Data Structures and Algorithms
Hashing

CS 225
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Department of Computer Science

Learning Objectives

Motivate and formally define a hash table

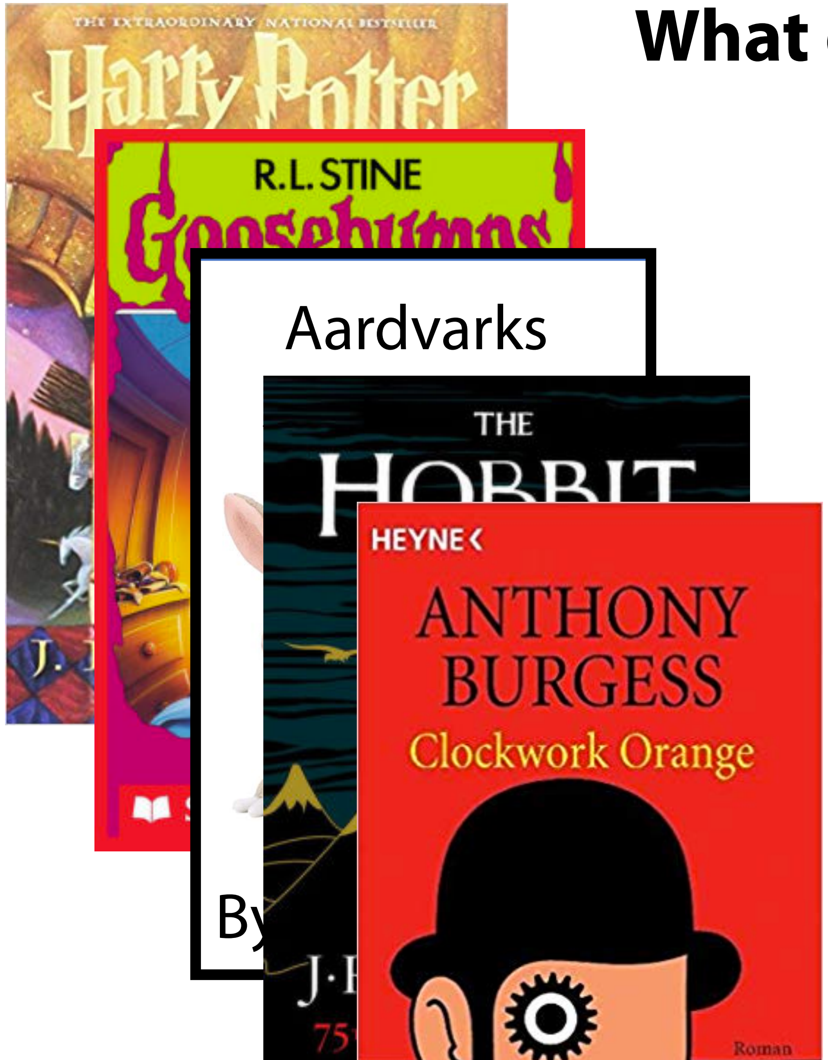
Discuss what a 'good' hash function looks like

Identify the key weakness of a hash table

Introduce strategies to "correct" this weakness

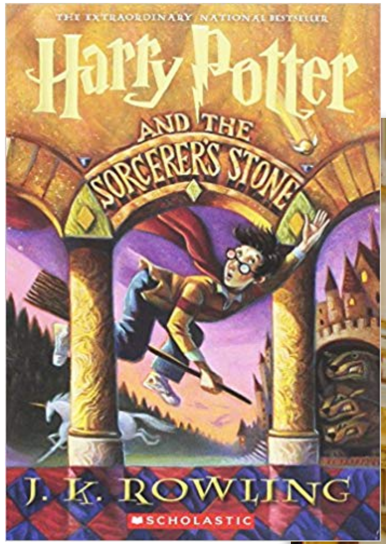
Data Structure Review

I have a collection of books and I want to store them in a dictionary!

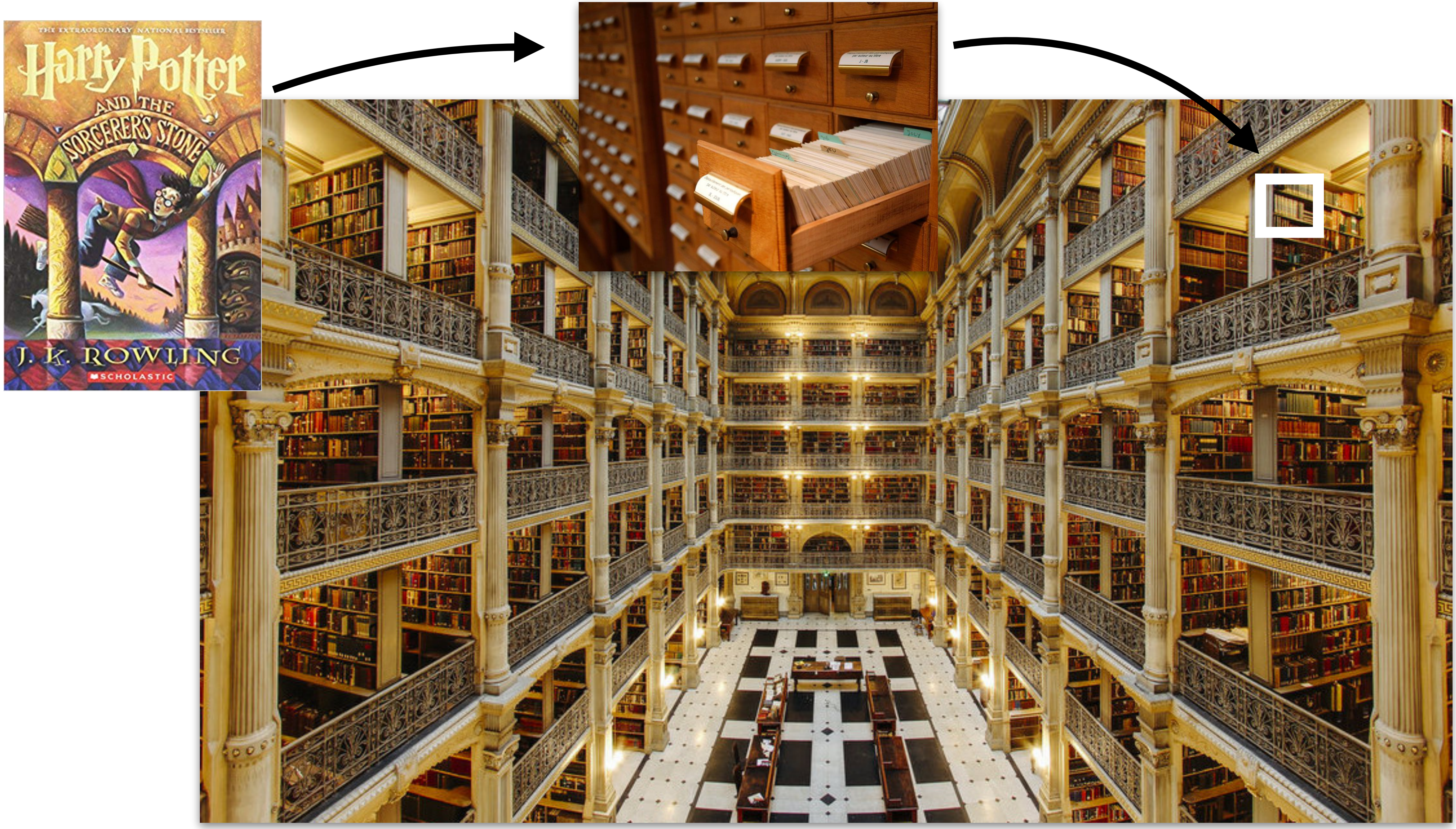


What data structures can I use here?

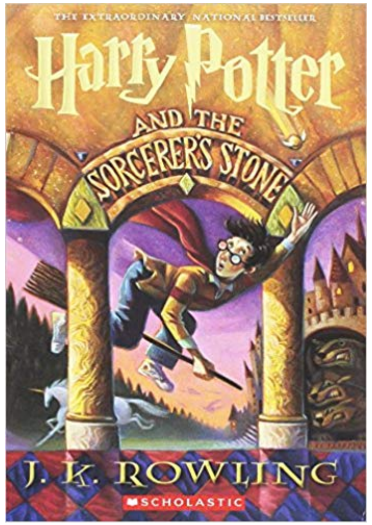
What if $O(\log n)$ isn't good enough?



What if $O(log\ n)$ isn't good enough?



A Hash Table based Dictionary



ISBN: 9781526602381

Call #: PR
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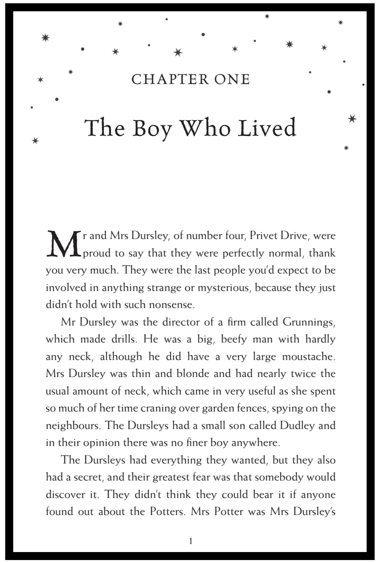


CHAPTER ONE

The Boy Who Lived

Mr and Mrs Dursley, of number four, Privet Drive, were proud to say that they were perfectly normal, thank you very much. They were the last people you'd expect to be involved in anything strange or mysterious, because they just didn't hold with such nonsense.

Mr Dursley was the director of a firm called Grunnings, which made drills. He was a big, beefy man with hardly any neck, although he did have a very large moustache. Mrs Dursley was thin and blonde and had nearly twice the usual amount of neck, which came in very useful as she spent so much of her time craning over garden fences, spying on the neighbours. The Dursleys had a small son called Dudley and in their opinion there was no finer boy anywhere.

The Dursleys had everything they wanted, but they also had a secret, and their greatest fear was that somebody would discover it. They didn't think they could bear it if anyone found out about the Potters. Mrs Potter was Mrs Dursley's

1

A Hash Table based Dictionary

Client Code:

```
Dictionary<KeyType, ValueType> d;
d[k] = v;
```

A **Hash Table** consists of three things:

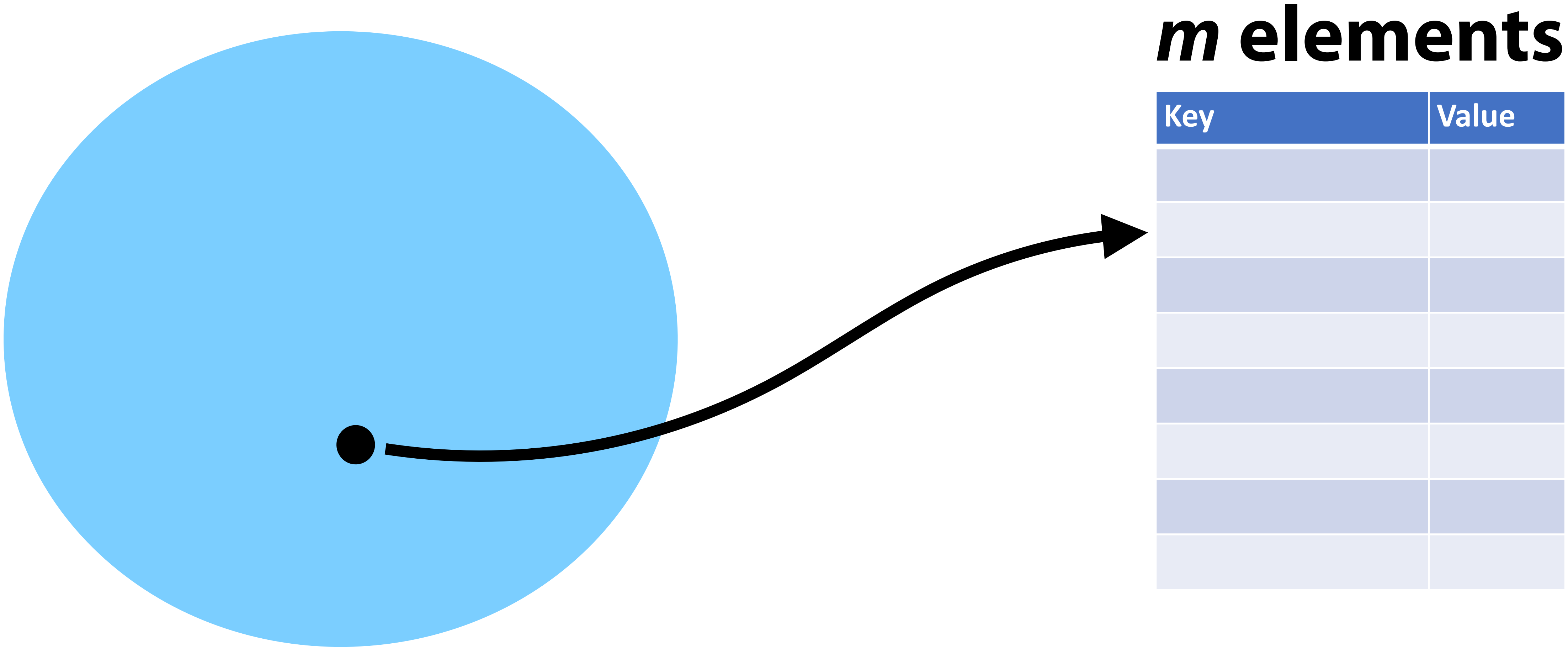
1.

2.

3.

Hash Function

Maps a **keyspace**, a (mathematical) description of the keys for a set of data, to a set of integers.



Hash Function

A hash function ***must*** be:

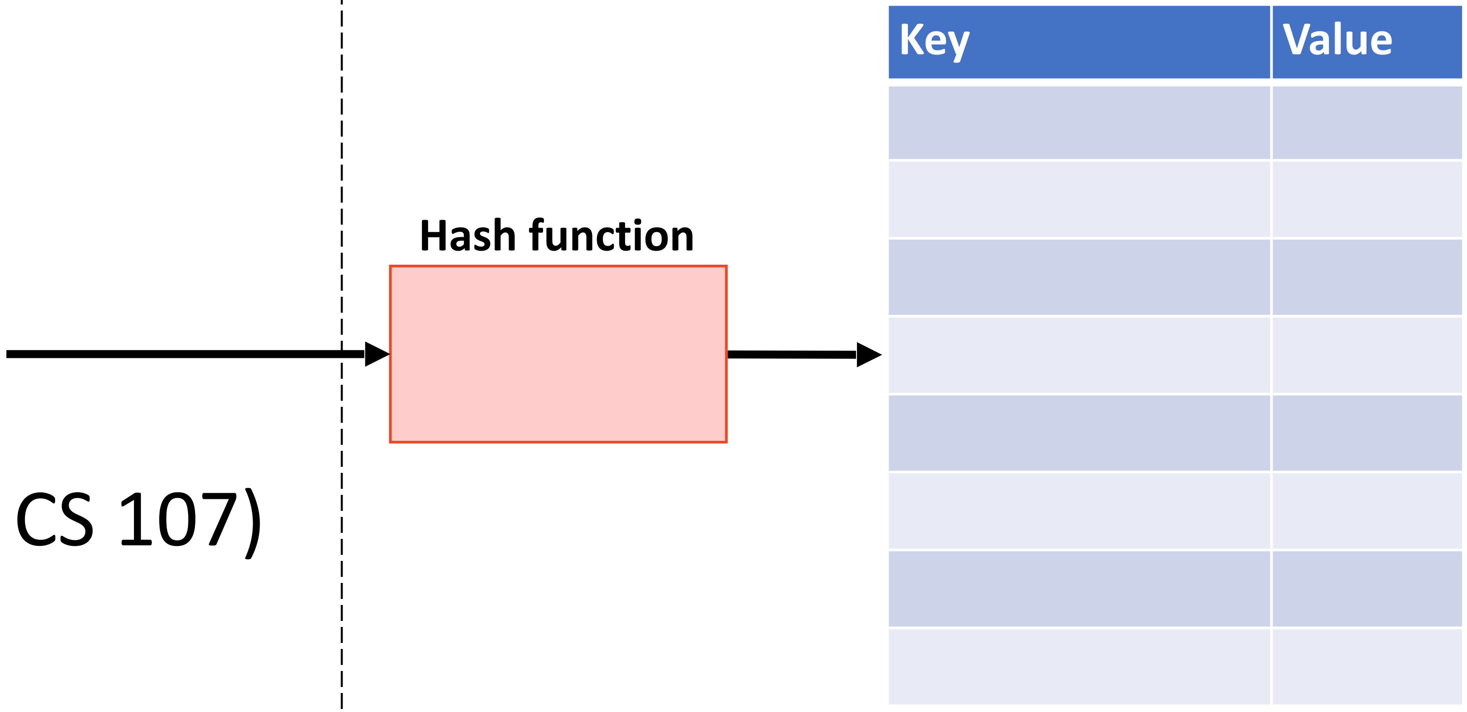
- **Deterministic**:

- **Efficient**:

- **Defined for a certain size table:**

Hash Function

(Angrave, CS 241)
(Beckman, CS 421)
(Challon, CS 125)
(Davis, CS 101)
(Evans, CS 225)
(Fagen-Ulmschneider, CS 107)
(Gunter, CS 422)
(Herman, CS 233)



Hash Function

(Angrave, CS 241)
(Beckman, CS 421)
(Challon, CS 125)
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(Evans, CS 225)
(Fagen-Ulmschneider, CS 107)
(Gunter, CS 422)
(Herman, CS 233)

Hash function

(key[0] - 'A')

Key	Value
Angrave	241
Beckman	421
Challon	125
Davis	101
Evans	225
Fagen-U	107
Gunter	422
Herman	233

General Hash Function

An $O(1)$ deterministic operation that maps all keys in a universe U to a defined range of integers $[0, \ldots, m-1]$

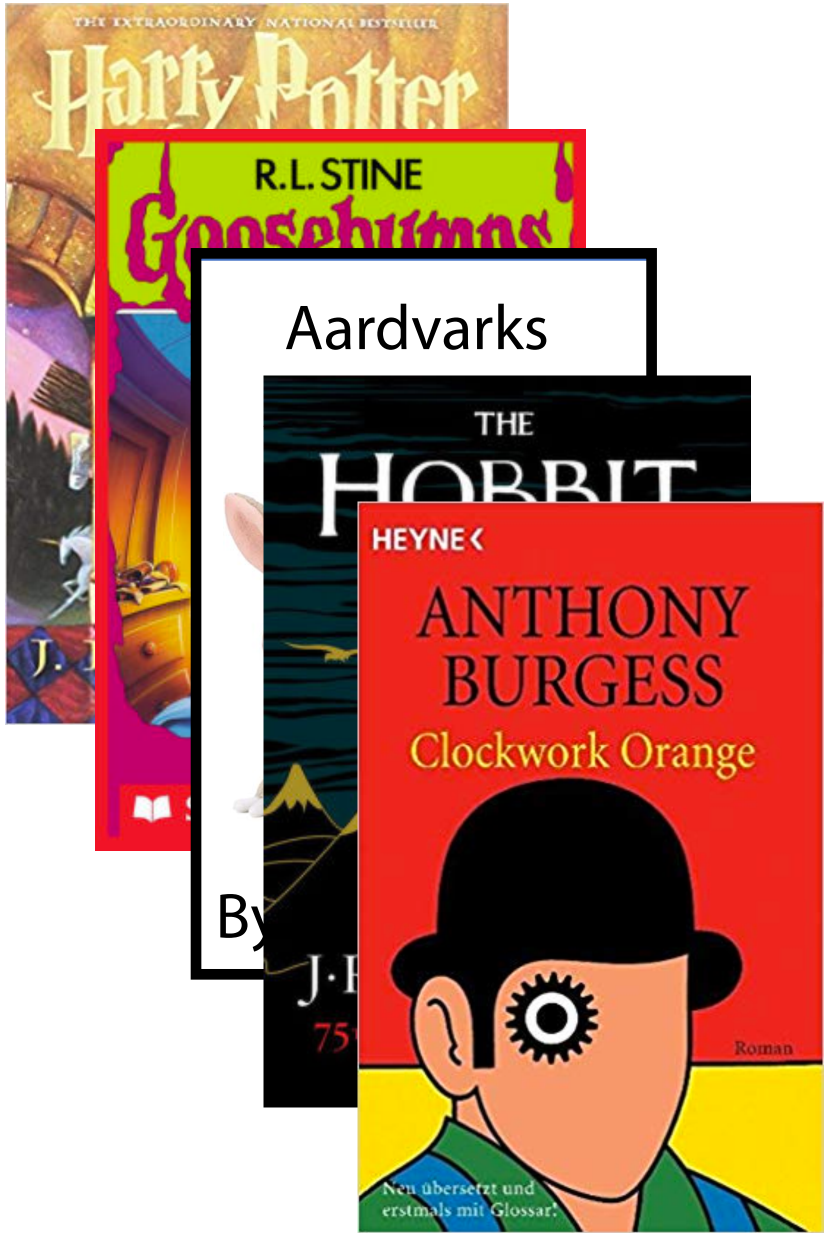
- A **hash**:

- A **compression**:

Choosing a good hash function is tricky…

- Don't create your own (yet*)

Hash Function

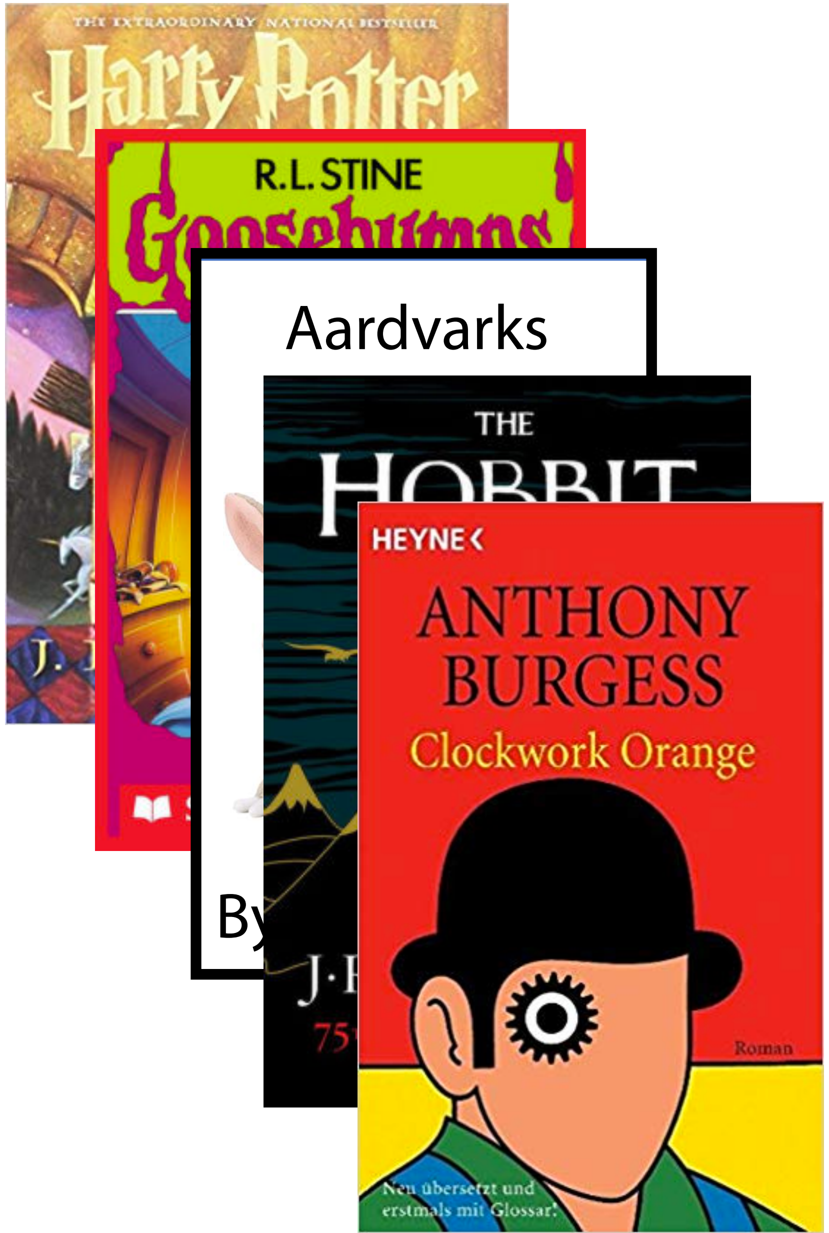


$$h(k) = \left(k.firstName[0] + k.lastName[0]\right) \% \, m$$

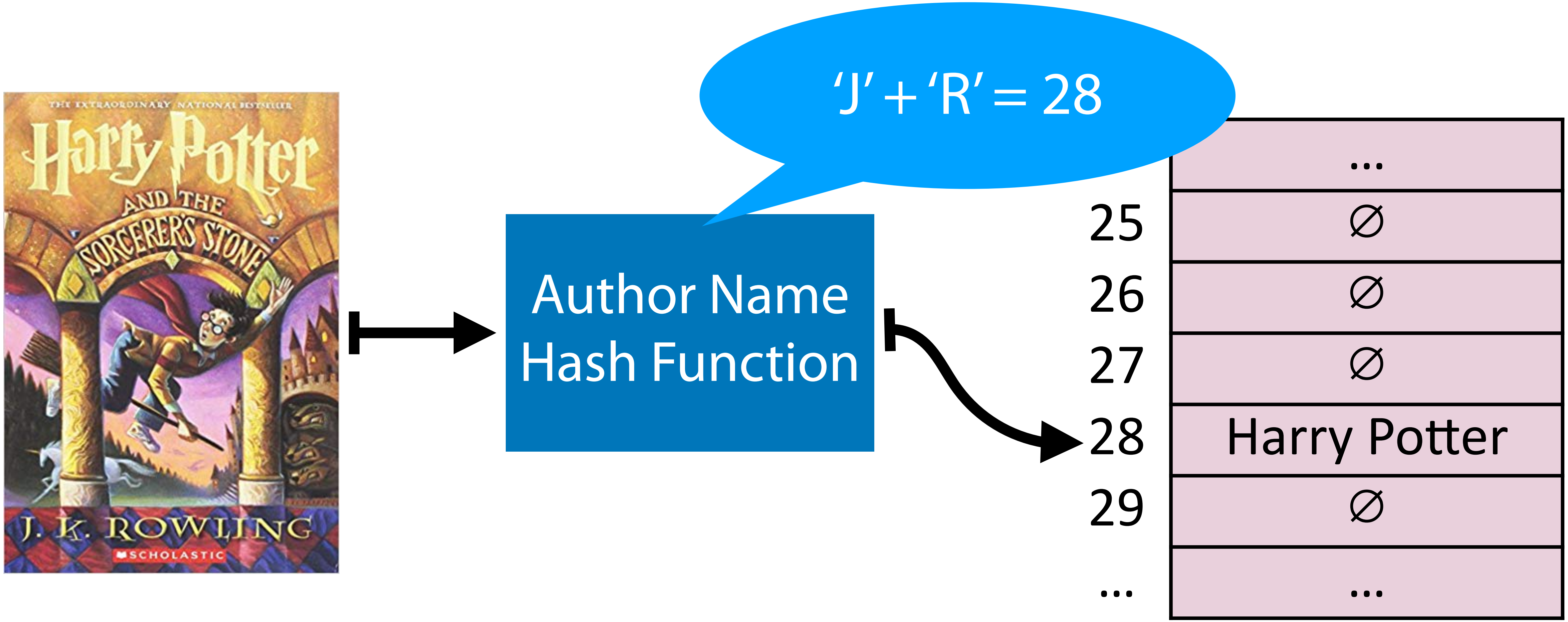
$$h(k) = \left(rand() * k.numPages\right) \% \, m$$

$$h(k) = (\text{Order I insert [Order seen]}) \% \, m$$

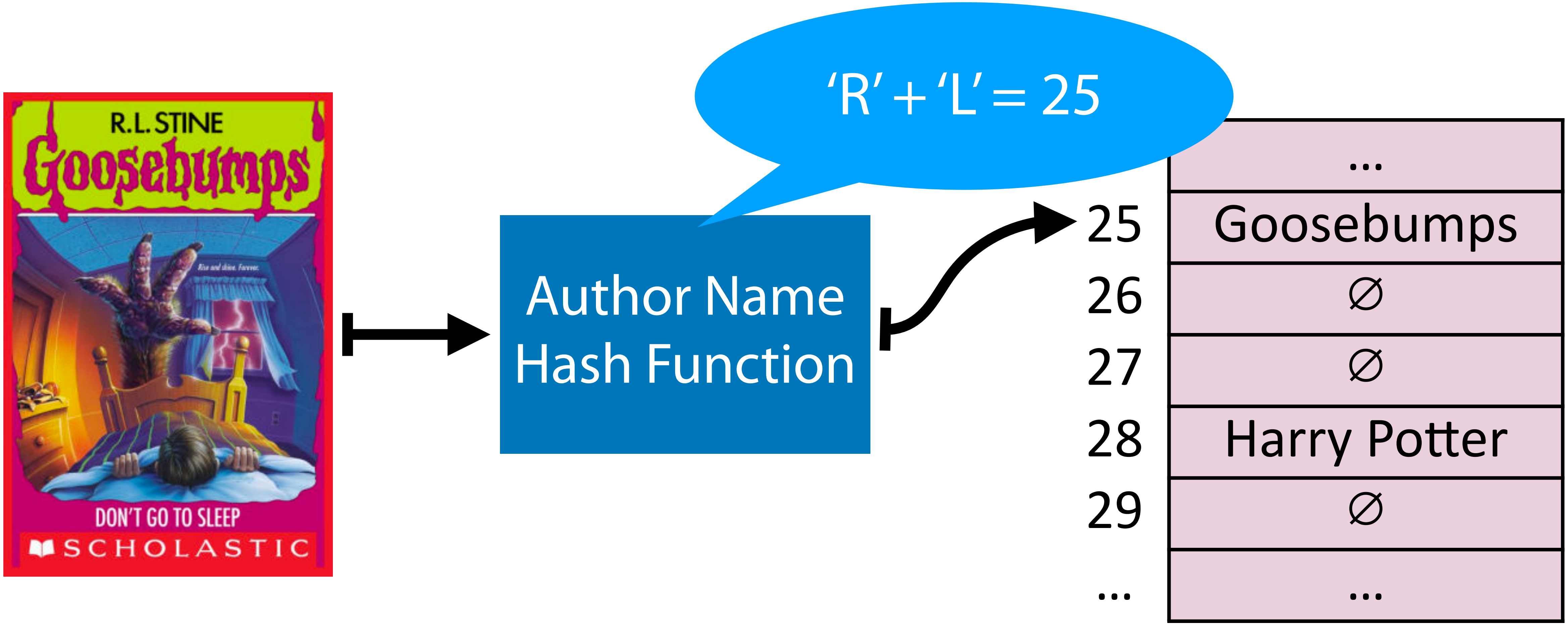
Hash Function



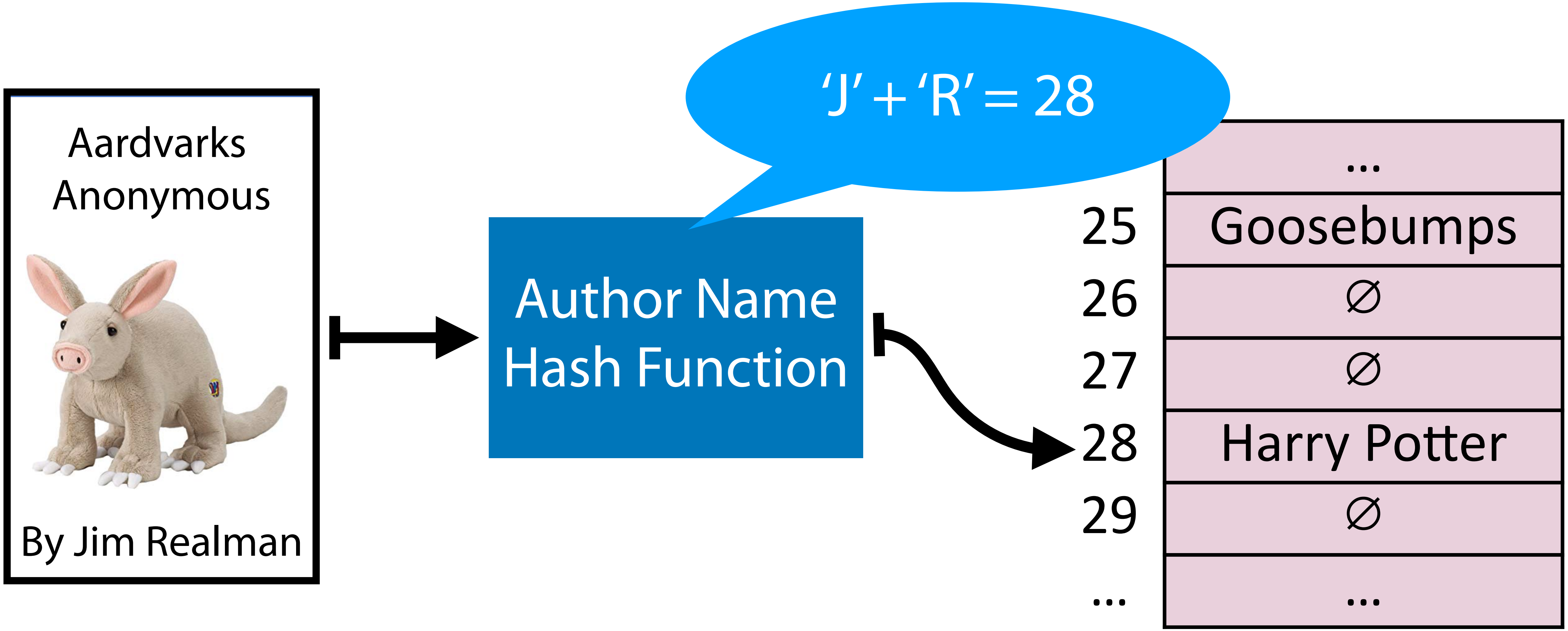
Hash Function



Hash Function



Hash Function

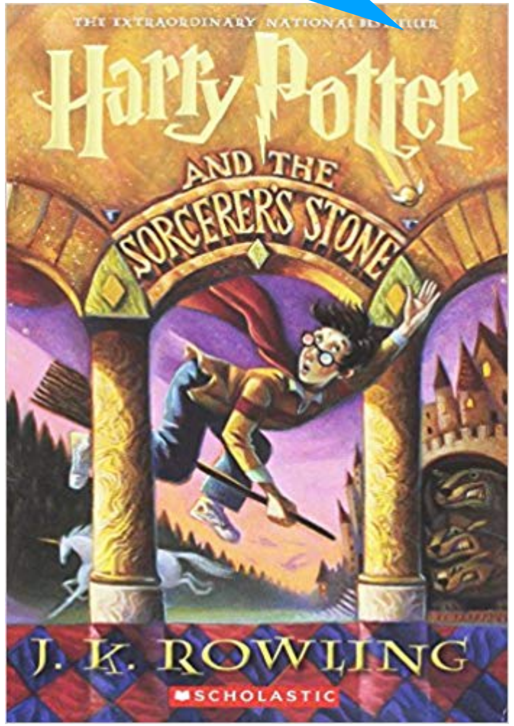


Hash Collision

A ***hash collision*** occurs when multiple unique keys hash to the same value

J.K Rowling = 28!

Jim Realman = 28!



