String Algorithms and Data Structures

Approximate Pattern Matching

CS 199-225

Brad Solomon

November 18, 2024

Department of Computer Science

the last 'main' lecture ^_^

Learning Objectives

Review approximate pattern matching

Formalize edit distance storage as an 'edit string'

Discuss strategies for efficient APM with edits

Introduce dynamic programming

Approximate Pattern Matching

Input: A text *T*, a pattern *P*, and a distance *d*

Output: All positions in *T* where *P* has at most *d* mismatches or edits

P: word

T: There would have been a time for such a word

Alignment 1:
```
would
word
```

~~Not a match!~~

Distance 2 match!

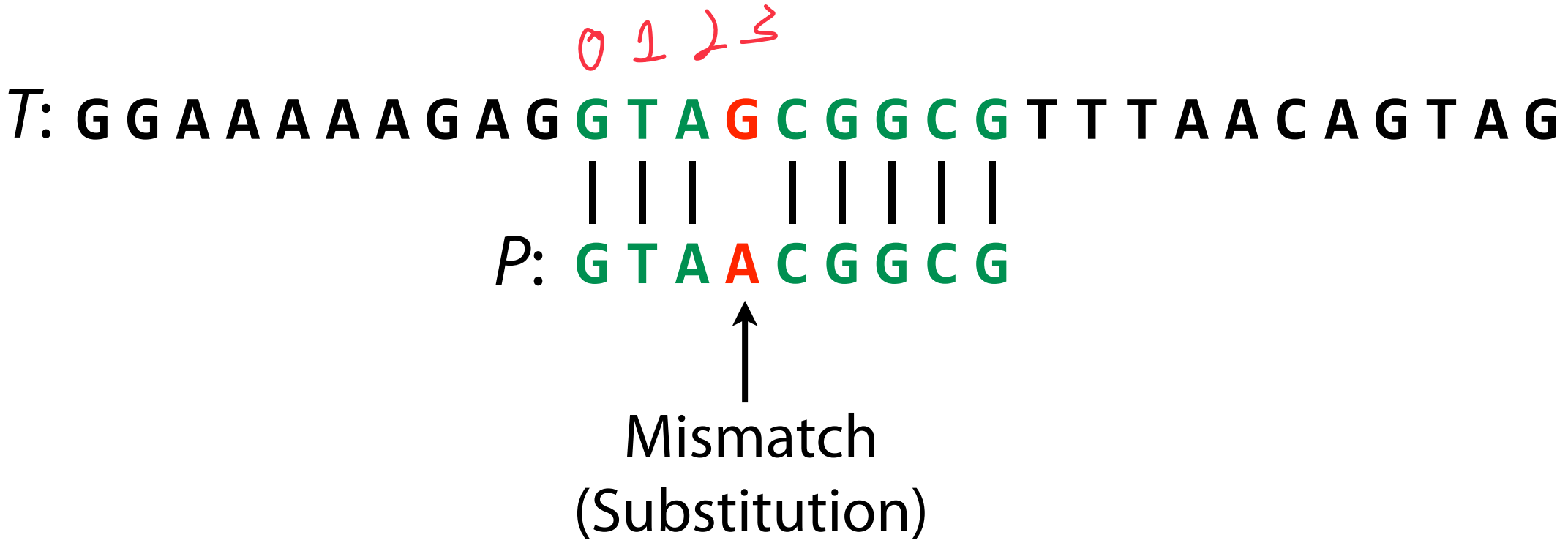
Alignment 2:
```
word
word
```

Match!

Distance 0 match!

Hamming Distance

The number of **substitutions** required to turn one string into another



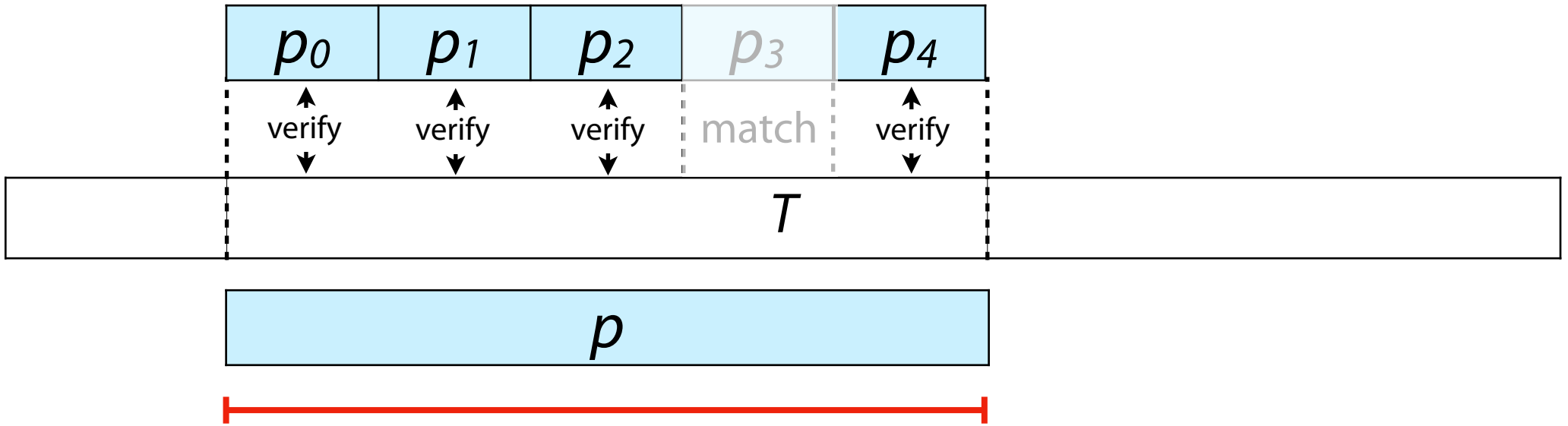
Approximate Pattern Matching

As a ***heuristic***, seed and extend reduces the overall search space

T: **`There would have been a time for such a word`**

`word`

`word`
`word`



Only consider mismatches while verifying a seed hit

Edit Distance

```
Score =  248 bits (129), Expect = 1e-63
Identities = 213/263 (80%), Gaps = 34/263 (12%)
Strand = Plus / Plus
```

Substitution

```
Query: 161 atatcaccacgtcaaaggtgactccaactcca---ccactccattttgttcagataatgc 217
           ||||||||||||||||||||||||||||  |     | |   || ||||||||||||||
Sbjct: 481 atatcaccacgtcaaaggtgactccaact-tattgatagtgtttttatgttcagataatgc 539

Query: 218 ccgatgatcatgtcatgcagctccaccgattgtgagaacgacagcgacttccgtcccagc 277
           |||||||   ||||||||||||||||||| || |           ||||||||||||
Sbjct: 540 ccgatgactttgtcatgcagctccaccgattttg-g----------ttccgtcccagc 586
```

Deletion

```
Query: 278 c-gtgcc--aggtgctgcctcagattcaggttatgccgctcaattcgctgcgtatatcgc 334
           |  ||  |  | ||||||||||||||||||||||||||||||||||||| |||||||||
Sbjct: 587 caatgacgta-gtgctgcctcagattcaggttatgccgctcaattcgctgggtatatcgc 645

Query: 335 ttgctgattacgtgcagctttcccttcaggcggga----------ccagccatccgtc 382
           ||||||||||||||||||||||||||||||||           ||||||||||||
Sbjct: 646 ttgctgattacgtgcagctttcccttcaggcgggattcatacagcggccagccatccgtc 705
```

Insertion

```
Query: 383 ctccatatc-accacgtcaaagg 404
           ||||||||  ||||||||||||
Sbjct: 706 atccatatcaaccacgtcaaagg 728
```

Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

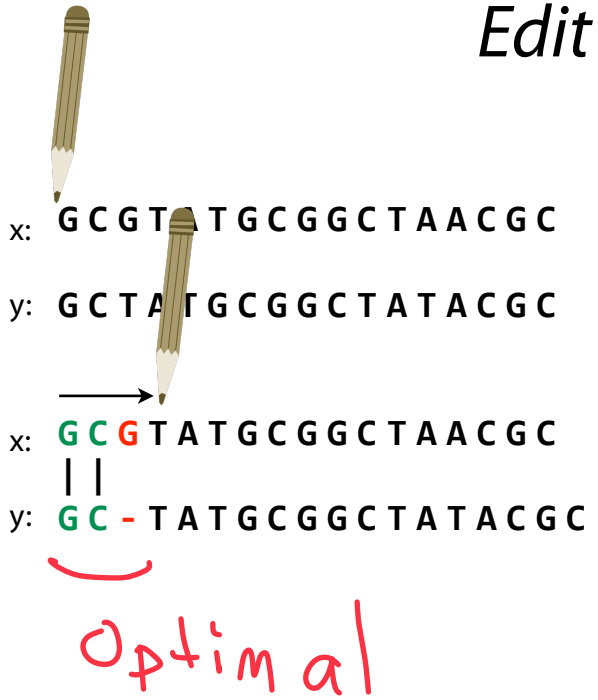
x: **G C G T A T G C G G C T A A C G C**

y: **G C T A T G C G G C T A T A C G C**

Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

Edit string summarizes how editor turns *x* into *y:*



Optimal

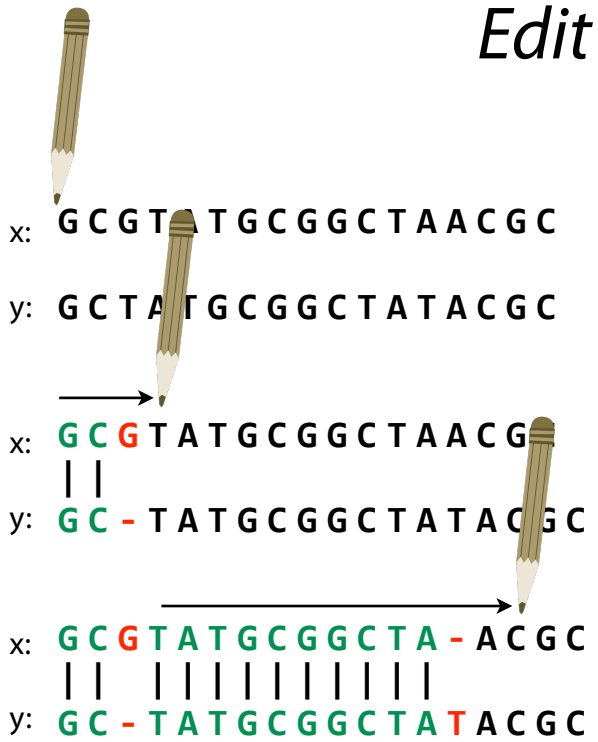
Operations:
M = match, **R** = replace (substitute),
I = insert into *x*, **D** = delete from *x*

M M

Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

Edit string summarizes how editor turns *x* into *y:*



Operations:

M = match, **R** = replace (substitute),

I = insert into *x*, **D** = delete from *x*

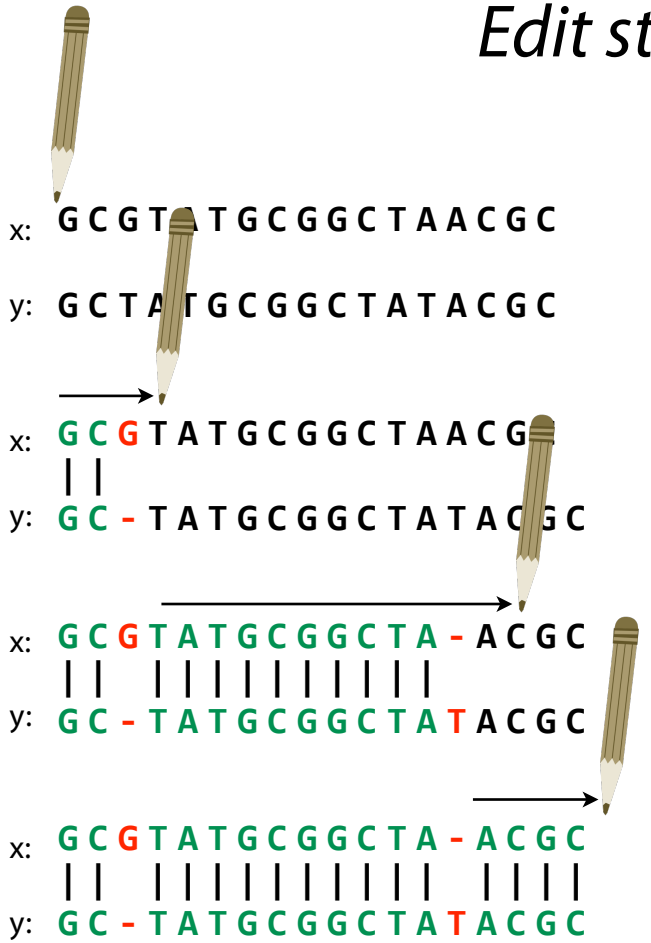
MMD

Reminder: this is **D**, not **I**, because we have to delete a character from *x* to make it more like *y*

Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

Edit string summarizes how editor turns *x* into *y:*



Operations:
M = match, **R** = replace (substitute),
I = insert into *x*, **D** = delete from *x*

MMD

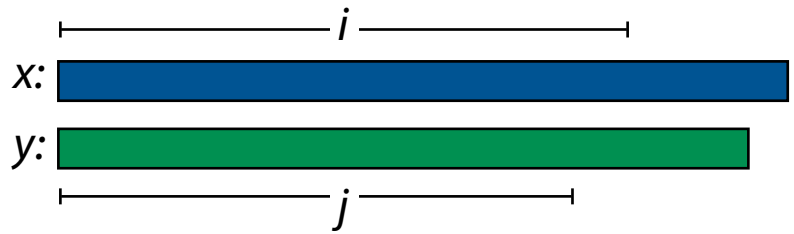
Reminder: this is **D**, not **I**, because we have to delete a character from *x* to make it more like *y*

MMDMMMMMMMMMMI

MMDMMMMMMMMMMIMMMM

Edit Distance

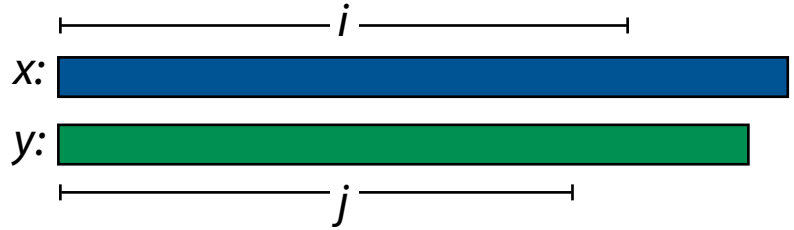
D[*i*, *j*]: edit distance between length-*i* prefix of *x* and length-*j* prefix of *y*



Optimal edit string for D[*i*, *j*] is built by ***extending a shorter optimal string by 1 operation***. 3 options:

Edit Distance

D[*i*, *j*]: edit distance between length-*i* prefix of *x* and length-*j* prefix of *y*



Optimal edit string for D[*i*, *j*] is built by ***extending a shorter optimal string by 1 operation***. 3 options:

Append **D** to transcript for D[*i*-1, *j*]

Append **I** to transcript for D[*i*, *j*-1]

Append **M** or **R** to transcript for D[*i*-1, *j*-1]

We choose based on whichever option has the fewest edits

Edit Distance

X: GTTTAA

Y: GGTTTA

D[5, 6]

GTTTA
GGTTTA

D[5, 5]

GTTTA
GGTTT

D[6, 5]

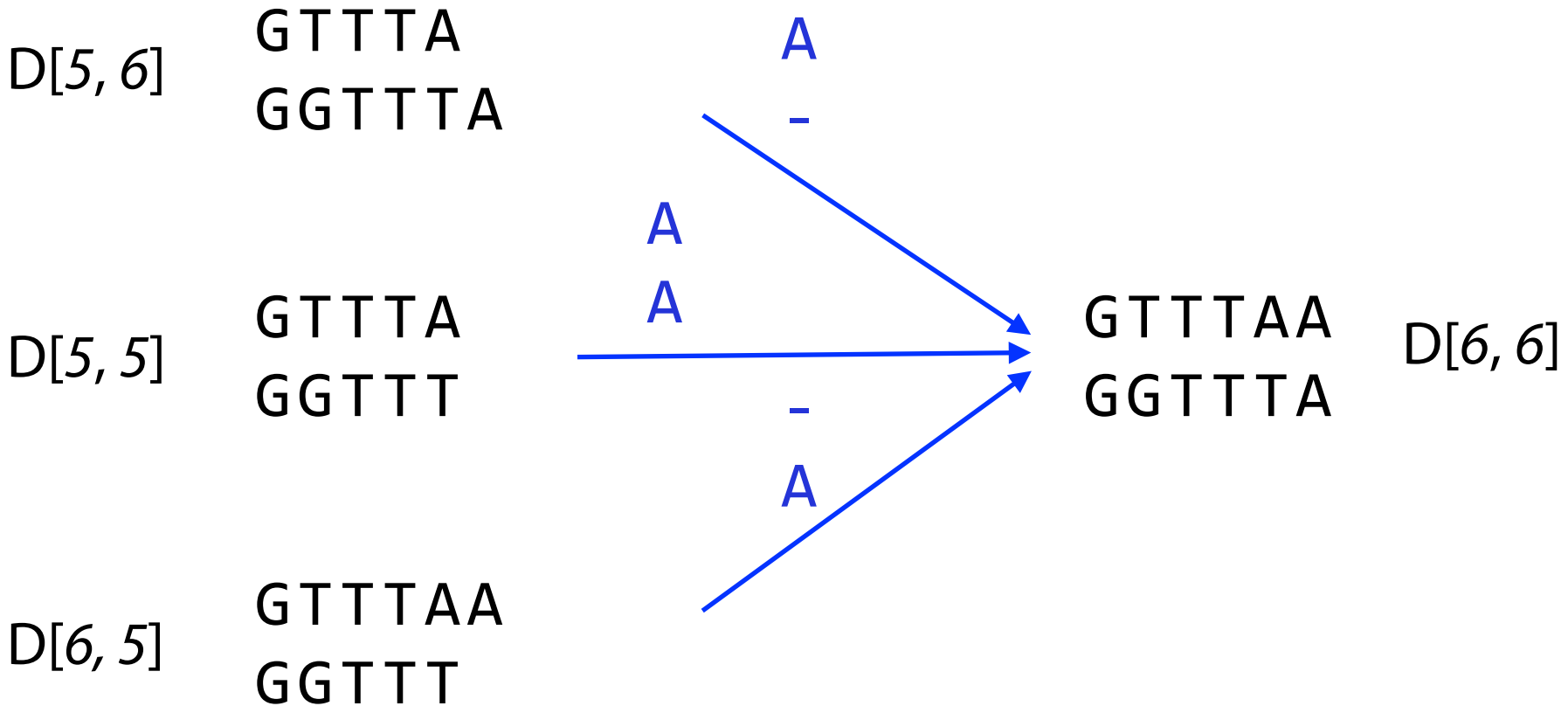
GTTTAA
GGTTT

D[6, 6]

GTTTAA
GGTTTA

Edit Distance

X: GTTTAA Y: GGTTTA



Edit Distance

X: GTTTAA Y: GGTTTA

D[*5*, *6*]

```
GTTTA
GGTTTA
```

D[*5*, *5*]

```
GTTTA
GGTTT
```

D[*6*, *5*]

```
GTTTAA
GGTTT
```

Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[*5*, *6*]

G	-	T	T	T	A
G	G	T	T	T	A

D[*5*, *5*]

GTTTA
GGTTT

D[*6*, *5*]

GTTTAA
GGTTT

Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[*5*, *6*]

G	-	T	T	T	A
G	G	T	T	T	A

MRMMR

D[*5*, *5*]

G	T	T	T	A
G	G	T	T	T

D[*6*, *5*]

GTTTAA
GGTTT

Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[*5*, *6*]

G	-	T	T	T	A
G	G	T	T	T	A

MRMMR

D[*5*, *5*]

G	T	T	T	A
G	G	T	T	T

MRMMRD

D[*6*, *5*]

G	T	T	T	A	A
G	G	T	T	T	-

Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[*5*, *6*]

G	-	T	T	T	A
G	G	T	T	T	A

G	-	T	T	T	A	
G	G	T	T	T	A	

D[*6*, *6*][1]

MRMMR

D[*5*, *5*]

G	T	T	T	A
G	G	T	T	T

G	T	T	T	A	
G	G	T	T	T	

D[*6*, *6*][2]

MRMMRD

D[*6*, *5*]

G	T	T	T	A	A
G	G	T	T	T	-

G	T	T	T	A	A	
G	G	T	T	T	-	

D[*6*, *6*][3]

Edit Distance

Proof by induction

Assume optimal for all smaller (i, j)

X: GTTTAA Y: GGTTTA

Deletion

D[5, 6]

MIMMMM

G	-	T	T	T	A
G	G	T	T	T	A

MIMMMMD

G	-	T	T	T	A	A
G	G	T	T	T	A	-

D[6, 6]¹ 2

Substitution

D[5, 5]

MRMMR

G	T	T	T	A
G	G	T	T	T

MRMMRM

G	T	T	T	A	A
G	G	T	T	T	A

D[6, 6]² 2

Insert

D[6, 5]

MRMMRD

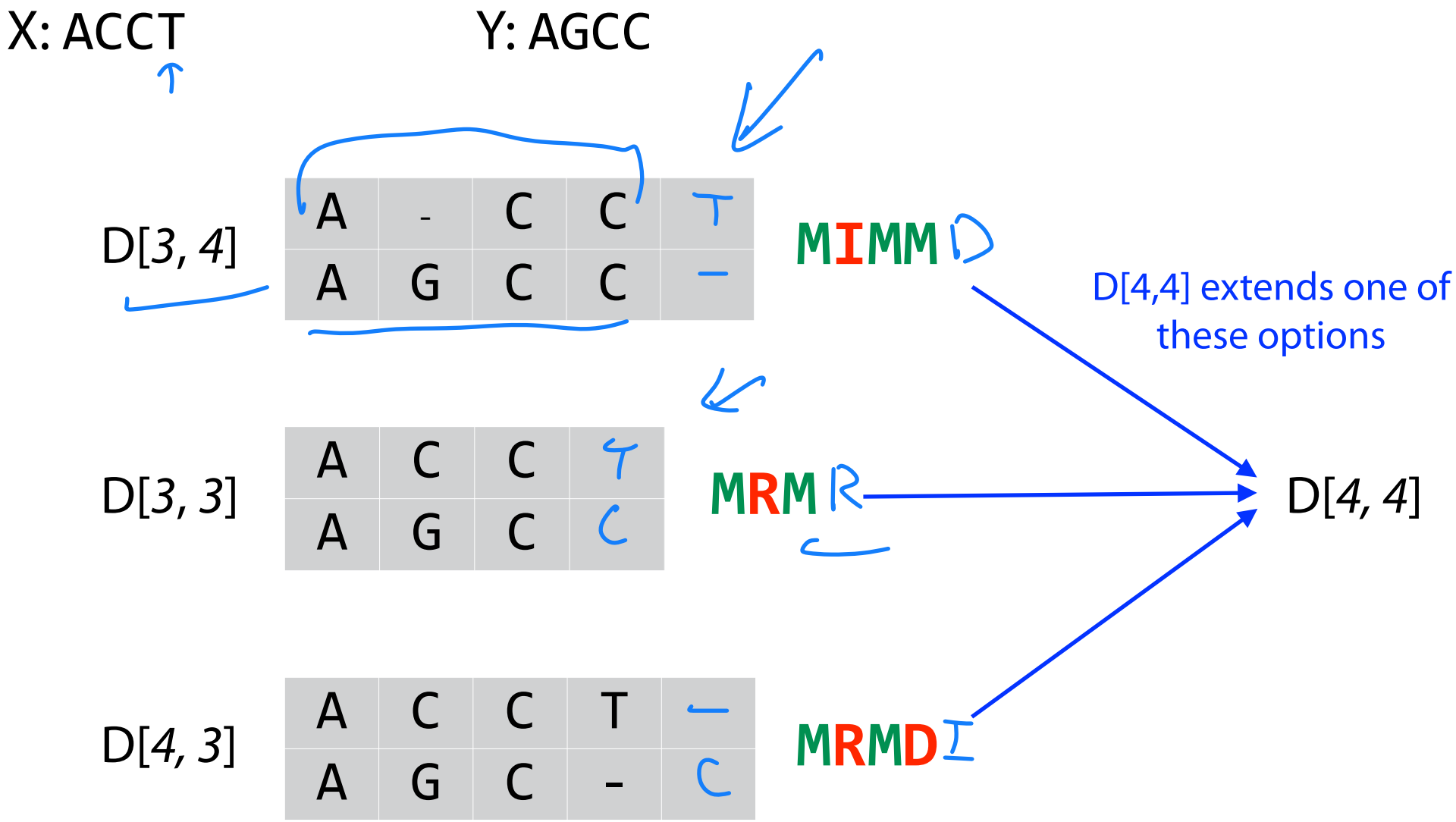
G	T	T	T	A	A
G	G	T	T	T	-

MRMMRDI

G	T	T	T	A	A	-
G	G	T	T	T	-	A

D[6, 6]³ 4

Edit Distance



Edit Distance

X: ACCT Y: AGCC

D[*3*, *4*]

A	-	C	C	T
A	G	C	C	-

MIMMD

D[3, 3]

A	C	C	T
A	G	C	C

MRMR

D[*4*, *4*]

D[4,4] extends one of these options

D[*4*, *3*]

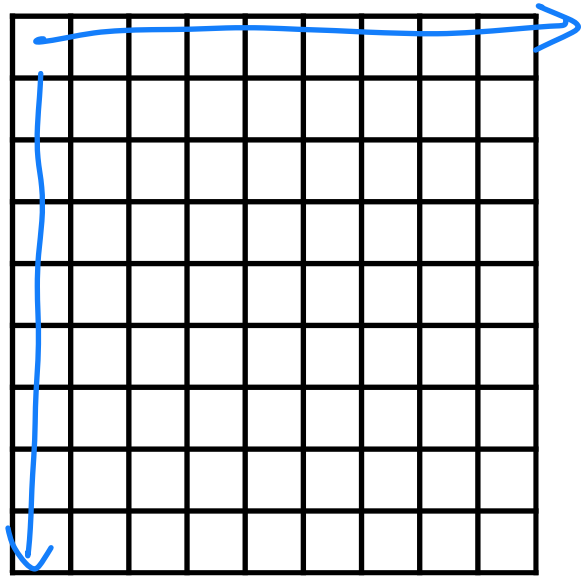
A	C	C	T	-
A	G	C	-	C

MRMDI

extra bad

Edit Distance

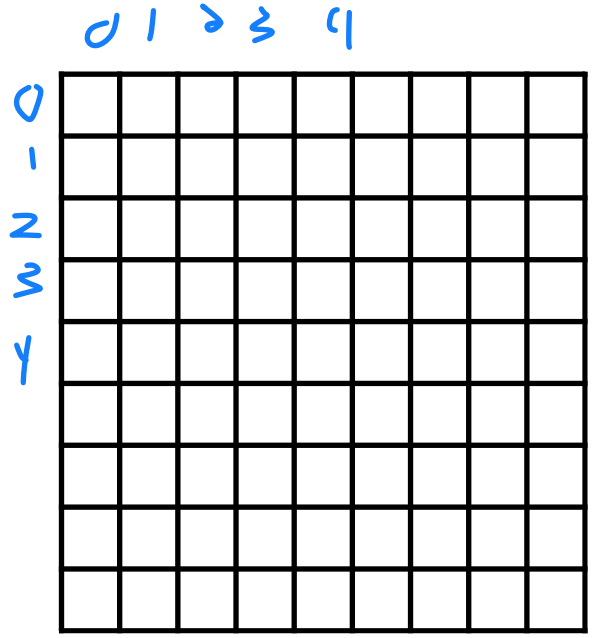
We can store D as a 2D matrix:



Let $D[0, j] = j$, and let $D[i, 0] = i$

Edit Distance

We can store D as a 2D matrix:



Is at beginning

Ds at beginning

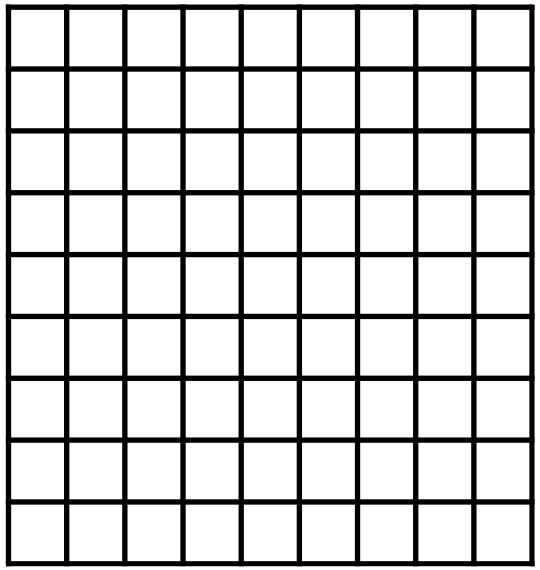
Let $D[0, j] = j$, and let $D[i, 0] = i$

```
----BBBBBBB
    |||||||
AAAABBBBBBB
```

```
the longest----
    |||||||
----longest day
```

Edit Distance

We can store D as a 2D matrix:



Is at beginning Ds at beginning

Let $D[0, j] = j$, and let $D[i, 0] = i$

Otherwise, let $D[i, j] = \min \begin{cases} D[i-1, j] + 1 \\ D[i, j-1] + 1 \\ D[i-1, j-1] + \delta(x[i-1], y[j-1]) \end{cases}$

D

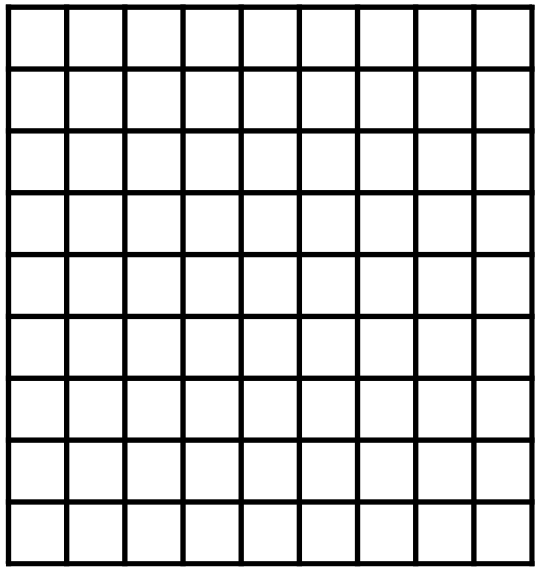
I

distance function

$\delta(a, b)$ is 0 if $a = b$, 1 otherwise

match replacement

Edit Distance



We can store D as a 2D matrix:

Is at beginning

Ds at beginning

Let $D[0, j] = j$, and let $D[i, 0] = i$

Otherwise, let $D[i, j] = \min \begin{cases} D[i-1, j] + 1 \\ D[i, j-1] + 1 \\ D[i-1, j-1] + \delta(x[i-1], y[j-1]) \end{cases}$

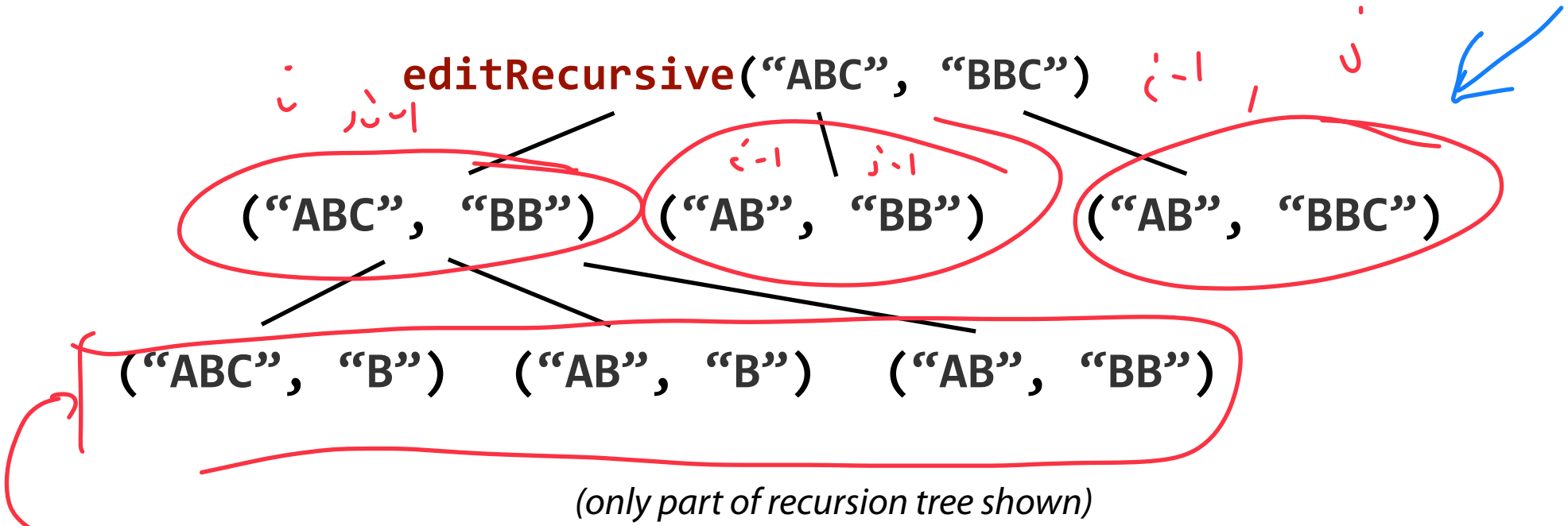
vertical (D)

horizontal (I)

diagonal (M or R)

