CS 225

#24: Disjoint Sets

Disjoint Sets

Let **R** be an equivalence relation. We represent R as disjoint sets

- Each element exists in exactly one set.
- Every set is an equitant representation.
  - Mathematically: $4 \in [0]_R \rightarrow 8 \in [0]_R$
  - Programmatically: find(4) == find(8)

Building Disjoint Sets:

- Maintain a collection $S = \{s_0, s_1, \ldots s_k\}$
- Each set has a representative member

void makeSet(const T & t);
void union(const T & k1, const T & k2);
T & find(const T & k);

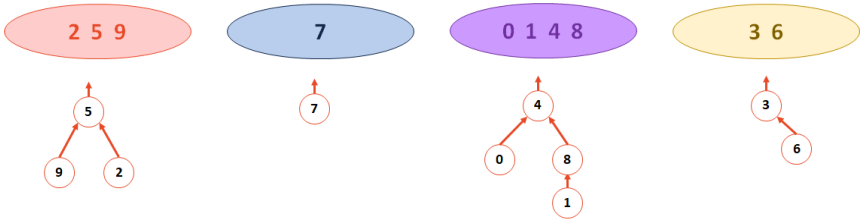
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

Operation: find(k)

Operation: union(k1, k2)

Implementation #2:

- Continue to use an array where the index is the key
- The value of the array is:
  - **-1**, if we have found the representative element
  - **The index of the parent**, if we haven't found the rep. element



4	8	5	6	-1	-1	-1	-1	4	5
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

Implementation – DisjointSets::find

DisjointSets.cpp (partial)

```
1  int DisjointSets::find(int i) {
2    if ( s[i] < 0 ) { return i; }
3    else { return _find( s[i] ); }
4  }
```

What is the running time of **find**?

What is the ideal UpTree?

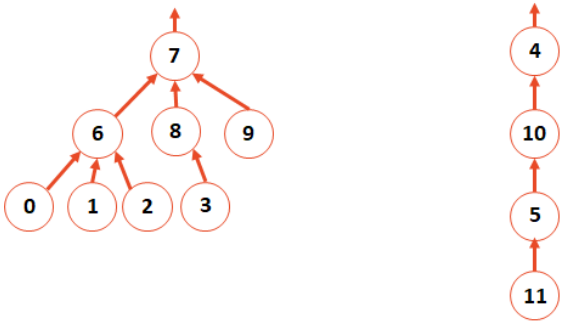
Implementation – DisjointSets::union

DisjointSets.cpp (partial)

```
1  void DisjointSets::union(int r1, int r2) {
2
3
4  }
```

How do we want to union the two UpTrees?

Building a Smart Union Function



The implementation of this visual model is the following:

6	6	6	8	-1	10	7	-1	7	7	4	5
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

What are possible strategies to employ when building a "smart union"?

Smart Union Strategy #1: _____
Idea: Keep the height of the tree as small as possible!

Metadata at Root:

After `union( 4, 7 )`:

6	6	6	8		10	7		7	7	4	5
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Smart Union Strategy #2: _____
Idea: Minimize the number of nodes that increase in height.
(Observe that the tree we union have all their nodes gain in height.)

Metadata at Root:

After `union( 4, 7 )`:

6	6	6	8		10	7		7	7	4	5
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Smart Union Implementation:

DisjointSets.cpp (partial)

```
1  void DisjointSets::unionBySize(int root1, int root2) {
2    int newSize = arr_[root1] + arr_[root2];
3
4    if ( arr_[root1] < arr_[root2] ) {
5      arr_[root2] = root1;  arr_[root1] = newSize;
6    } else {
7      arr_[root1] = root2;  arr_[root2] = newSize;
8    }
9  }
```

How do we improve this?

DisjointSets.cpp (partial)

```
1  int DisjointSets::find(int i) {
2    if ( arr_[i] < 0 ) { return i; }
3    else { return _find( arr_[i] ); }
4  }
```

DisjointSets.cpp (partial)

```
1  void DisjointSets::unionBySize(int root1, int root2) {
2    int newSize = arr_[root1] + arr_[root2];
3
4    // If arr_[root1] is less than (more negative), it is the
5    // larger set; we union the smaller set, root2, with root1.
6    if ( arr_[root1] < arr_[root2] ) {
7      arr_[root2] = root1;
8      arr_[root1] = newSize;
9    }
10   // Otherwise, do the opposite:
11   else {
12     arr_[root1] = root2;
13     arr_[root2] = newSize;
14   }
15 }
```

Running Time:

- Worst case running time of find(k)**:**
- Worst case running time of union(r1, r2), given roots:
- New function: "Iterated Log":
  log*(n) :=
- Overall running time:
  - A total of **m** union/find operation runs in: