 + 2N}{4}$

Resize Strategy: +2 elements every time

Avg case is best



Total copies for N inserts:

$$\frac{N^2 + 2N}{4}$$

↳ is not amortized!

Amortized: Total work over N inserts

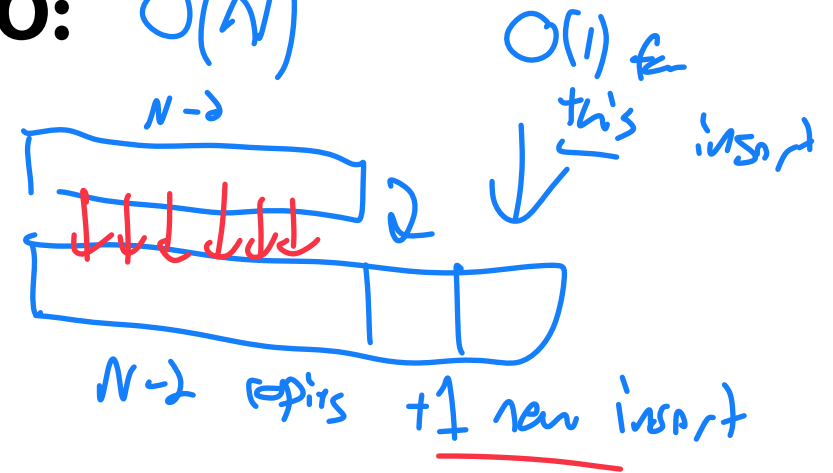
Big O: $O(N)$

Estimate work for one insert

↳ $\frac{N^2 + 2N}{4N} \Rightarrow O(N)$

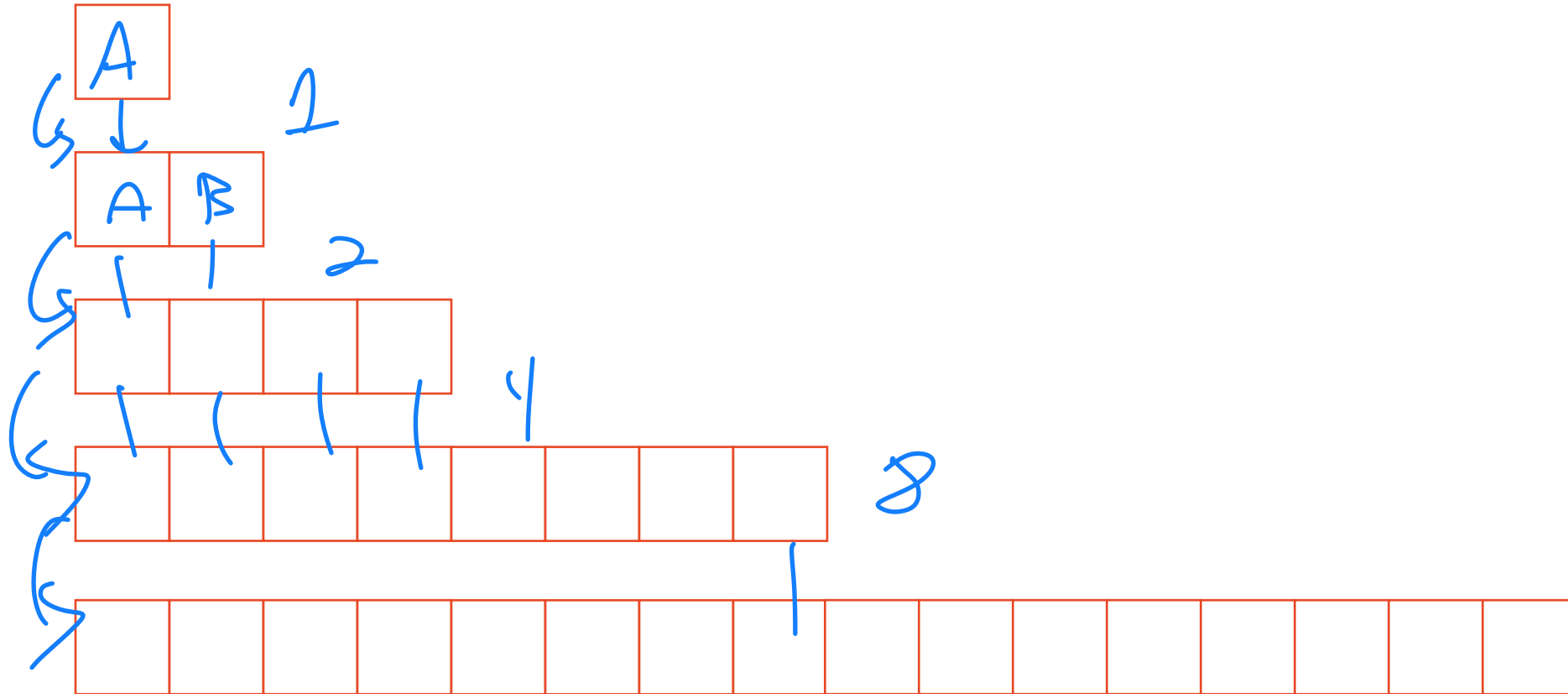
Compare & contrast

Divide by N to get amortized for one insert



$N-1$ th insert $O(N)$

Resize Strategy: x2 elements every time



Resize Strategy: x2 elements every time

1) How many copy calls per reallocation?

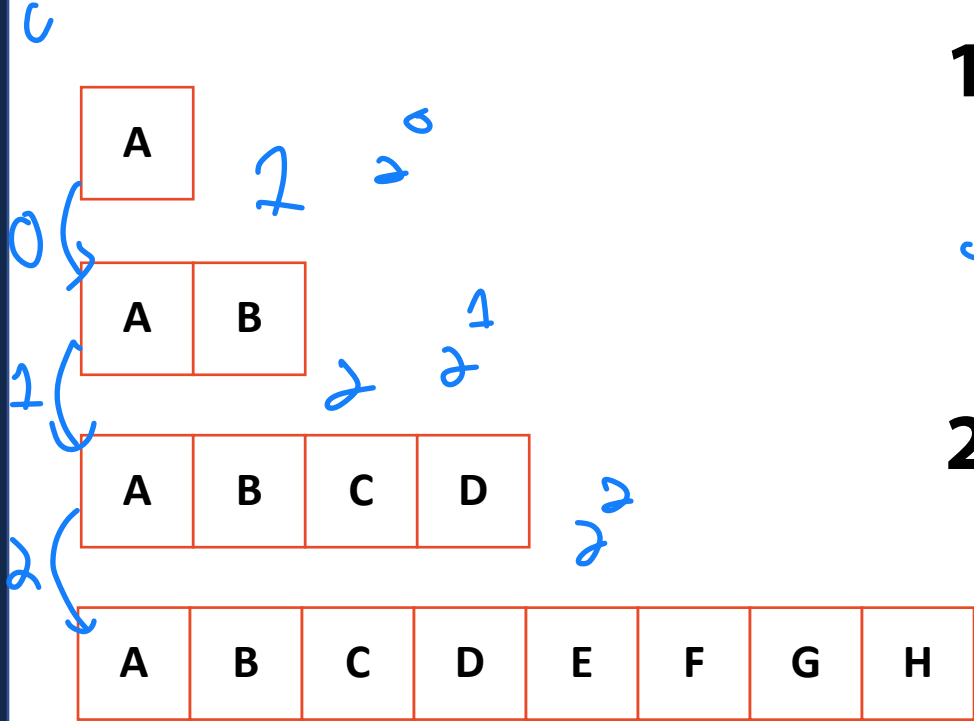
for realloc i , 2^i copies More copies

2) Total reallocations for N objects?

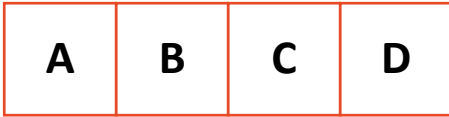
$$N \leq \underline{2^k} \rightarrow k = \lceil \log_2 N \rceil$$

is larger or equal to N

Less
realloc



Resize Strategy: x2 elements every time



Total number of copy calls:

1) How many copy calls per reallocation?

For reallocation i , 2^i copy calls are made

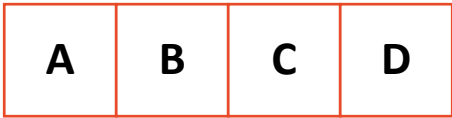
2) Total reallocations for N objects?

$k = \text{final realloc needed} = \lceil \log_2 n \rceil$

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

Resize Strategy: x2 elements every time

1) How many copy calls per reallocation?



For reallocation i , 2^i copy calls are made

2) Total reallocations for N objects?