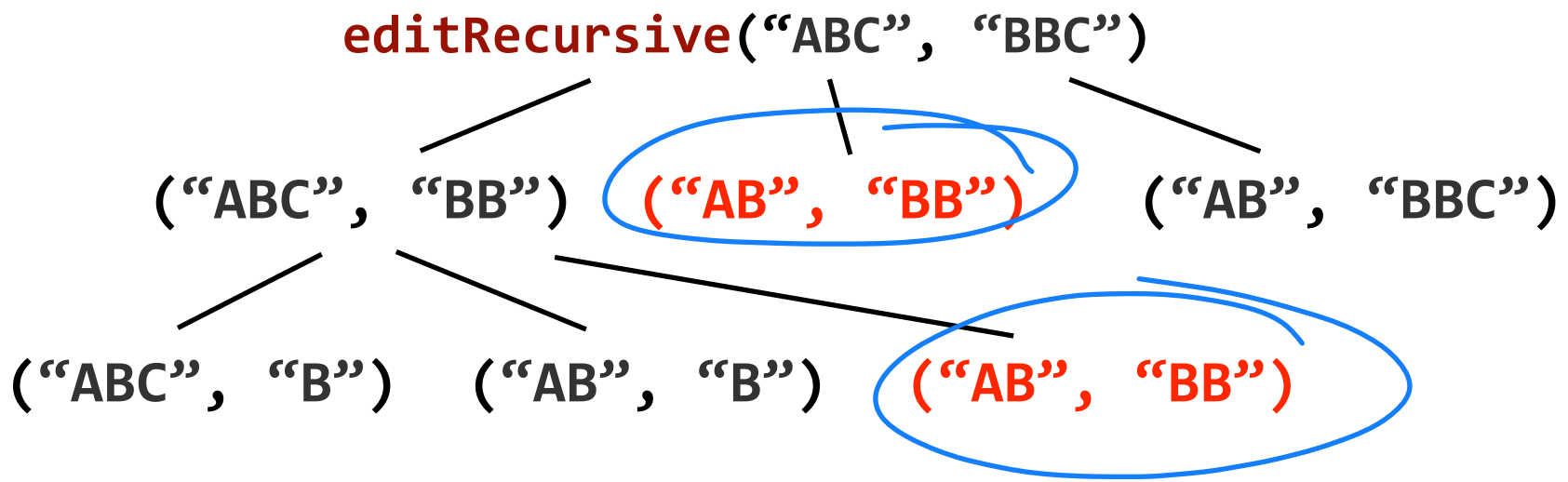
$\delta(a, b)$ is 0 if $a = b$, 1 otherwise

Edit Distance



(only part of recursion tree shown)

Edit Distance



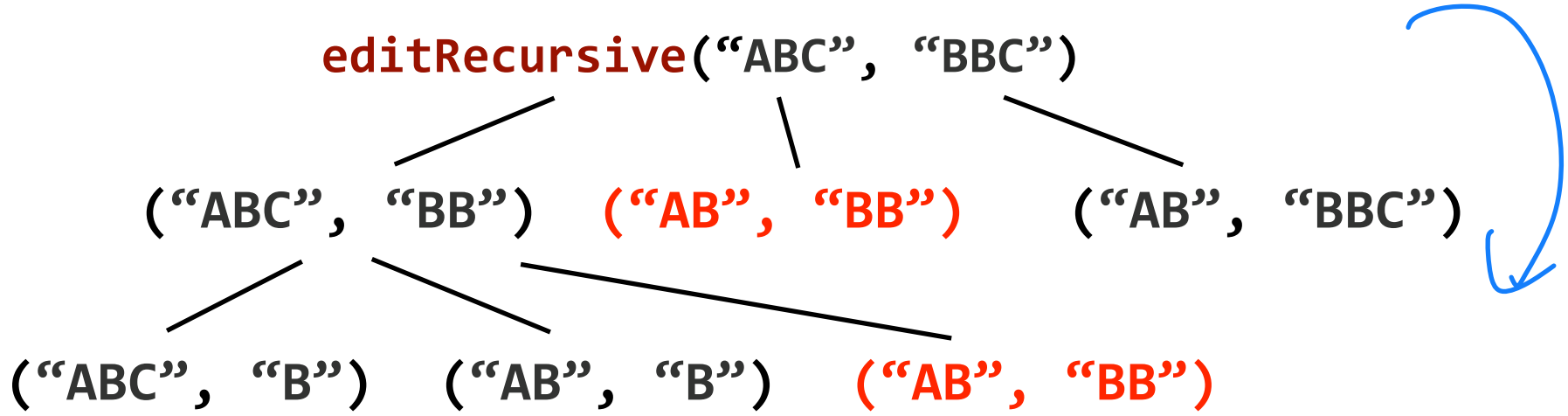
(only part of recursion tree shown)

`editRecursive("Shakespeare", "shake spear")`

`Calculate ("Shake", "shake") 8989 times`

How can we address this problem?

Edit Distance

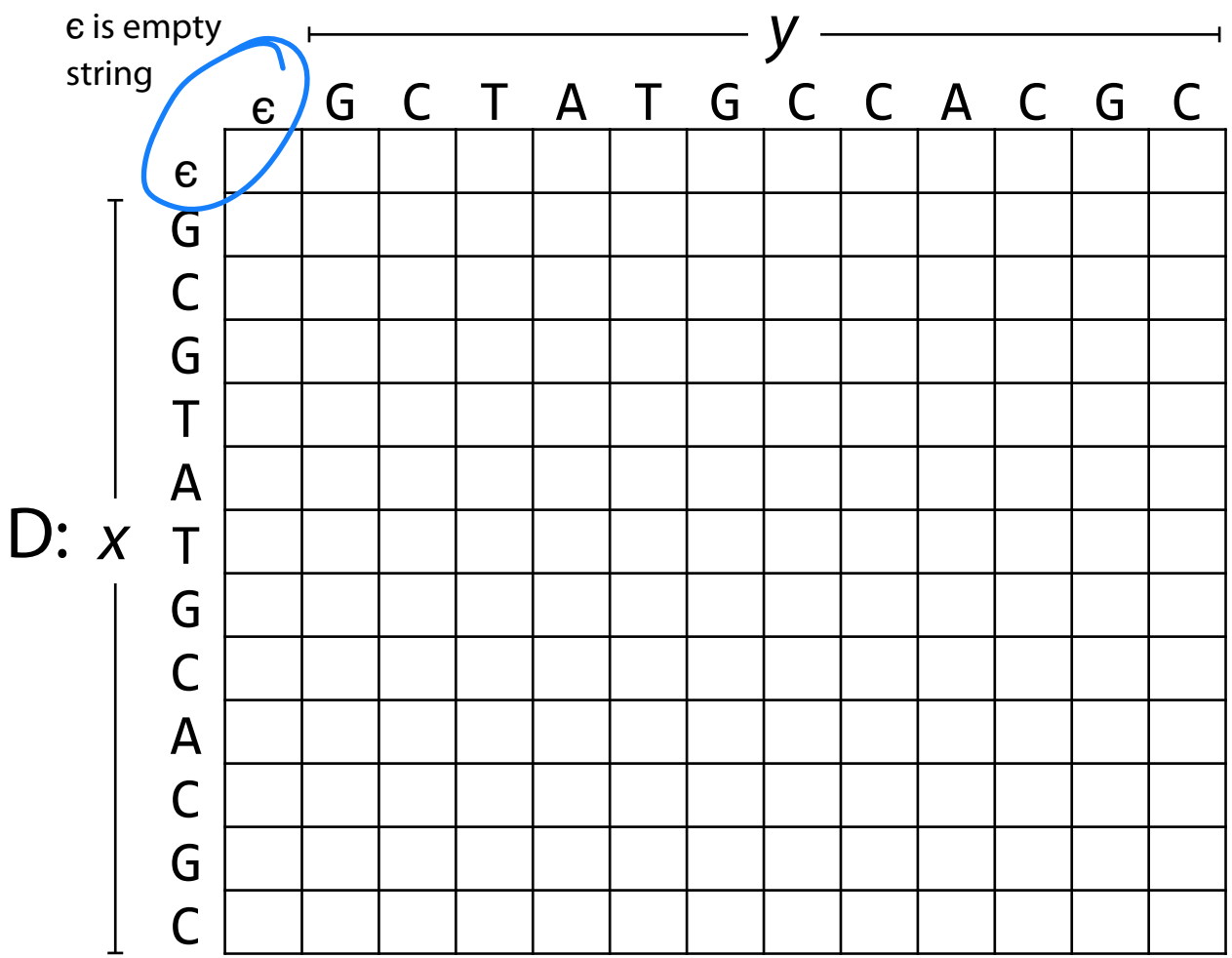


Memoization: Top-down

Dynamic Programming: Bottom-up

Both: Solve individual sub-problems once

Edit Distance: dynamic programming



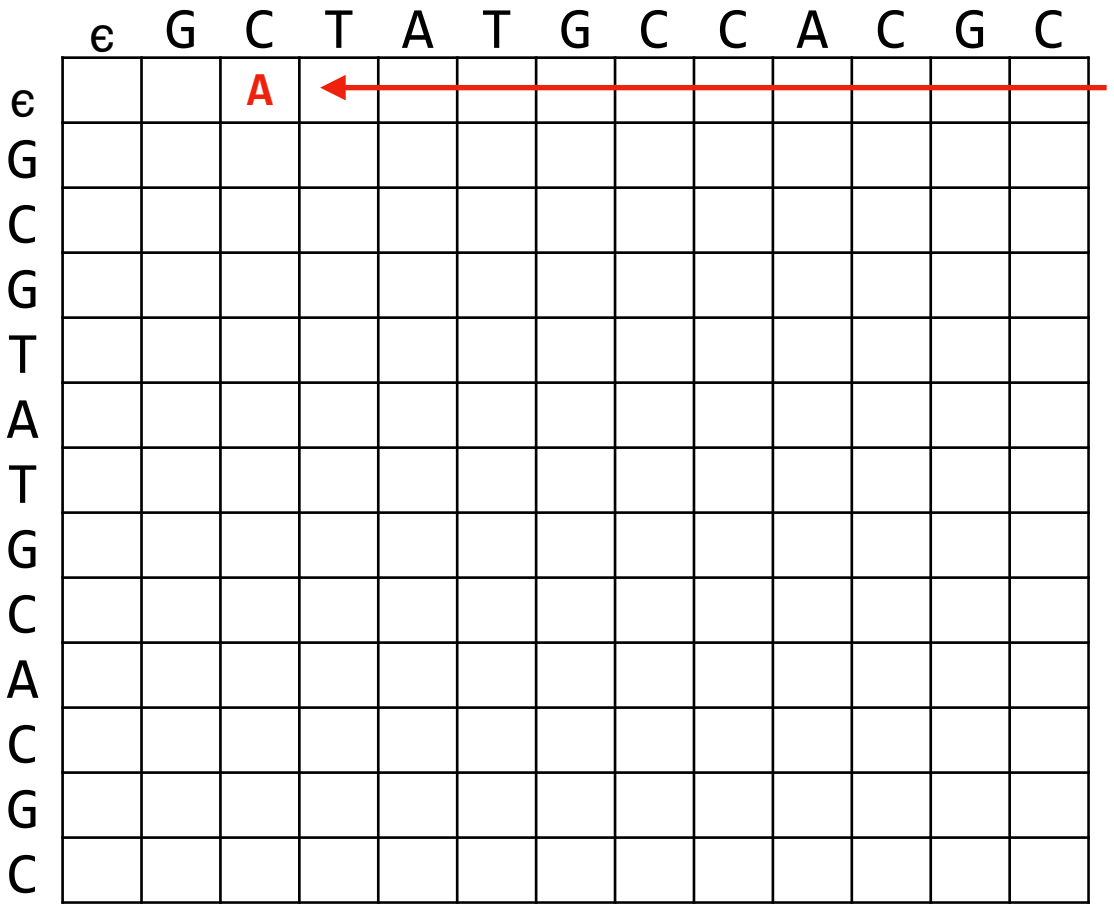
Let $n = |x|$, $m = |y|$

D: $(n+1) \times (m+1)$ matrix

D[i, j] = edit distance b/t length-i prefix of x and length-j prefix of y

Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

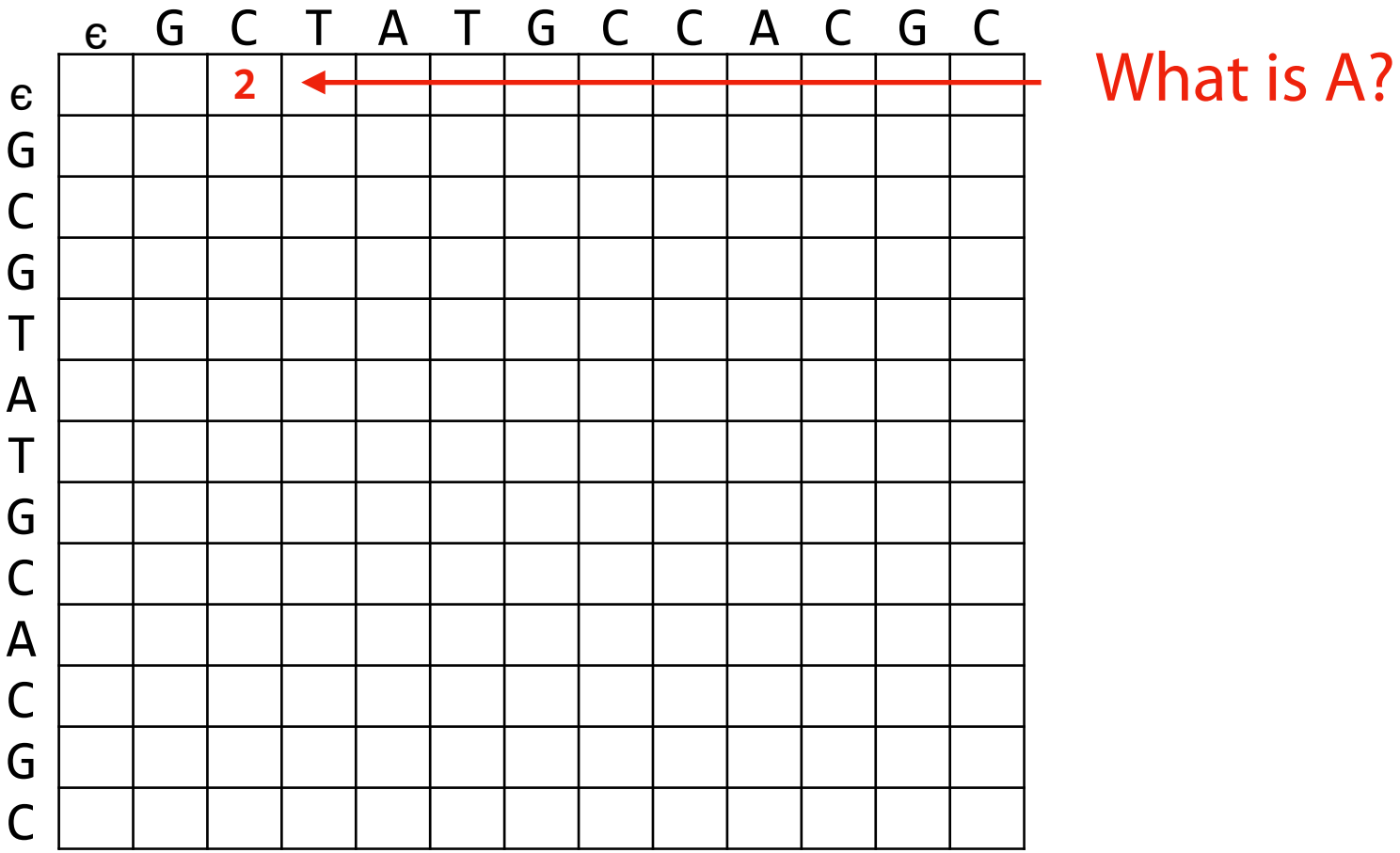


What is A?

D[*i*, *j*] = edit distance b/t length-*i* prefix of *x* and length-*j* prefix of *y*

Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$



Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1												
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1												
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

- - -

G C T

Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1												
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

G C G T A T

- - - - - -

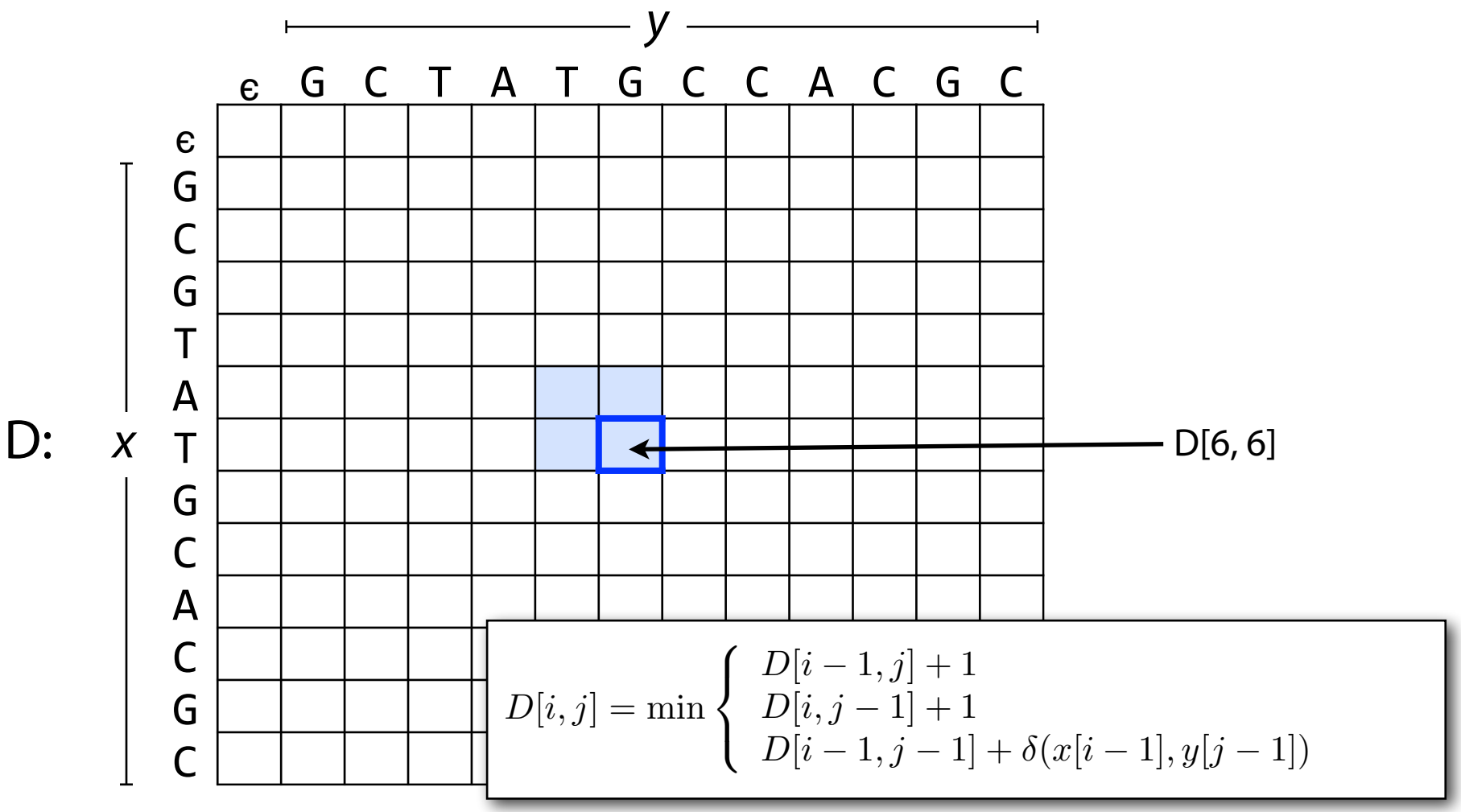
Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
	0	1	2	3	4	5	6	7	8	9	10	11	12
ϵ	0												
G	1												
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

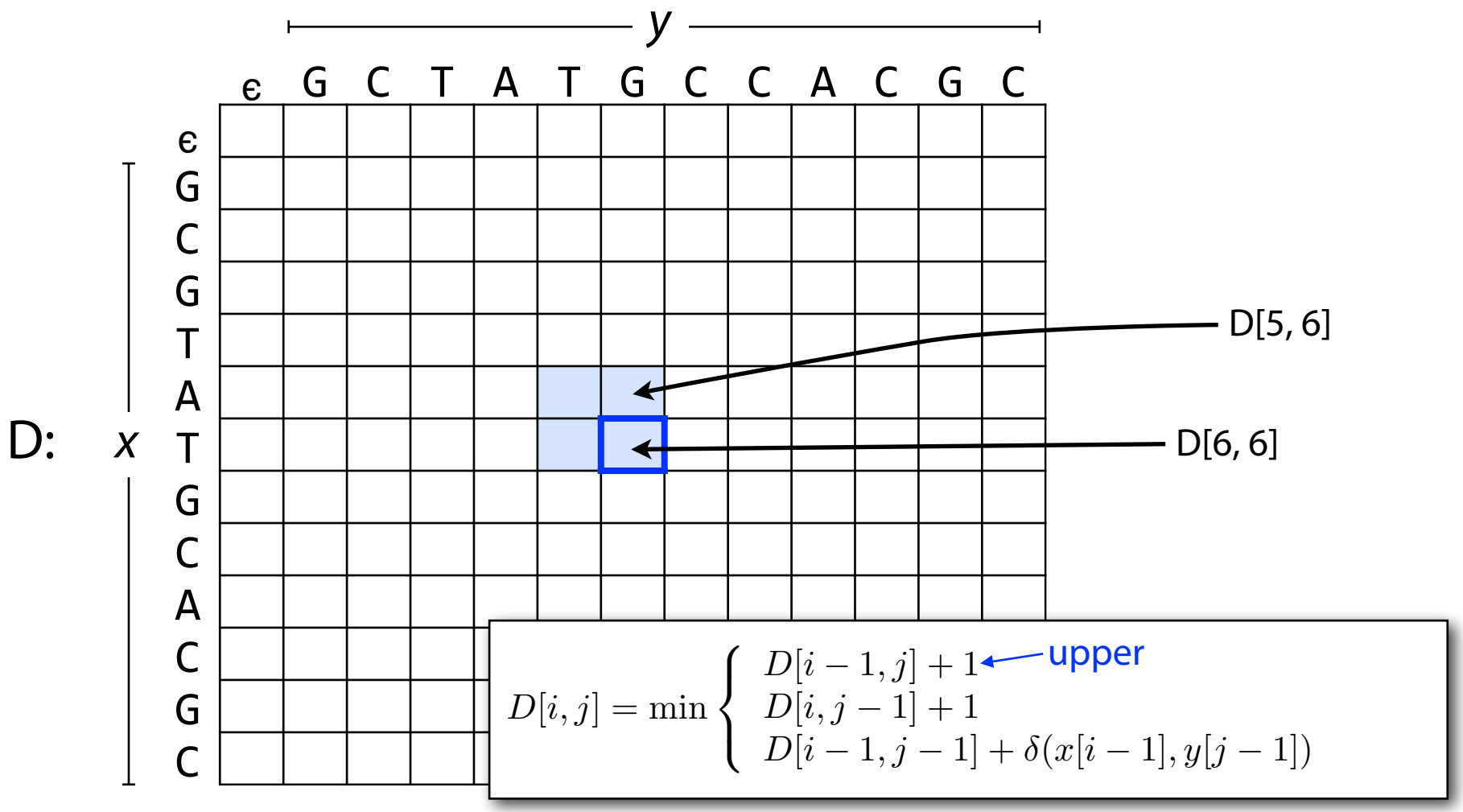
D is one row / column larger than x / y!

Edit Distance: dynamic programming



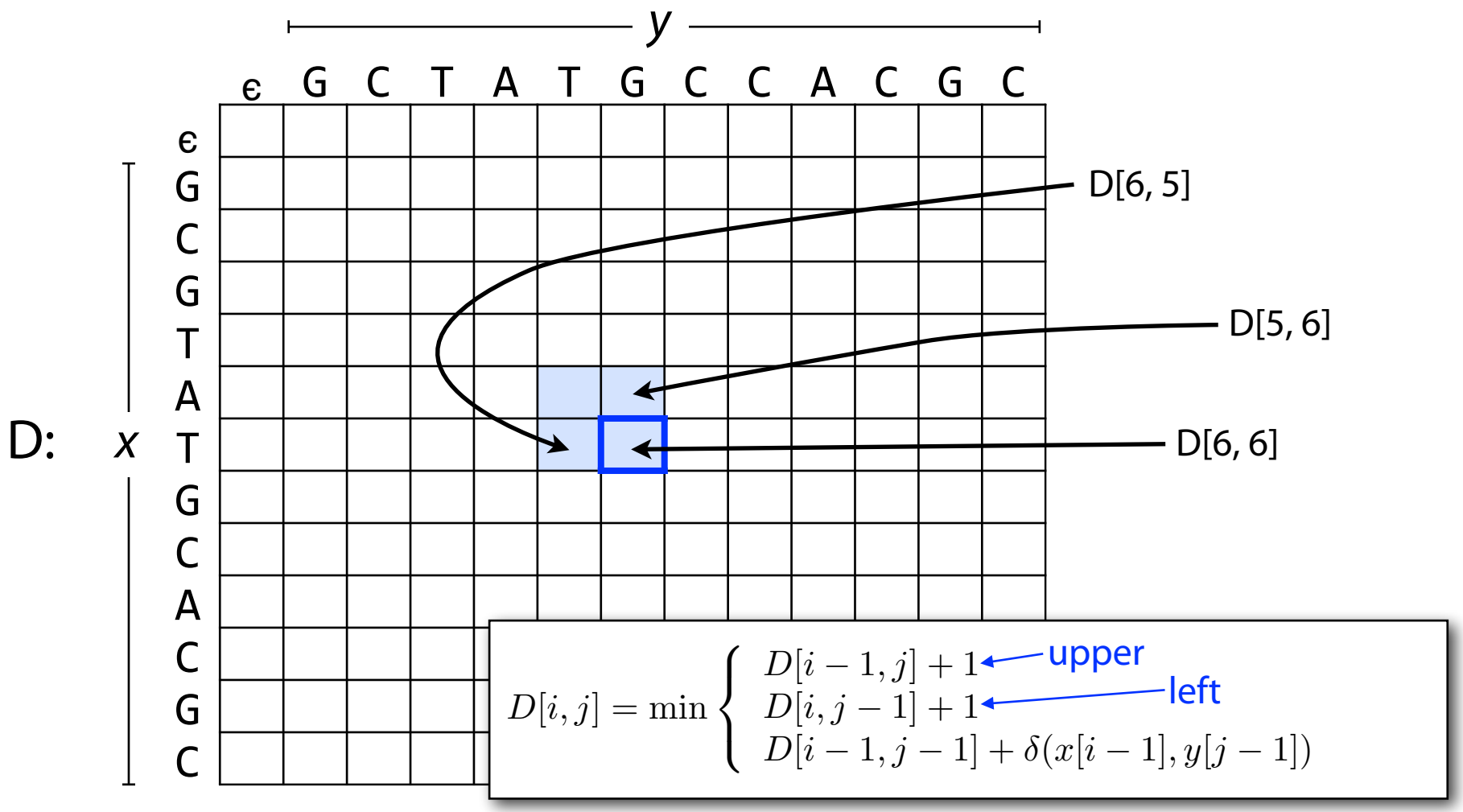
Cell depends upon its upper, left, and upper-left neighbors

Edit Distance: dynamic programming



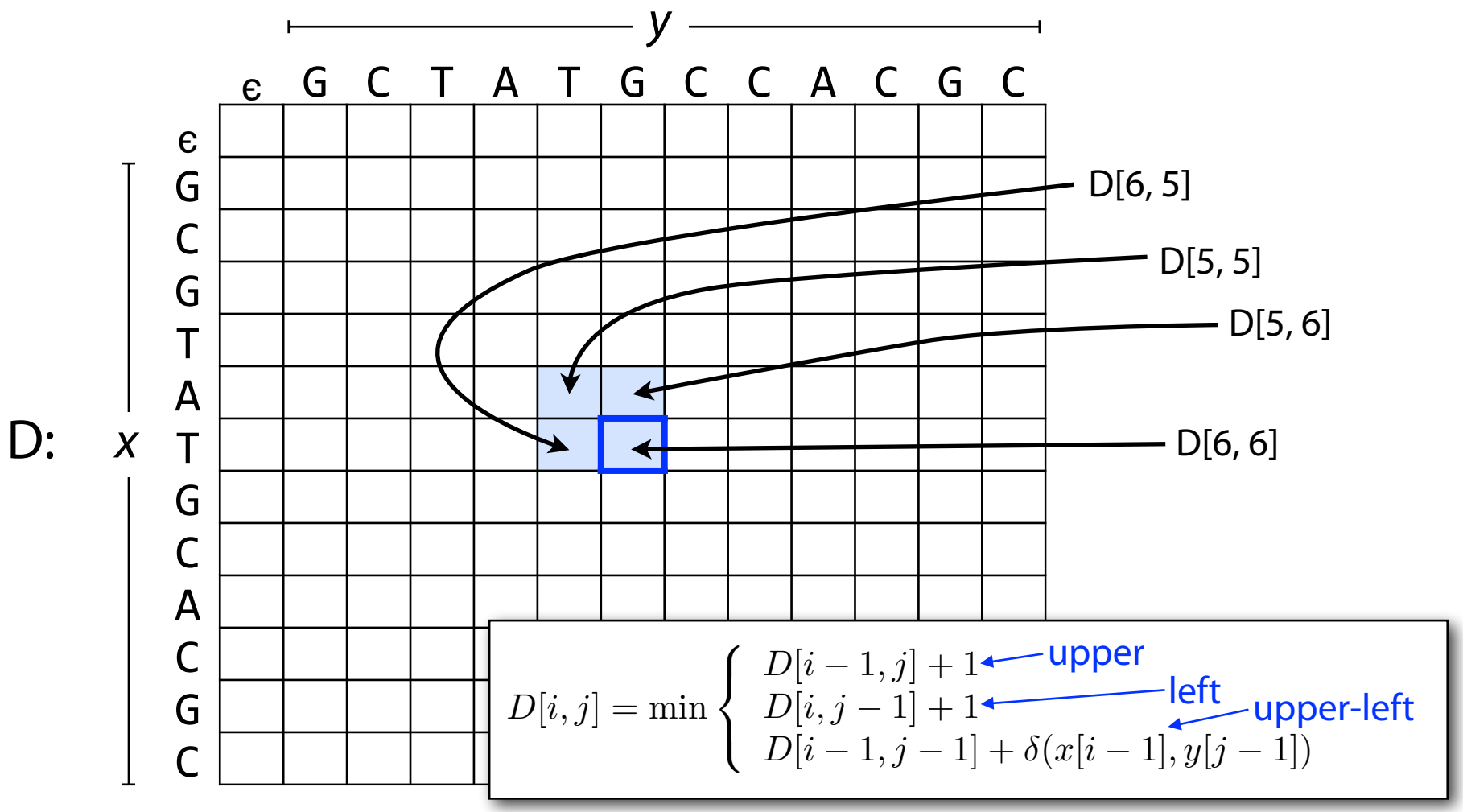
Cell depends upon its upper, left, and upper-left neighbors

Edit Distance: dynamic programming



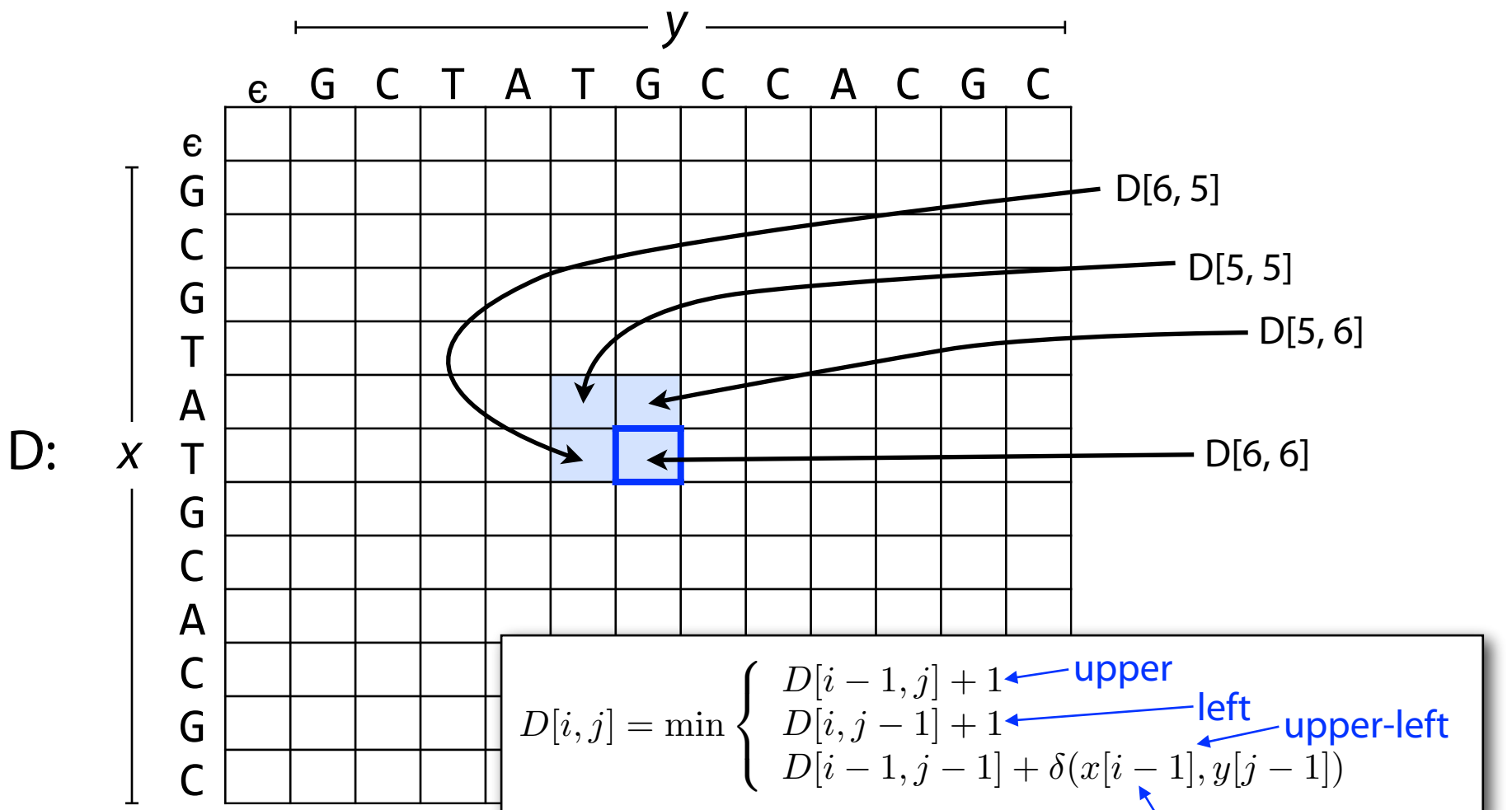
Cell depends upon its upper, left, and upper-left neighbors

Edit Distance: dynamic programming



Cell depends upon its upper, left, and upper-left neighbors

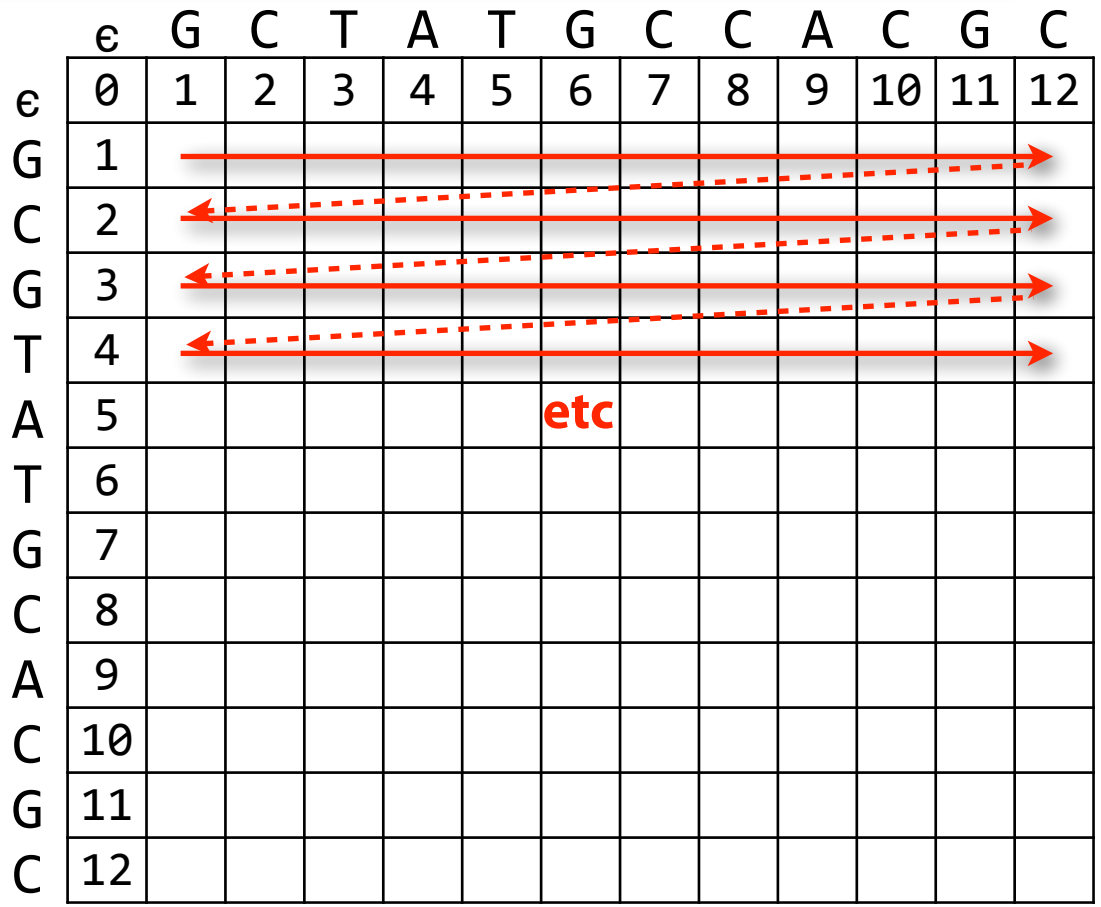
Edit Distance: dynamic programming



Cell depends upon its upper, left, and upper-left neighbors

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$



Fill remaining cells from top row to bottom and from left to right

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	?											
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=1,j=1**?

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	?											
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=1,j=1**?

x[i-1] = '**G**',
y[j-1] = '**G**',
so **delt = 0**

- G	G -	G
G -	- G	G
ID	**DI**	**M**

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
	0	1	2	3	4	5	6	7	8	9	10	11	12
ϵ													
G	1	?											
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=1,j=1**?

x[i-1] = '**G**',
y[j-1] = '**G**',
so **delt = 0**

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(1 + 1, 1 + 1, 0 + 0)
        = 0
```

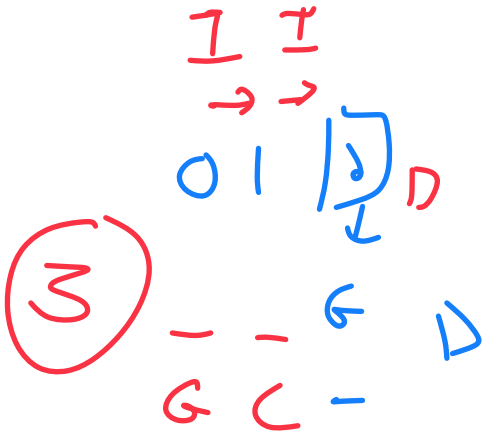
- G	G -	G |
G -	- G	G
ID	DI	M

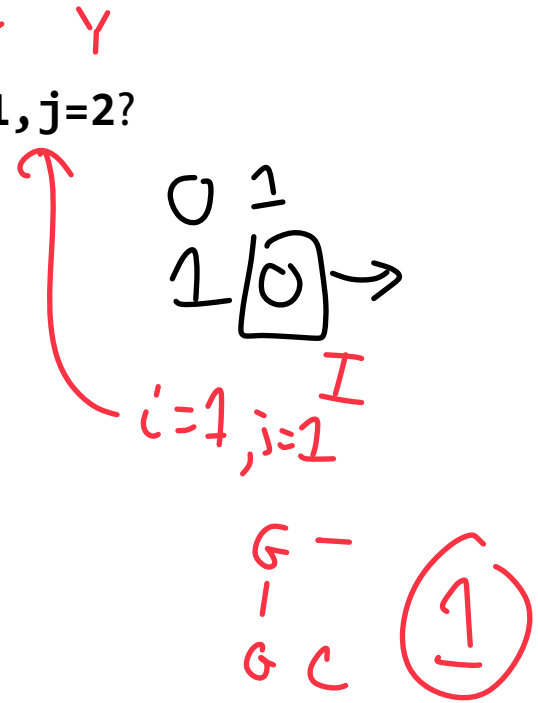
Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	?										
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

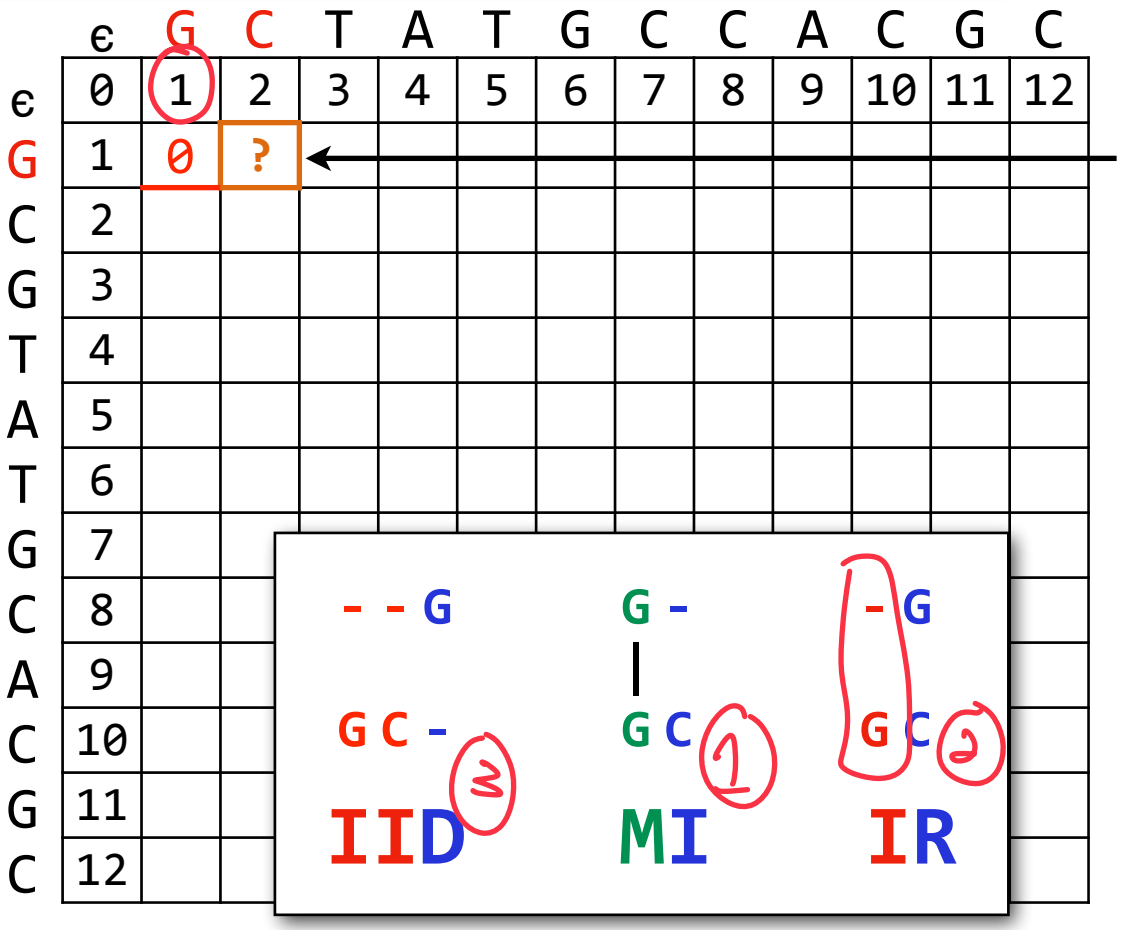
What goes here in **i=1, j=2**?





Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$



What goes here in **i=1,j=2**?

x[i-1] = '**G**',
y[j-1] = '**C**',
so **delt = 1**

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
	0	1	2	3	4	5	6	7	8	9	10	11	12
ϵ	0												
G	1	0	?										
C	2												
G	3												
T	4												
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=1,j=2**?

x[i-1] = '**G**',
y[j-1] = '**C**',
so **delt = 1**

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(2 + 1, 0 + 1, 1 + 1)
        = 1
```

--G	G-	-G
GC-	GC	GC
IID	MI	IR

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

$\delta(T, C) \Rightarrow 1$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6					
C	2	1	0	1	2	3	4	5					
G	3	2	1	1	2	3	3	4					
T	4	3	2	1	2	2	3	?					
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=4,j=7**?

$3+\delta$ $\quad 4+1$

$3+1 \rightarrow 4$

Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

0 or 1?