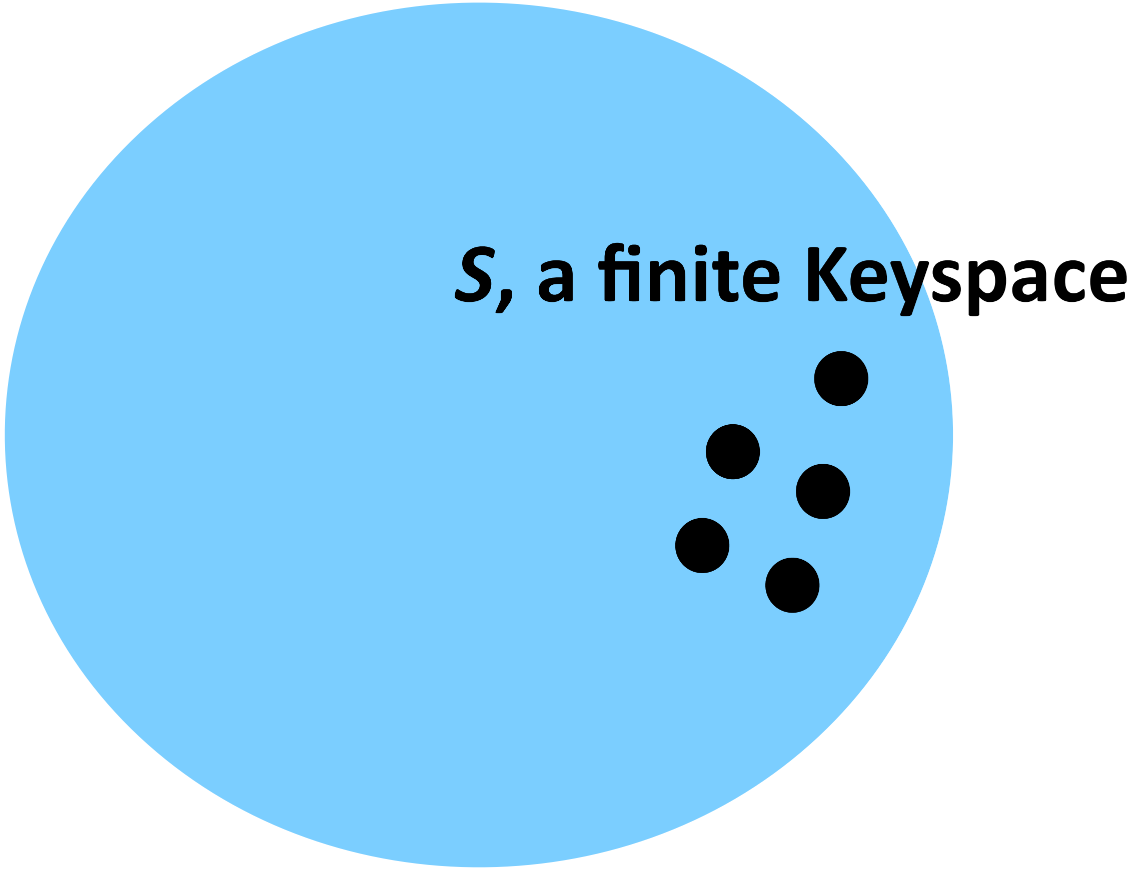
Aardvarks Anonymous

By Jim Realman

…	…
25	Goosebumps
26	∅
27	∅
28	???
29	∅
…	…

Perfect Hashing

If $m \geq S$, we can write a *perfect* hash with no collisions

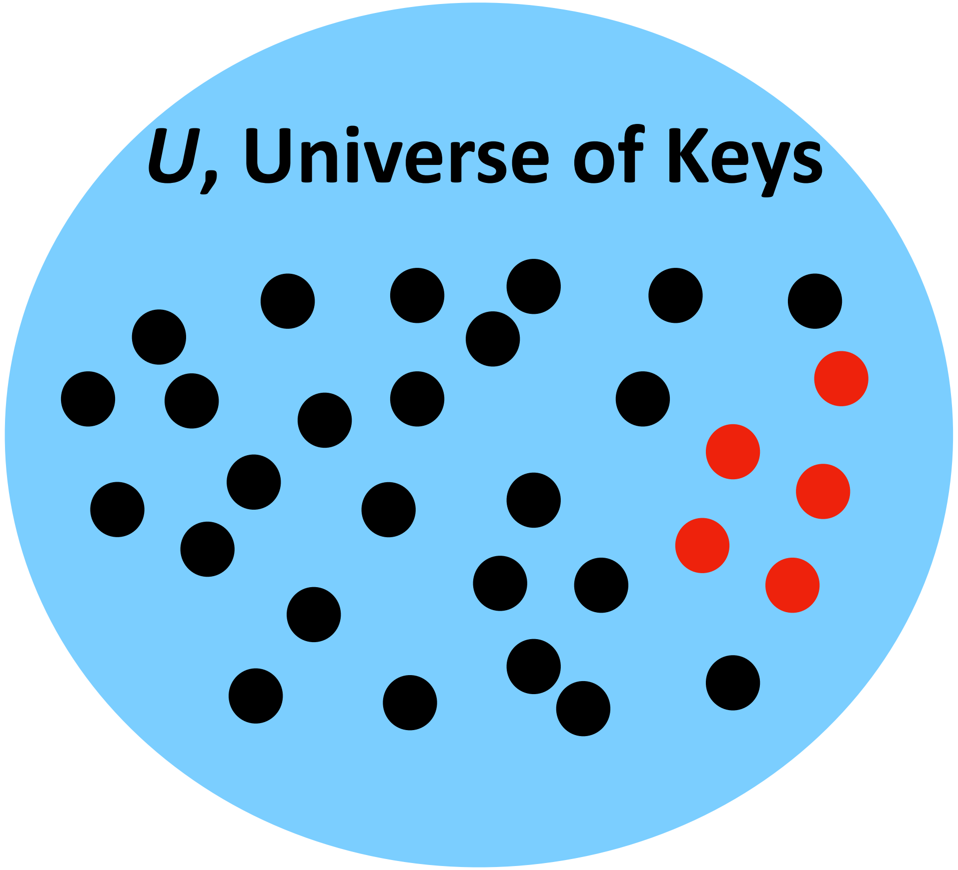


***m* elements**

Key	Value

General Purpose Hashing

In CS 225, we want our hash functions to work *in general*.

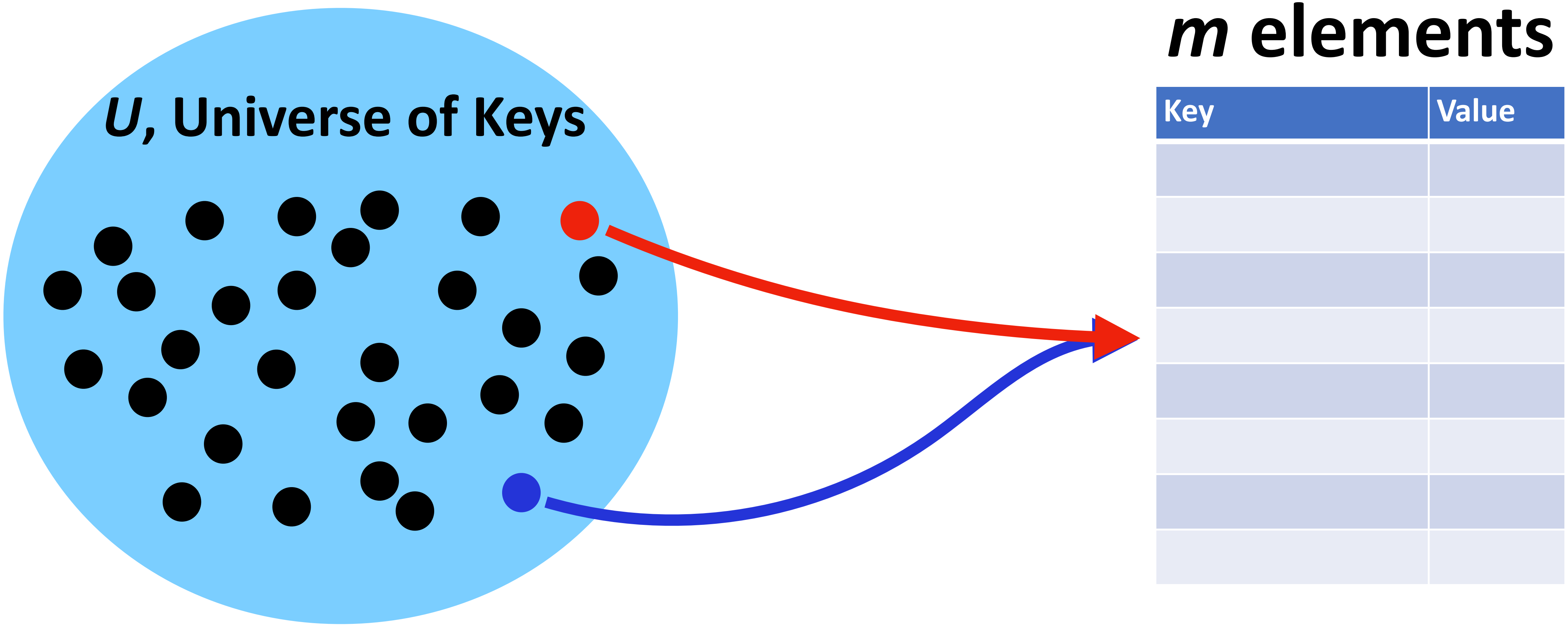


***m* elements**

Key	Value

General Purpose Hashing

If $m < U$, there must be at least one hash collision.



General Purpose Hashing

By fixing h, we open ourselves up to adversarial attacks.

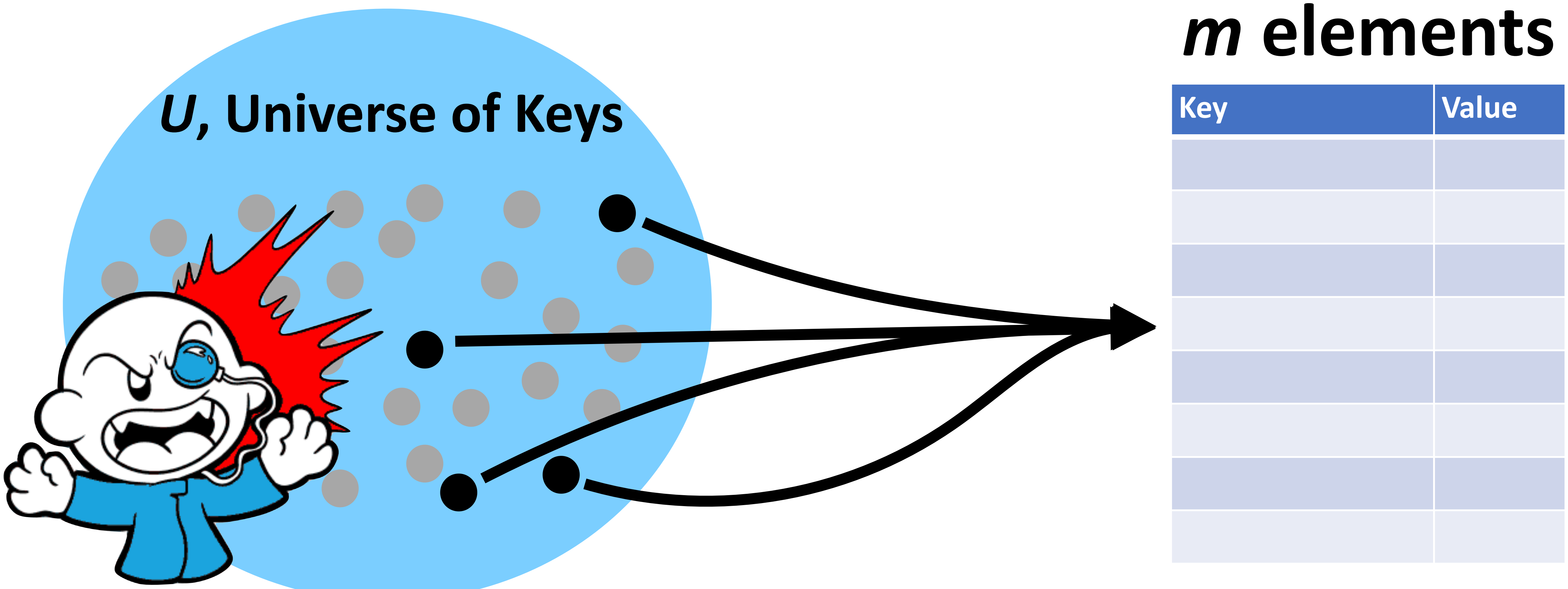


Image by Matthew Loffhagen

A Hash Table based Dictionary

Client Code:

```
Dictionary<KeyType, ValueType> d;
d[k] = v;
```

A **Hash Table** consists of three things:

1. A hash function

2. A data storage structure

3. **A method of addressing *hash collisions***

Open vs Closed Hashing

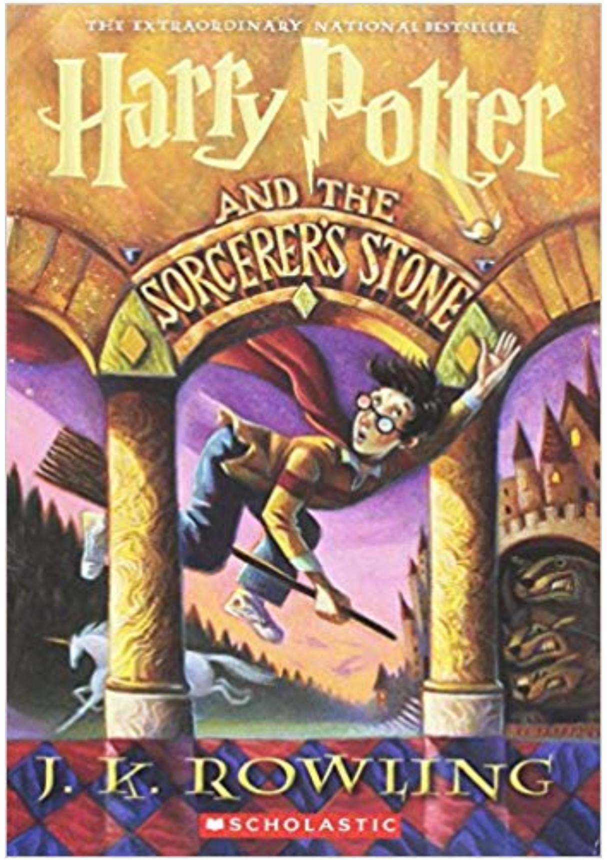
Addressing hash collisions depends on your storage structure.

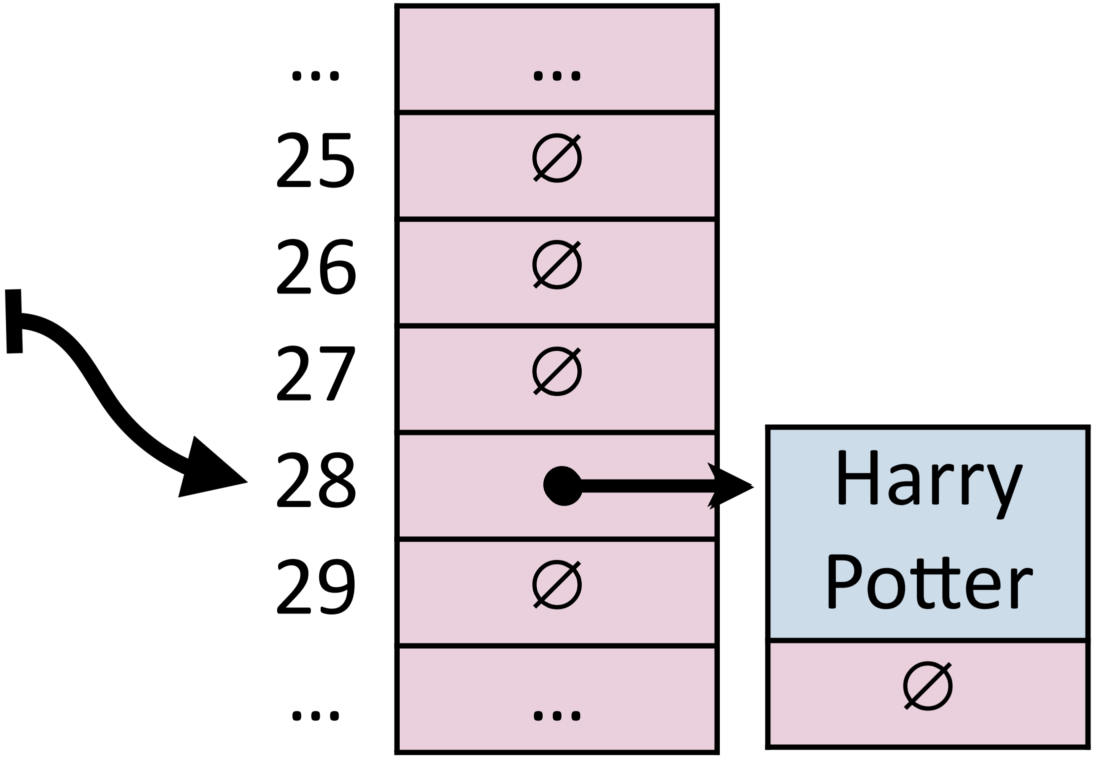
- **Open Hashing:**

- **Closed Hashing:**

Open Hashing

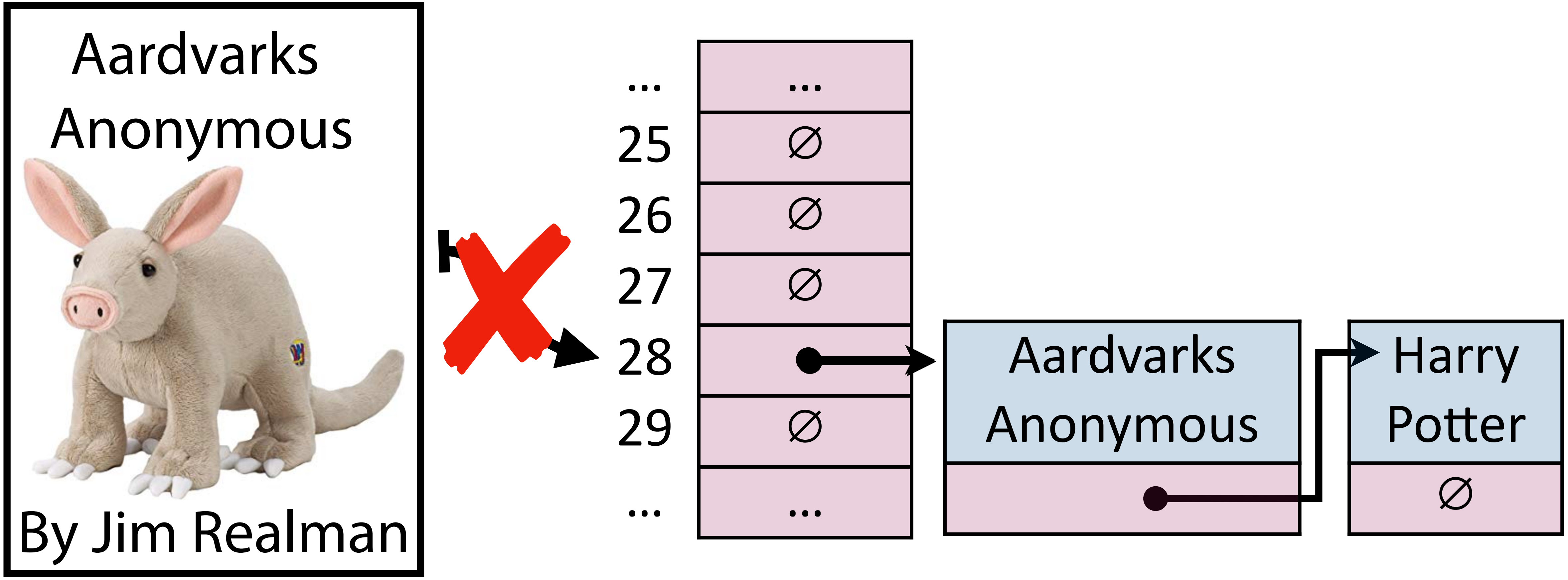
In an ***open hashing*** scheme, key-value pairs are stored externally (for example as a linked list).





Hash Collisions (Open Hashing)

A ***hash collision*** in an open hashing scheme can be resolved by ______________________________. This is called ***separate chaining***.