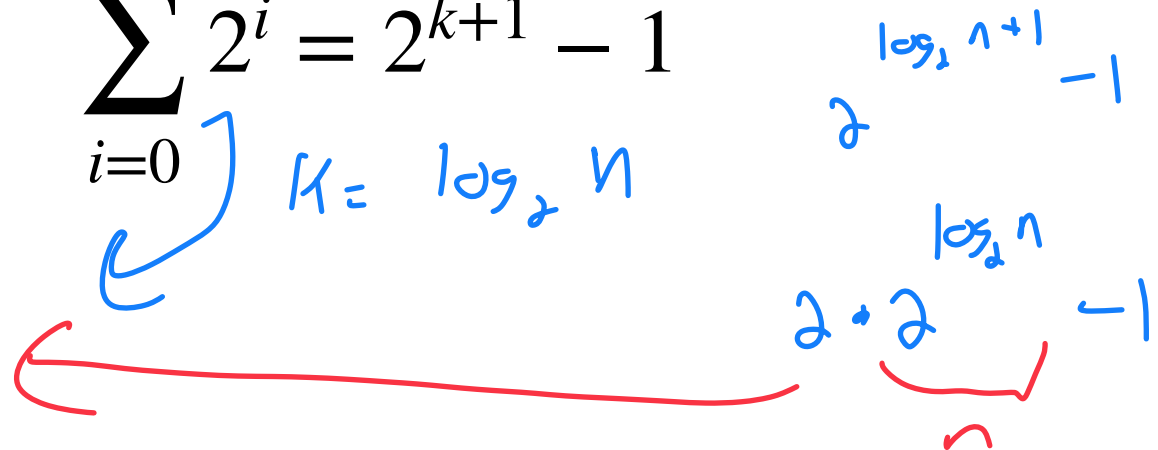
$k = \text{final realloc needed} = \lceil \log_2 n \rceil$

Total number of copy calls:

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$

$k = \log_2 n$

... For N objects: $2n - 1$



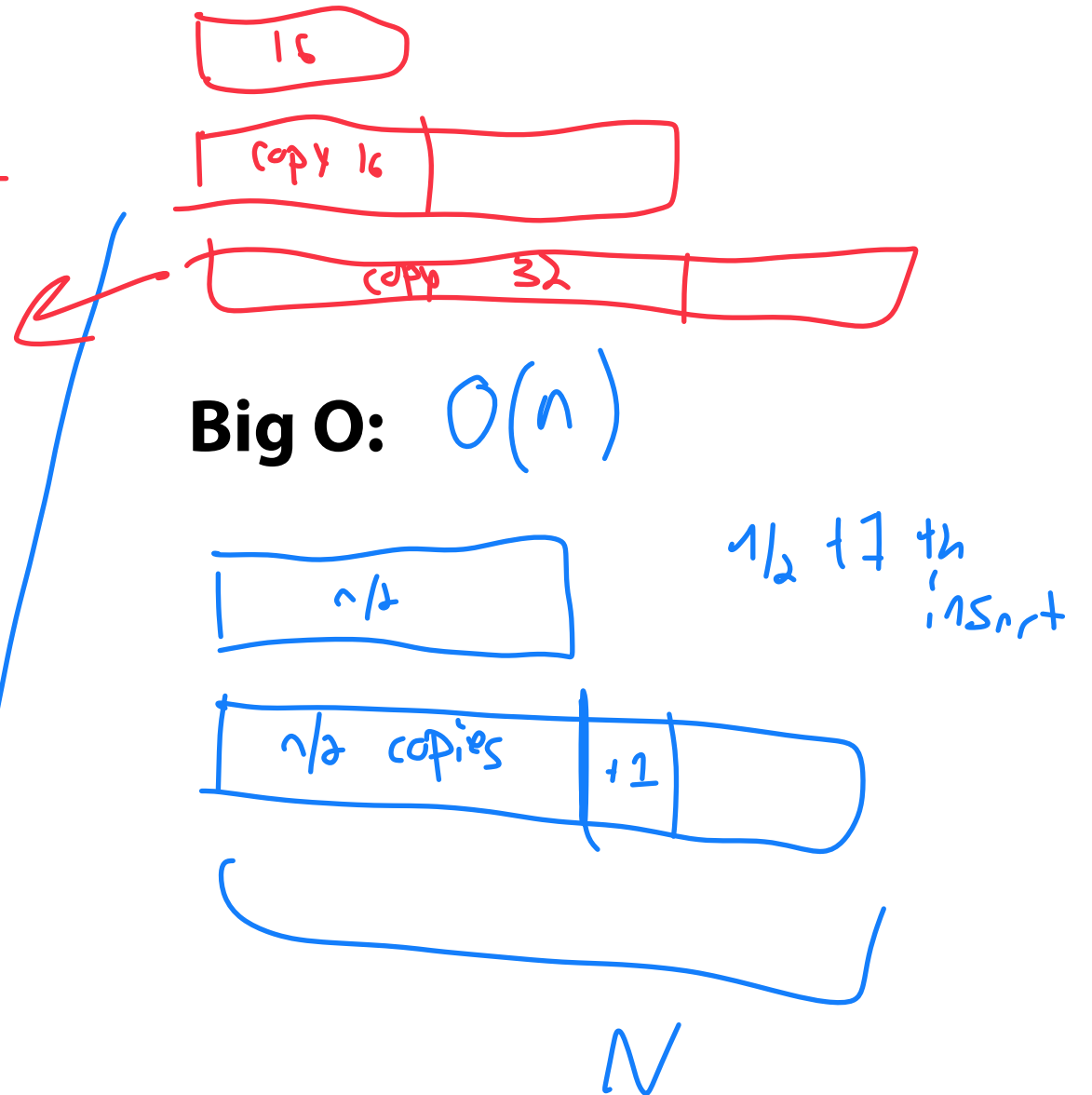
Resize Strategy: x2 elements every time

Total copies for n inserts: $2n - 1$

Amortized:

To do one insert $\frac{2n-1}{n}$
 $\hookrightarrow \underline{O(1)^*}$

Diff b/w amortized / average?
- Assumptions made
(No prob distribution here!)



List Implementation



	Singly Linked List	Array
Look up arbitrary location ↳ index	$O(n)$	$O(1)$ 😊
Insert after given element ↳ ref pointer	$O(1)$ 😊	$O(n)$
Remove after given element	$O(1)$ 😊	$O(n)$
Insert at arbitrary location ↳	Find $O(n)$ change $O(1)$ $O(n)$	Find $O(1)$ change $O(n)$ $O(n)$
Remove at arbitrary location	$O(n)$	$O(n)$
Search for an input value _____	$O(n)$	$O(n)$

Special cases

insert / remove front $O(1)$

if array not full
always amortized

insert Back
remove is $O(1)$ *

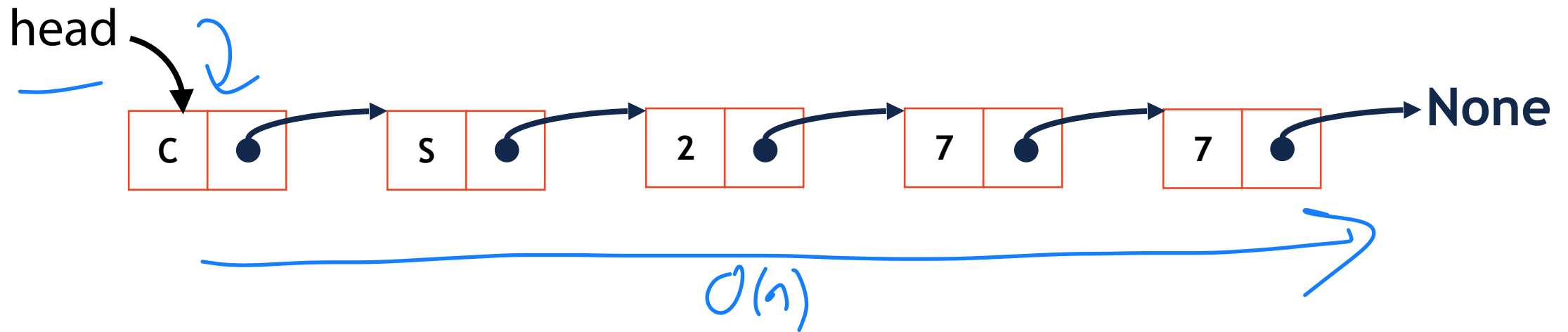
Thinking critically about lists: tradeoffs

The implementations shown are foundational (simple).

Can we make our lists better at some things? What is the cost?