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6					
C	2	1	0	1	2	3	4	5					
G	3	2	1	1	2	3	3	4					
T	4	3	2	1	2	2	3	4					
A	5												
T	6												
G	7												
C	8												
A	9												
C	10												
G	11												
C	12												

What goes here in **i=4,j=7**?

x[i-1] = 'T',
y[j-1] = 'C',
so **delt = 1**

```
G C - - - G T
G C T A T G C
M M I I I M R

G C - G T - -
G C T A T G C
M M I R M I I
```

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(4 + 1, 3 + 1, 3 + 1)
        = 4
```

Edit Distance: dynamic programming

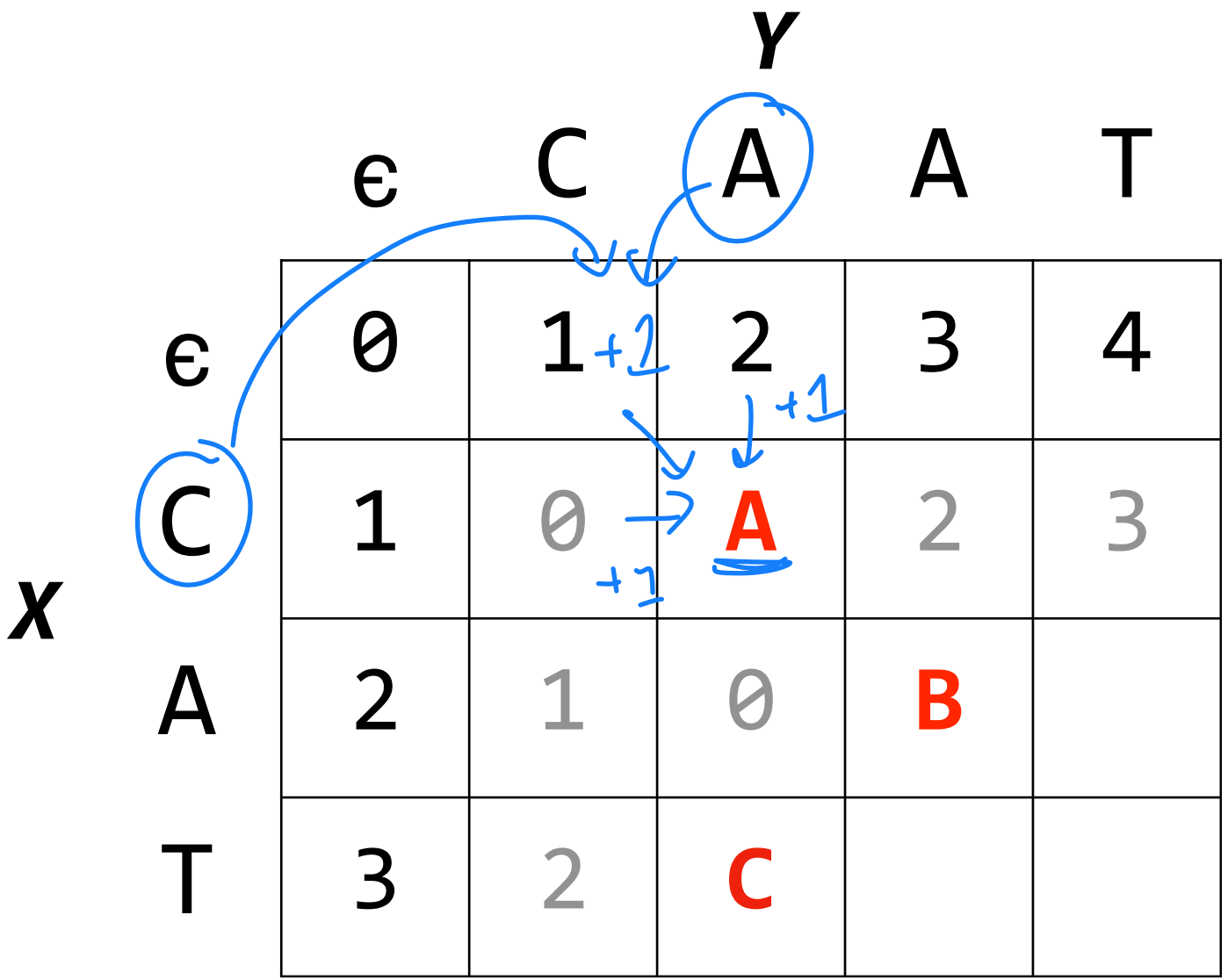
$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

Fill remaining cells from top row to bottom and from left to right

Edit distance for x, y

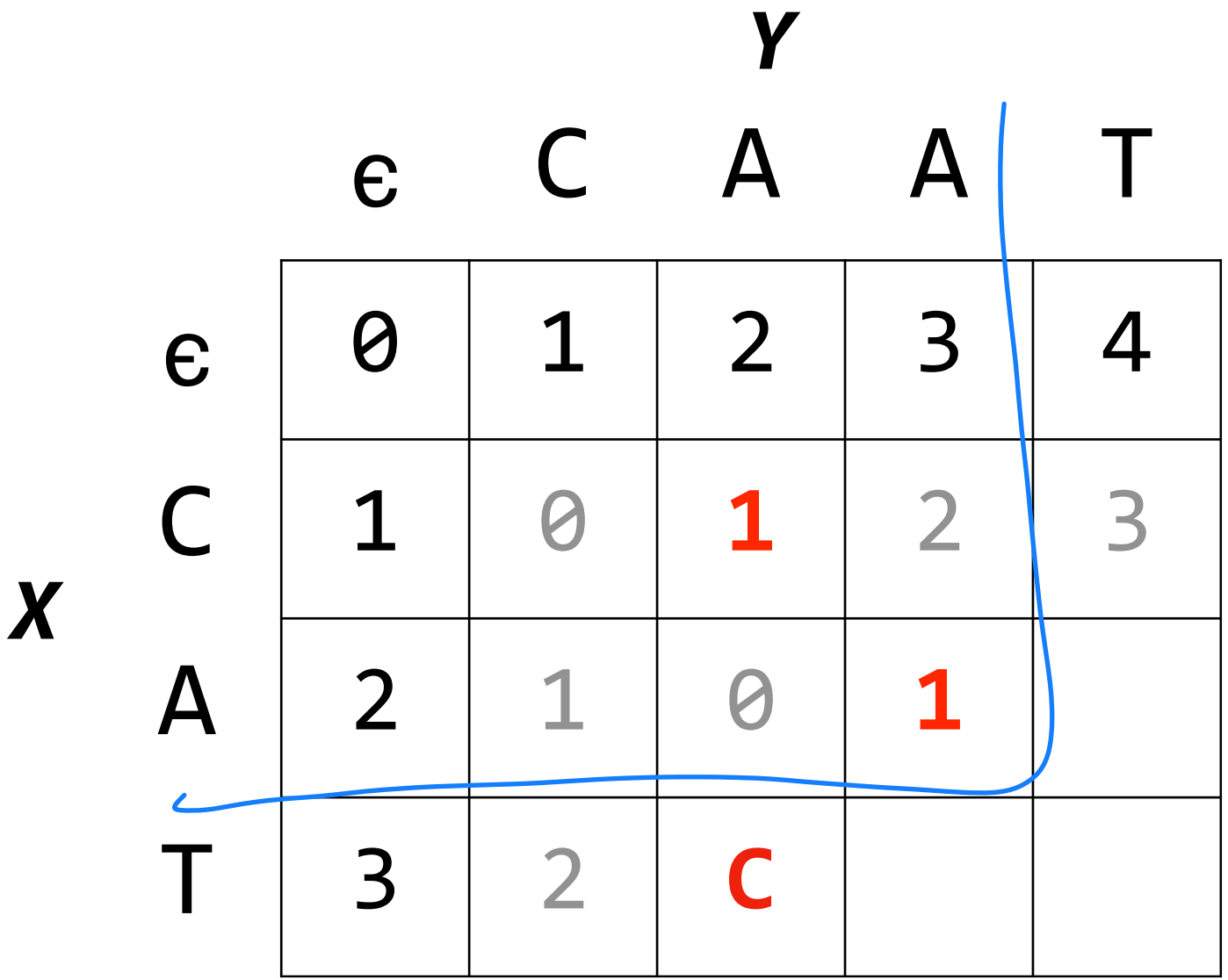
Edit Distance



Edit Distance

X \ Y	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	**1**	2	3
A	2	1	0	**B**	
T	3	2	**C**		

Edit Distance



Edit Distance

		Y				
		ϵ	C	A	A	T
X	ϵ	0	1	2	3	4
	C	1	0	**1**	2	3
	A	2	1	0	**1**	
	T	3	2	**1**		

Edit Distance

		Y				
		ϵ	C	A	A	T
X	ϵ	0	1	2	3	4
	C	1	0	1	2	3
	A	2	1	0	1	2
	T	3	2	1	1	1

eMatrix buildEditMatrix(X, Y)

Input:

string X: Input string X

string Y: Input string Y

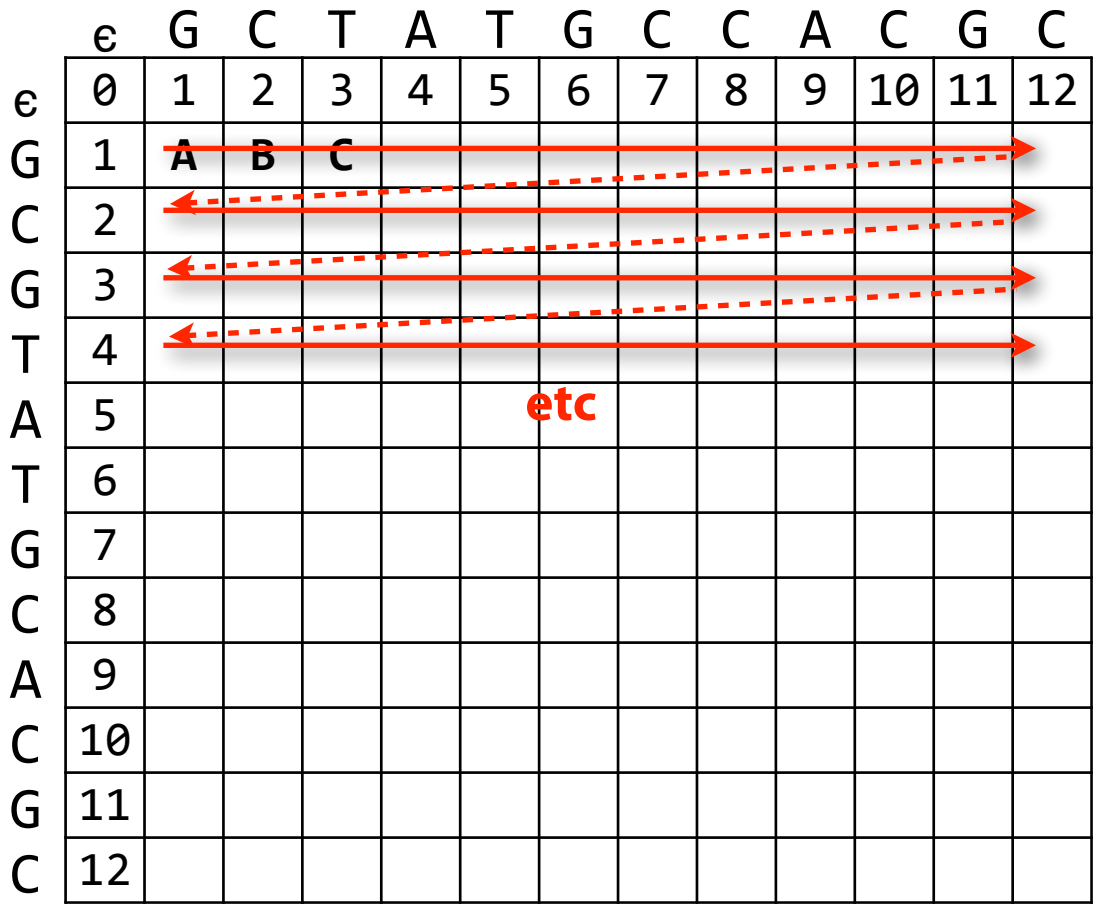
One letter smaller than D

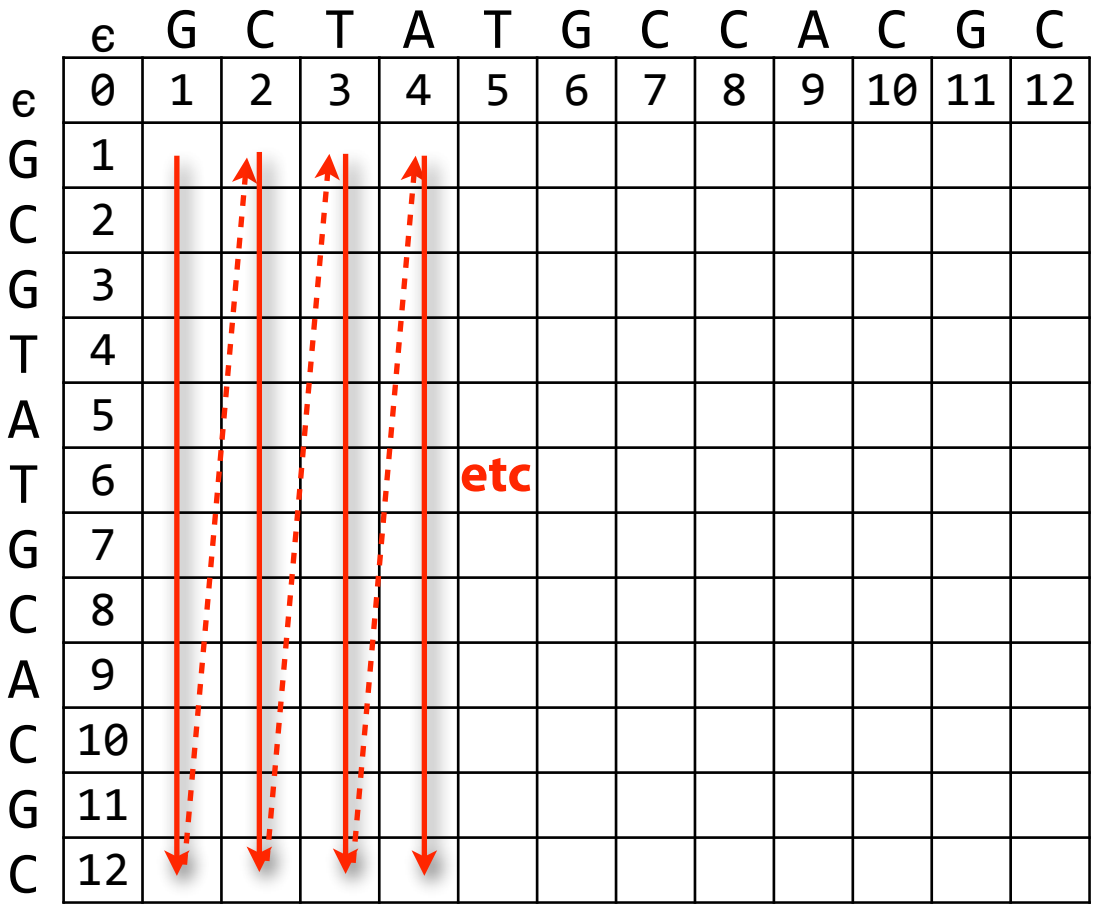
Output:

eMatrix : vector<vector<int>> storing all optimal edit distances

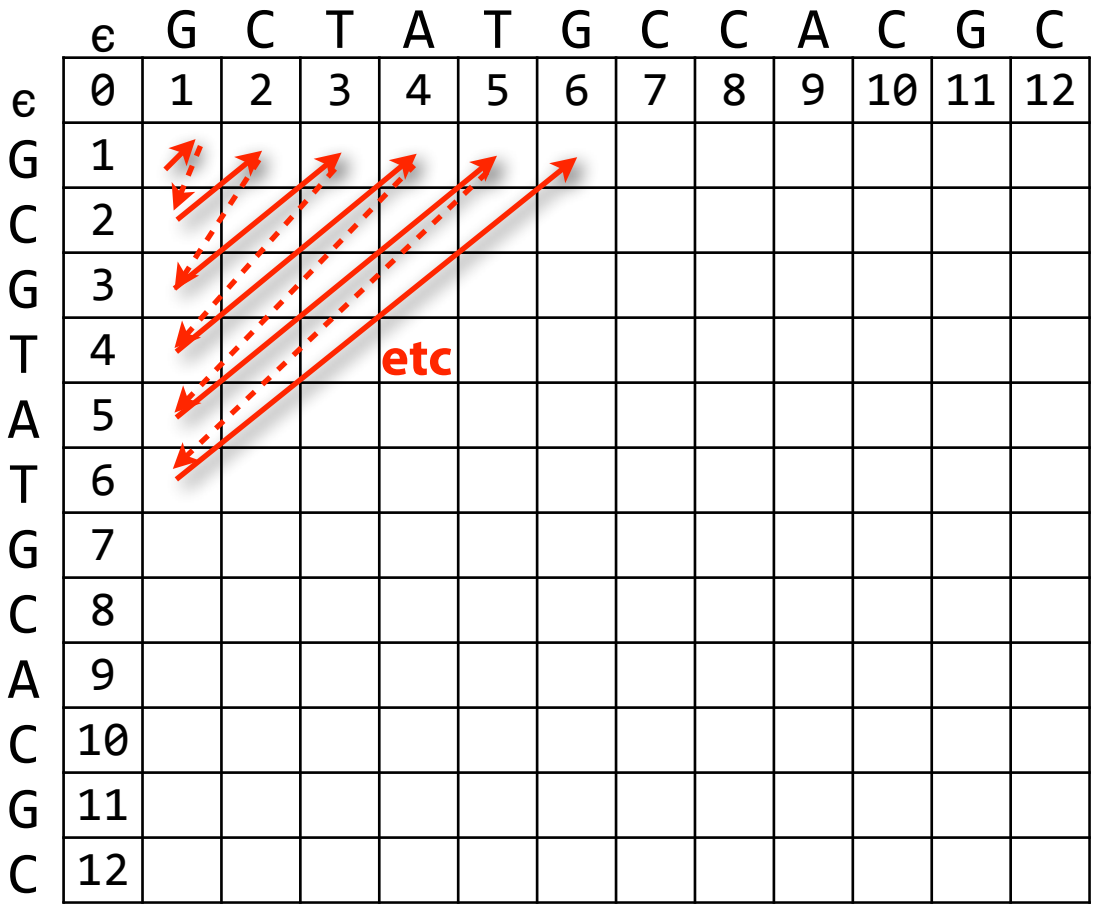
	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	1

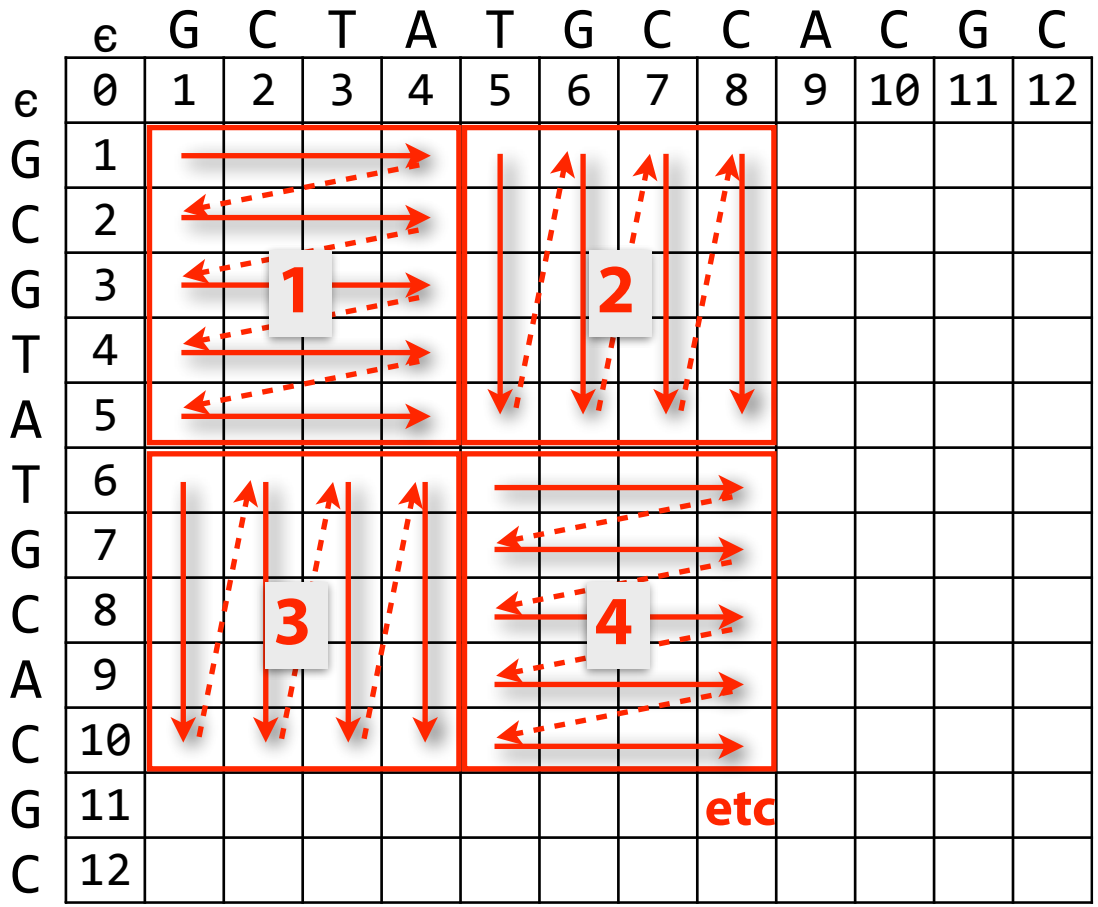
Edit Distance: dynamic programming



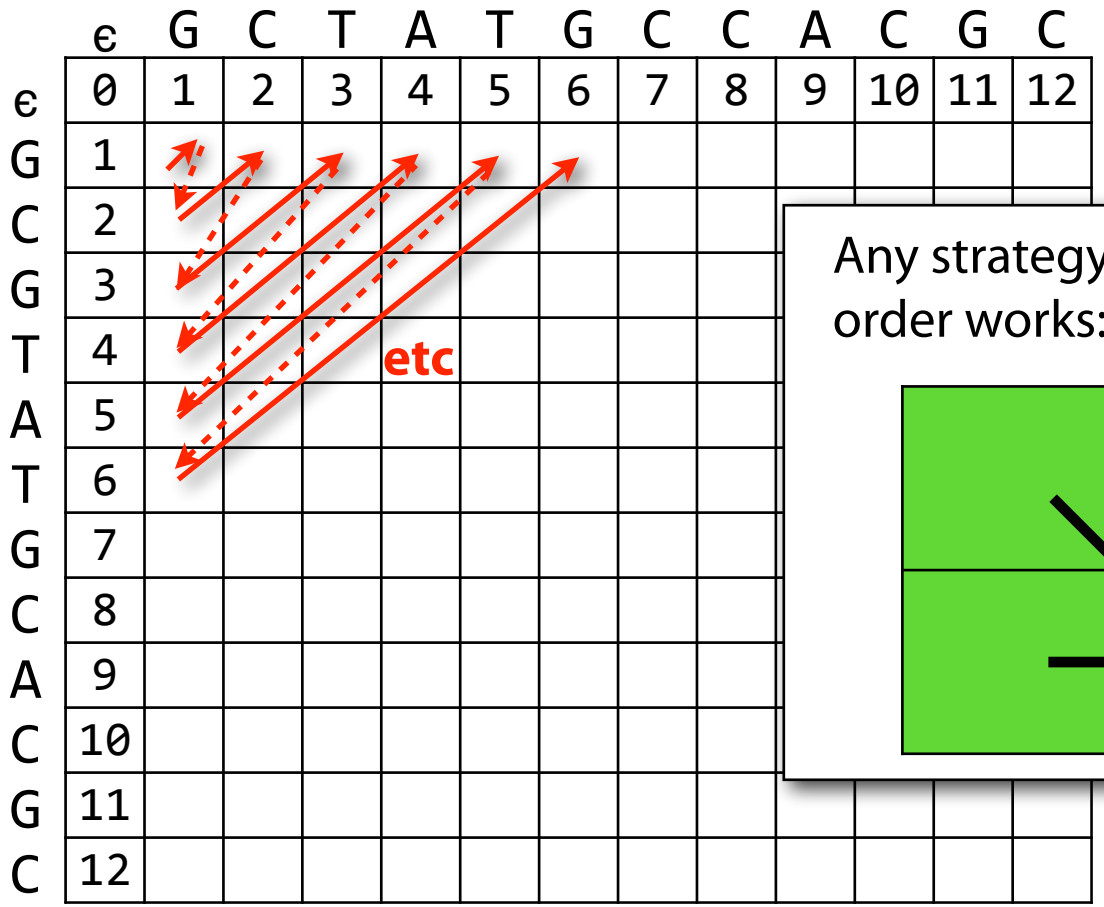


Edit Distance: dynamic programming

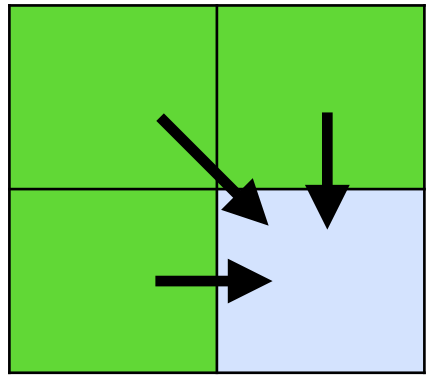


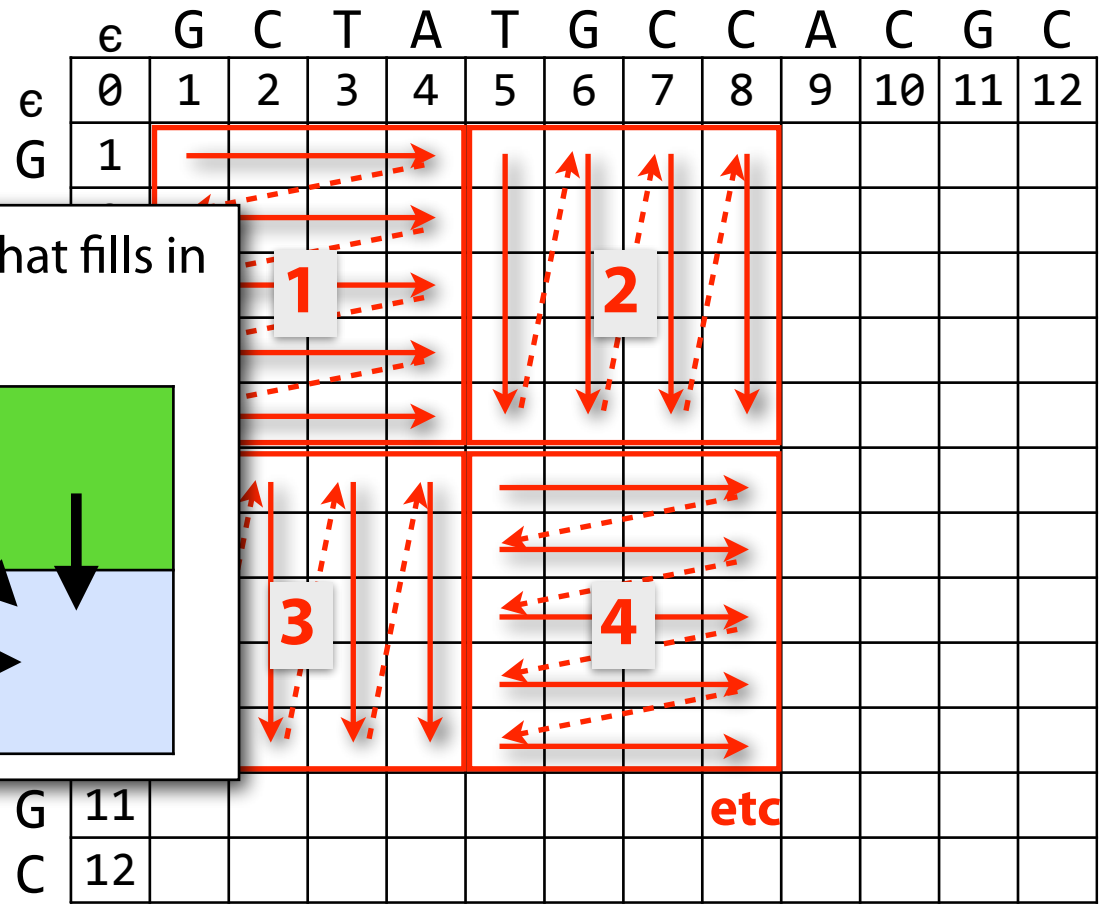


Edit Distance: dynamic programming



Any strategy that fills in order works:





Assignment 11: a_edist

Learning Objective:

Use dynamic programming to build an edit distance matrix

Construct an optimal edit string from the edit matrix

Consider: Does substitution, insertion, and deletion need to have the same 'weight' as a penalty? How could you modify the code to take a user-specified input for each?

Edit Distance

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	1

← Edit distance for x, y

But where and what is the edit?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	**1**

← Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	**2**
T	3	2	1	1	1

$D[2,\ 4] =$ 2 + 1

3?

Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	1

$D[3, 3] =$ 1 + 1

Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	**1**	2
T	3	2	1	1	1

$D[2,\ 3] =$ 1 + 0 ✓

← Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	1

$D[1, 3] = \frac{2+1}{}$

$D[1, 2] = \frac{1+0}{}$

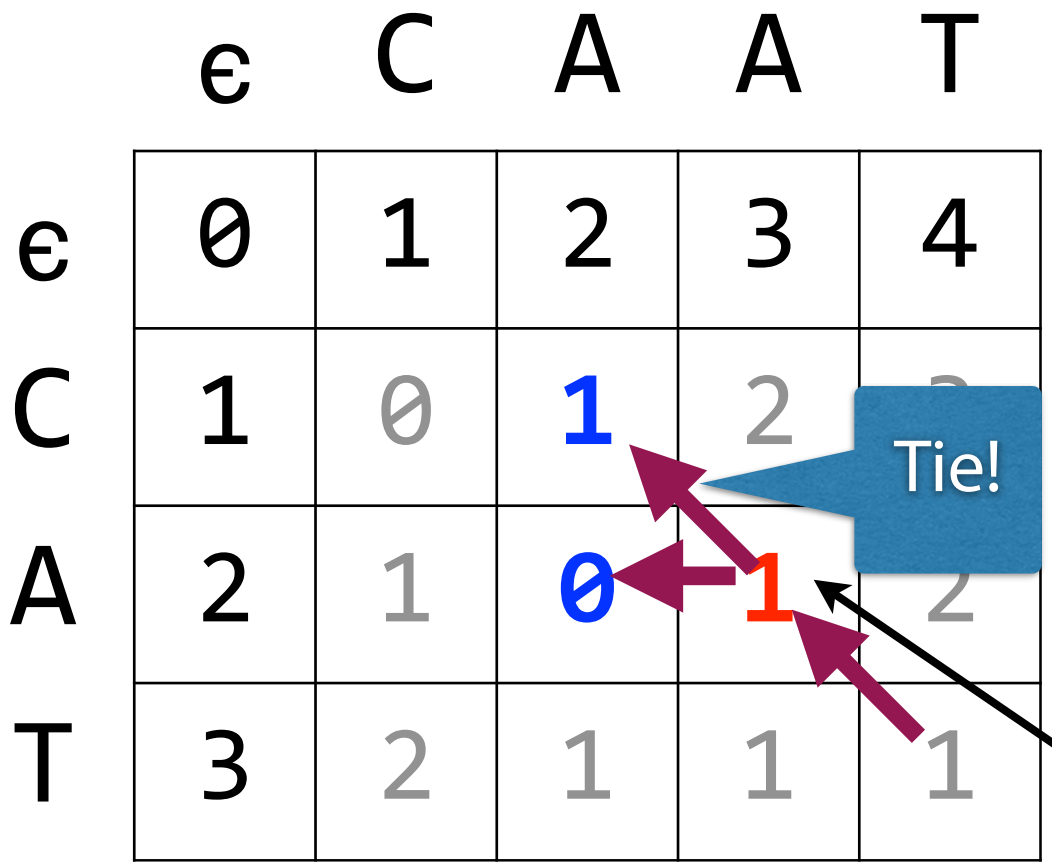
$D[2, 2] = \frac{0+1}{}$

Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



$$D[1,\ 3] = 2 + 1$$

$$D[1,\ 2] = 1 + 0$$

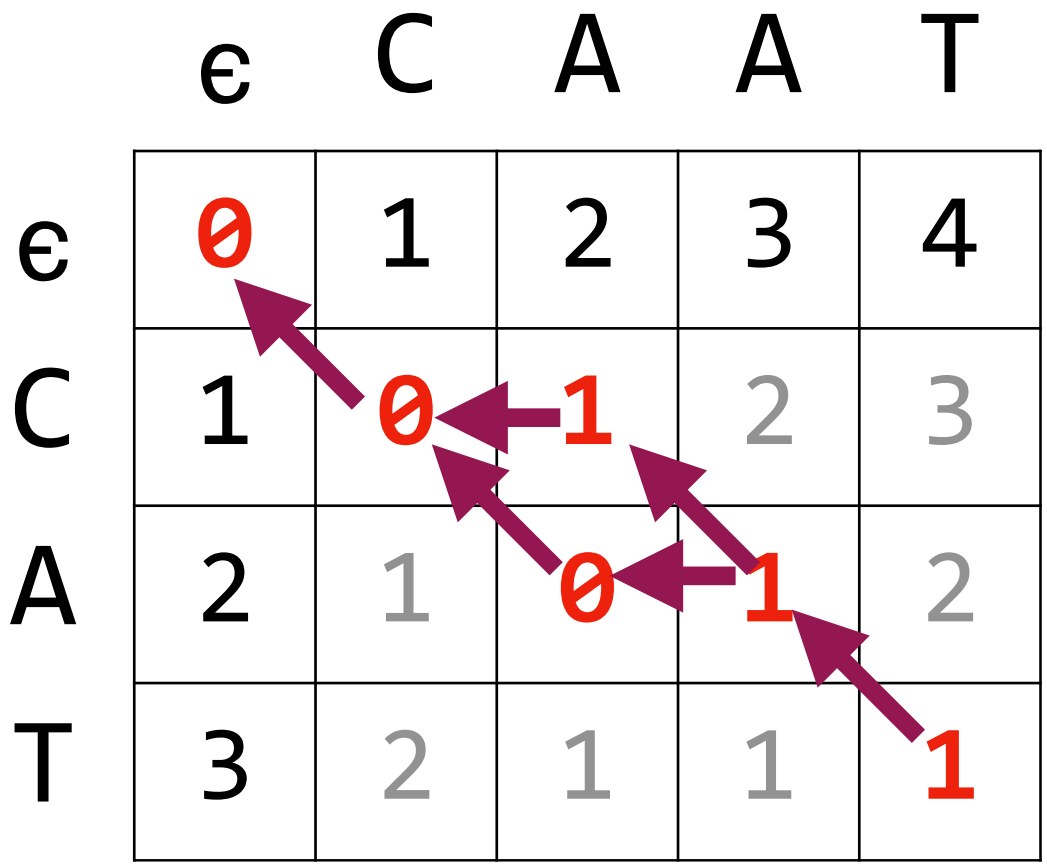
$$D[2,\ 2] = 0 + 1$$

Q: How did I get here?

Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

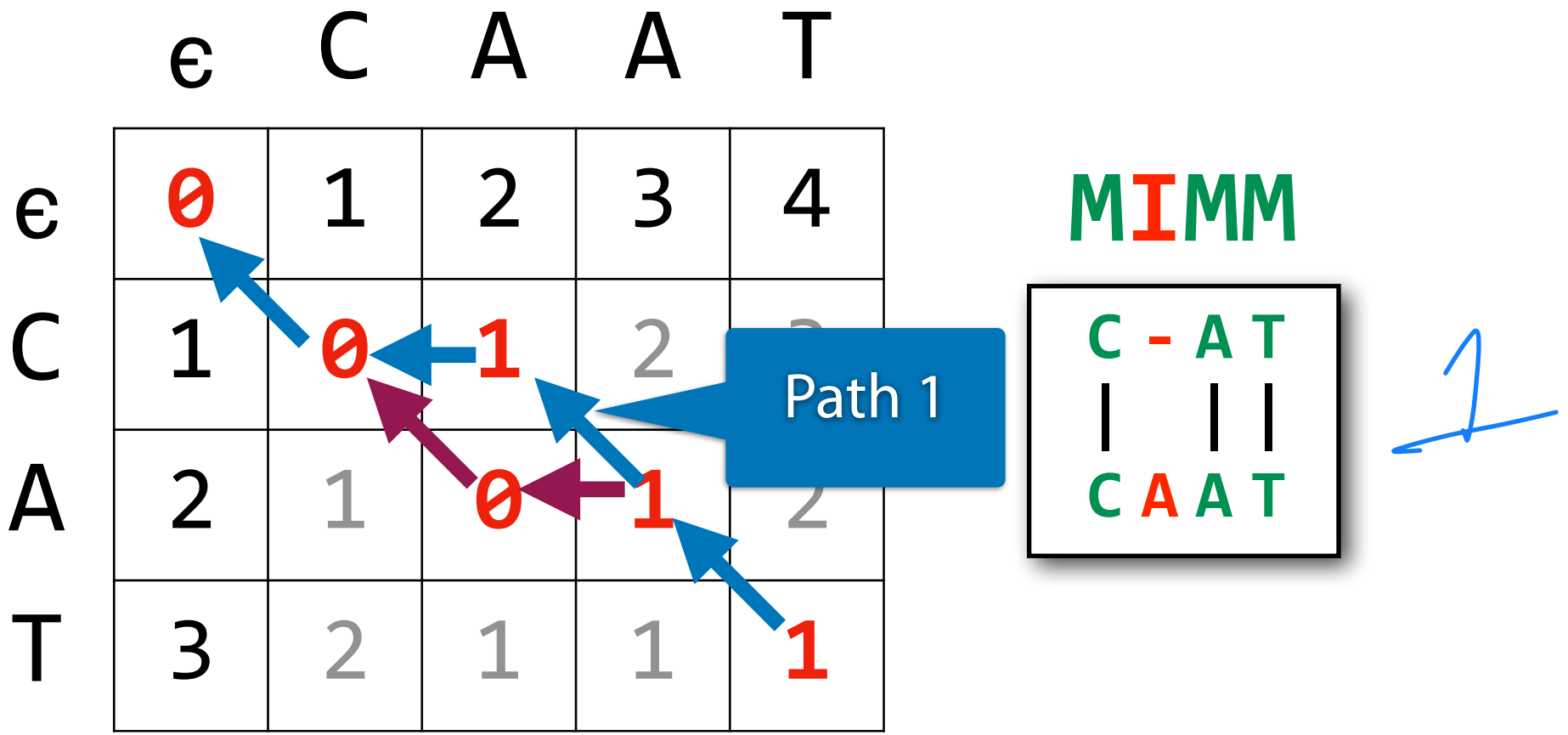
At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

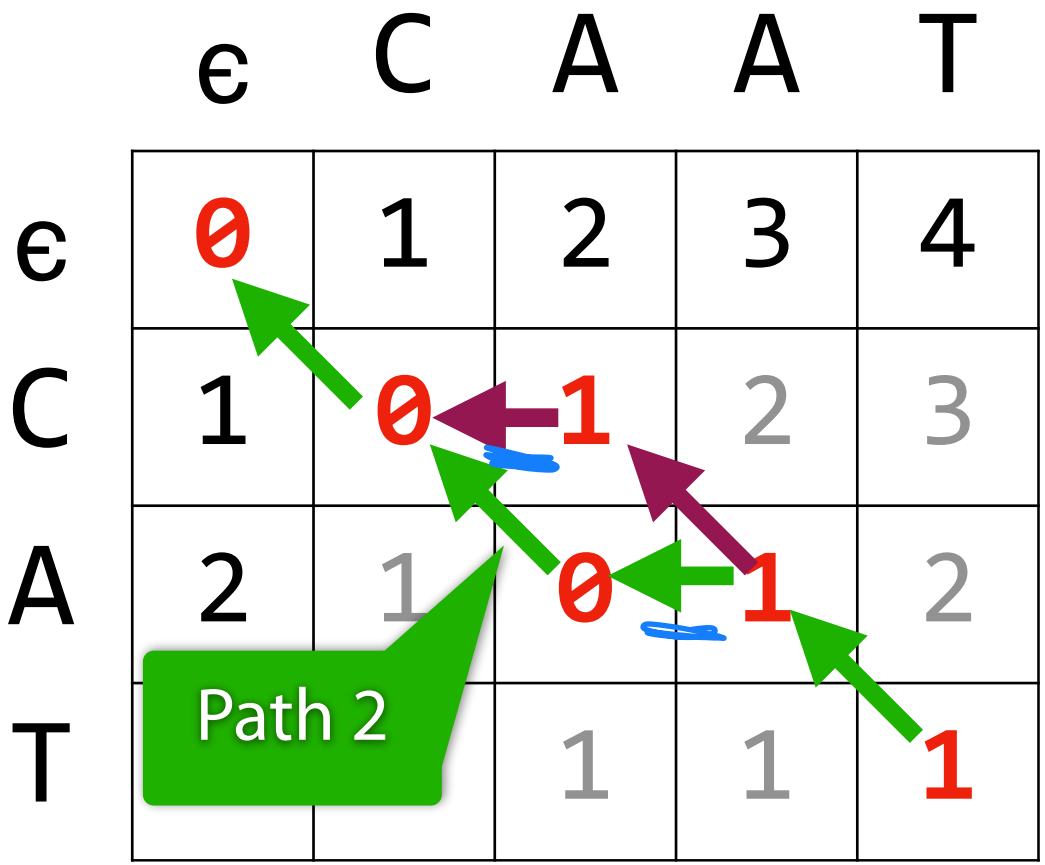
At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



Edit Distance

Traceback corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



MIMM