Thinking critically about lists: tradeoffs

Getting the size of a linked list has a Big O of: $O(n)$

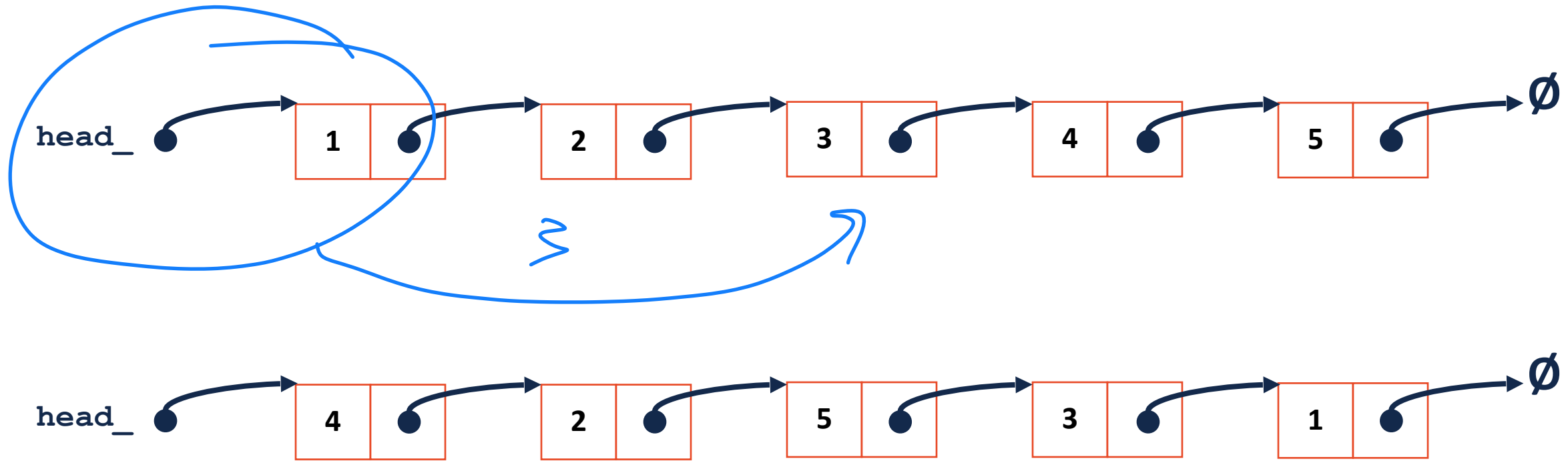


why not add size integer to list class?

+ 1 int in memory cost

+ 1 op to increment/dec on insert/remove

Thinking critically about lists: tradeoffs



if I only want to touch the min val
↳ each min val in order

Thinking critically about lists: tradeoffs

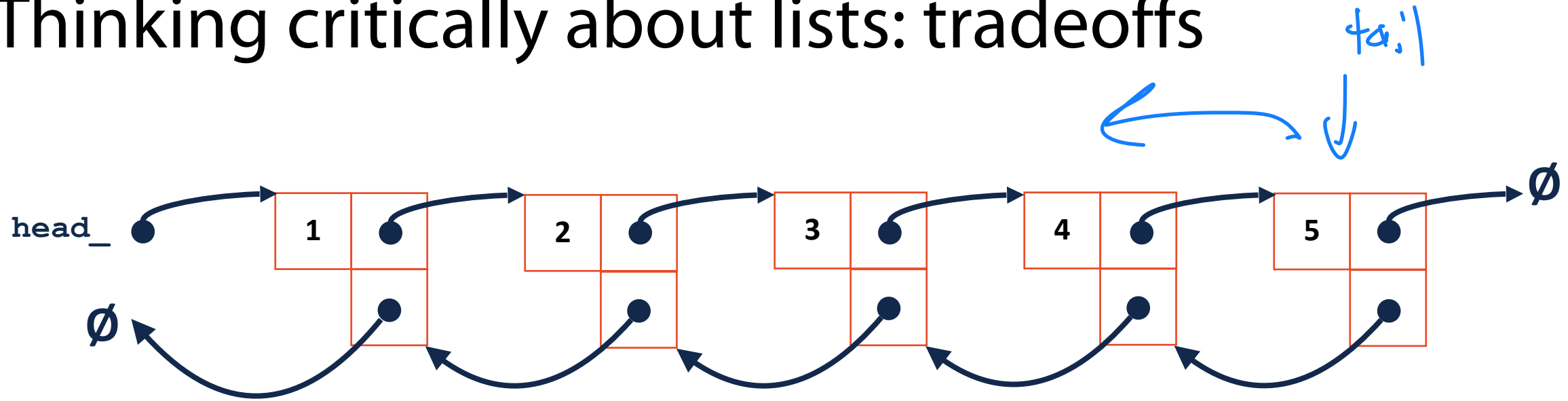
2	7	5	9	7	14	1	0	8	3
---	---	---	---	---	----	---	---	---	---

0	1	2	3	5	7	7	8	9	14
---	---	---	---	---	---	---	---	---	----

Binary search! 😊

Insert Slower

Thinking critically about lists: tradeoffs



$O(n)$ memory increase

Size = #



Thinking critically about lists: tradeoffs

When we discuss data structures, consider how they can be modified or improved!

Next time: Can we make a 'list' that is $O(1)$ to insert and remove? What is our tradeoff in doing so?