```
C - A T
|   | |
C A A T
```

MMIM

```
C A - T
| |   |
C A A T
```

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

← Q: How did I get here?

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

$$D[11,\ 12] = 3 + 1$$

$$D[11,\ 11] = 2 + 0$$

$$D[12,\ 11] = 3 + 1$$

A: From here

Q: How did I get here?

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

Q: How did I get here?

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

$$D[10,\ 11] = 3 + 1$$

$$D[10,\ 10] = 2 + 0$$

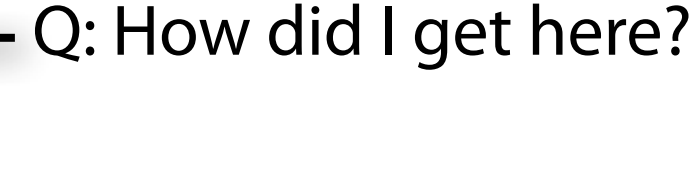
$$D[11,\ 10] = 3 + 1$$

A: From here

Q: How did I get here?

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2



Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

$$D[9,\ 10] = 3 + 1$$

$$D[9,\ 9] = 2 + 0$$

$$D[10,\ 9] = 3 + 1$$

A: From here

Q: How did I get here?

Edit Distance

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

Alignment:

```
G C G T A T G - C A C G C
|| |  |||| |  |||||
G C - T A T G C C A C G C
```

MMDMMMMIMMMMM

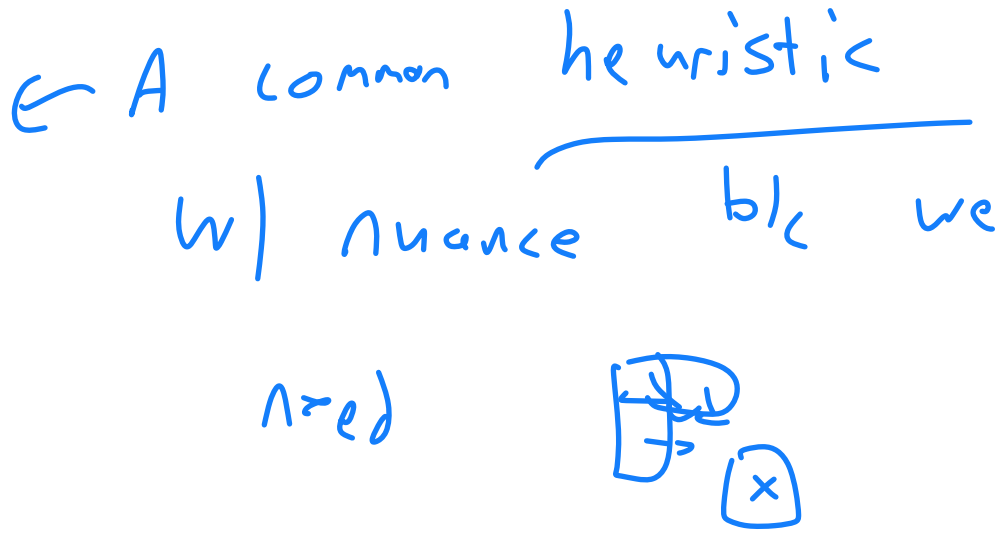
Edit Distance

Dynamic Programming fills our table with optimal distances

Traceback identifies the optimal *edit string* to convert *X* to *Y*

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

What is our efficiency? ($|X| = m, |Y| = n$)



Edit Distance

Dynamic Programming fills our table with optimal distances

Traceback identifies the optimal *edit string* to convert *X* to *Y*

	ϵ	G	C	T	A	T	G	C	C	A	C	G	C
ϵ	0	1	2	3	4	5	6	7	8	9	10	11	12
G	1	0	1	2	3	4	5	6	7	8	9	10	11
C	2	1	0	1	2	3	4	5	6	7	8	9	10
G	3	2	1	1	2	3	3	4	5	6	7	8	9
T	4	3	2	1	2	2	3	4	5	6	7	8	9
A	5	4	3	2	1	2	3	4	5	5	6	7	8
T	6	5	4	3	2	1	2	3	4	5	6	7	8
G	7	6	5	4	3	2	1	2	3	4	5	6	7
C	8	7	6	5	4	3	2	1	2	3	4	5	6
A	9	8	7	6	5	4	3	2	2	2	3	4	5
C	10	9	8	7	6	5	4	3	2	3	2	3	4
G	11	10	9	8	7	6	5	4	3	3	3	2	3
C	12	11	10	9	8	7	6	5	4	4	3	3	2

What is our efficiency? ($|X| = m$, $|Y| = n$)

Table filling: Filling (m + 1) × (n + 1) ~~(n + 1)~~ cells, each requiring constant work, so $O(mn)$

Traceback: Each step goes ⇖, ⇐ or ⇑. Worst case: traceback never moves diagonally, requiring m ⇑'s and n ⇐'s, so $O(m + n)$

`string buildEditString(X, Y)`

Input: **`string X:`** Input string X (edits with respect to X)

`string Y:` Input string Y (edits turn X into Y)

Output: **`string`**: An optimal edit string produced by the matrix

	ϵ	C	A	A	T
ϵ	0	1	2	3	4
C	1	0	1	2	3
A	2	1	0	1	2
T	3	2	1	1	1

On tie: prioritize diagonal, then vertical, then horizontal

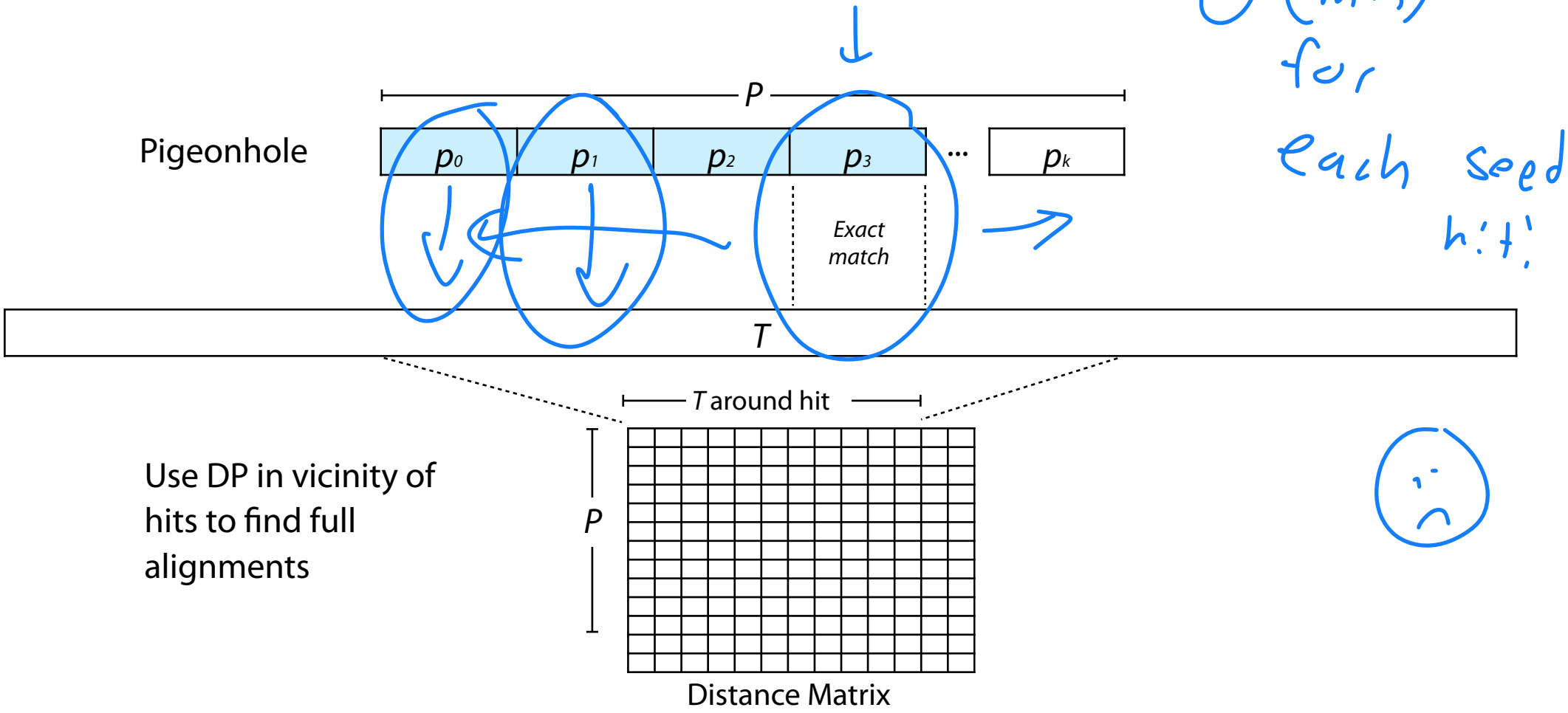
More than one Optimal

MIMM

C - A T
| | | |
C A A T

Approximate Pattern Matching

"Seed and extend" works for edit distance too!



Use DP in vicinity of hits to find full alignments

O(mn) for each seed hit!