Insertion (Separate Chaining)

`_insert("Bob")`

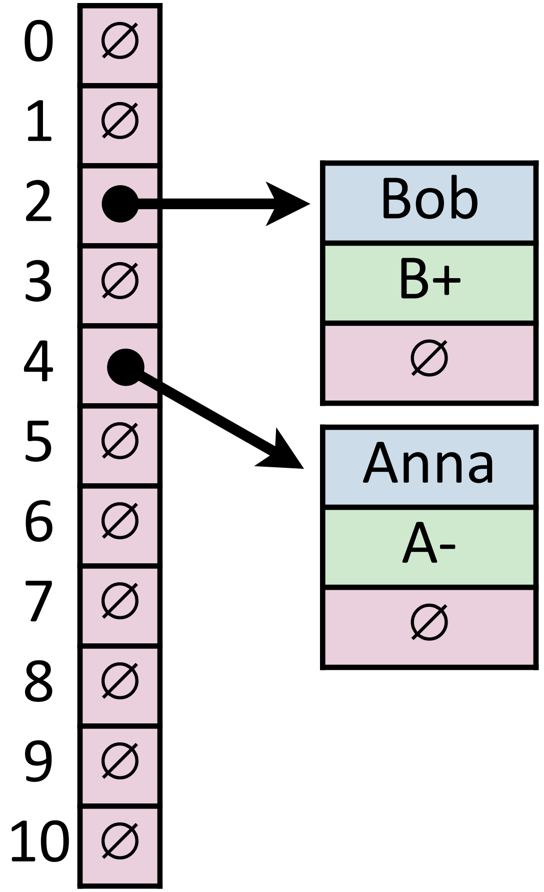
`_insert("Anna")`

Key	Value	Hash
Bob	**B+**	**2**
Anna	**A-**	**4**
Alice	A+	4
Betty	B	2
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7

0	∅
1	∅
2	∅
3	∅
4	∅
5	∅
6	∅
7	∅
8	∅
9	∅
10	∅

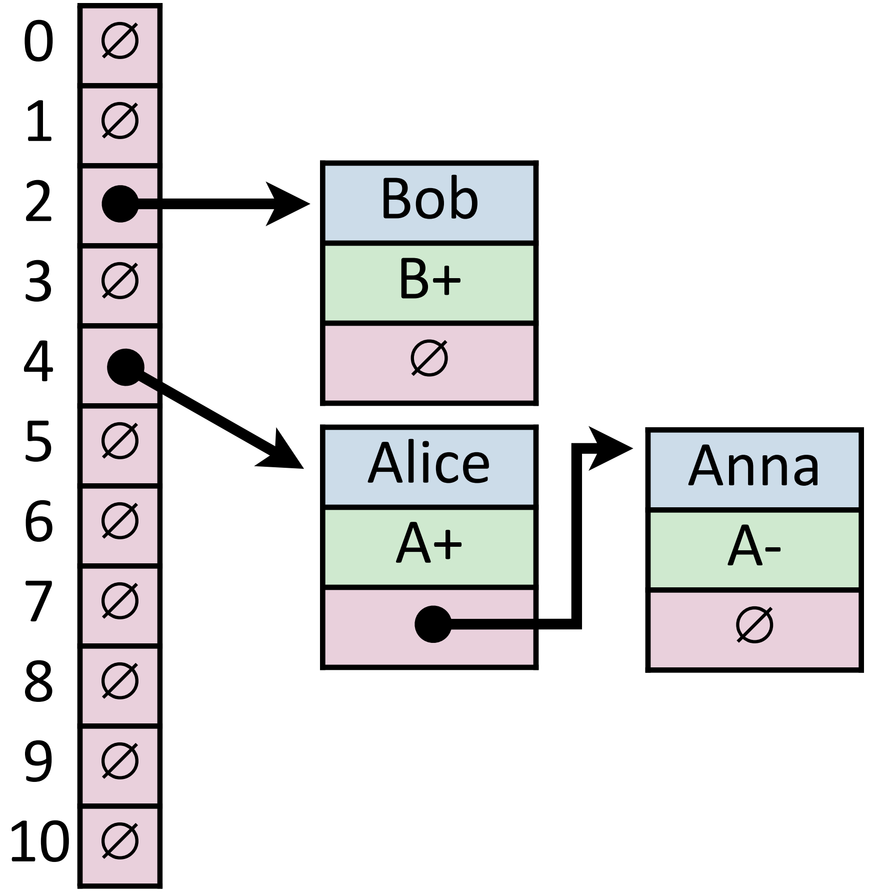
Insertion (Separate Chaining) _insert("Alice")

Key	Value	Hash
Bob	B+	2
Anna	A-	4
Alice	**A+**	**4**
Betty	B	2
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7



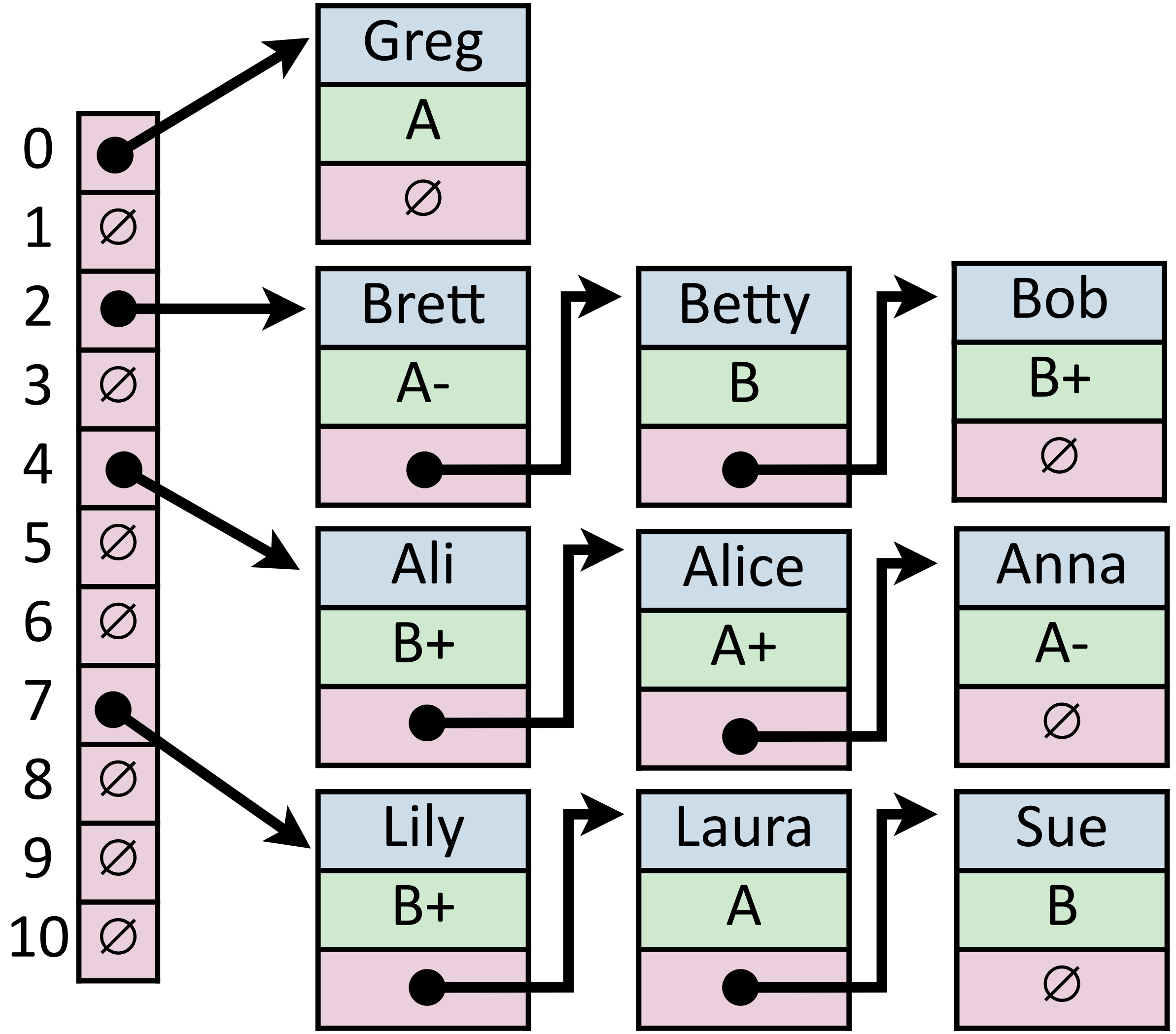
Insertion (Separate Chaining)

Key	Value	Hash
Bob	B+	2
Anna	A-	4
Alice	A+	4
Betty	**B**	**2**
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7



Insertion (Separate Chaining)

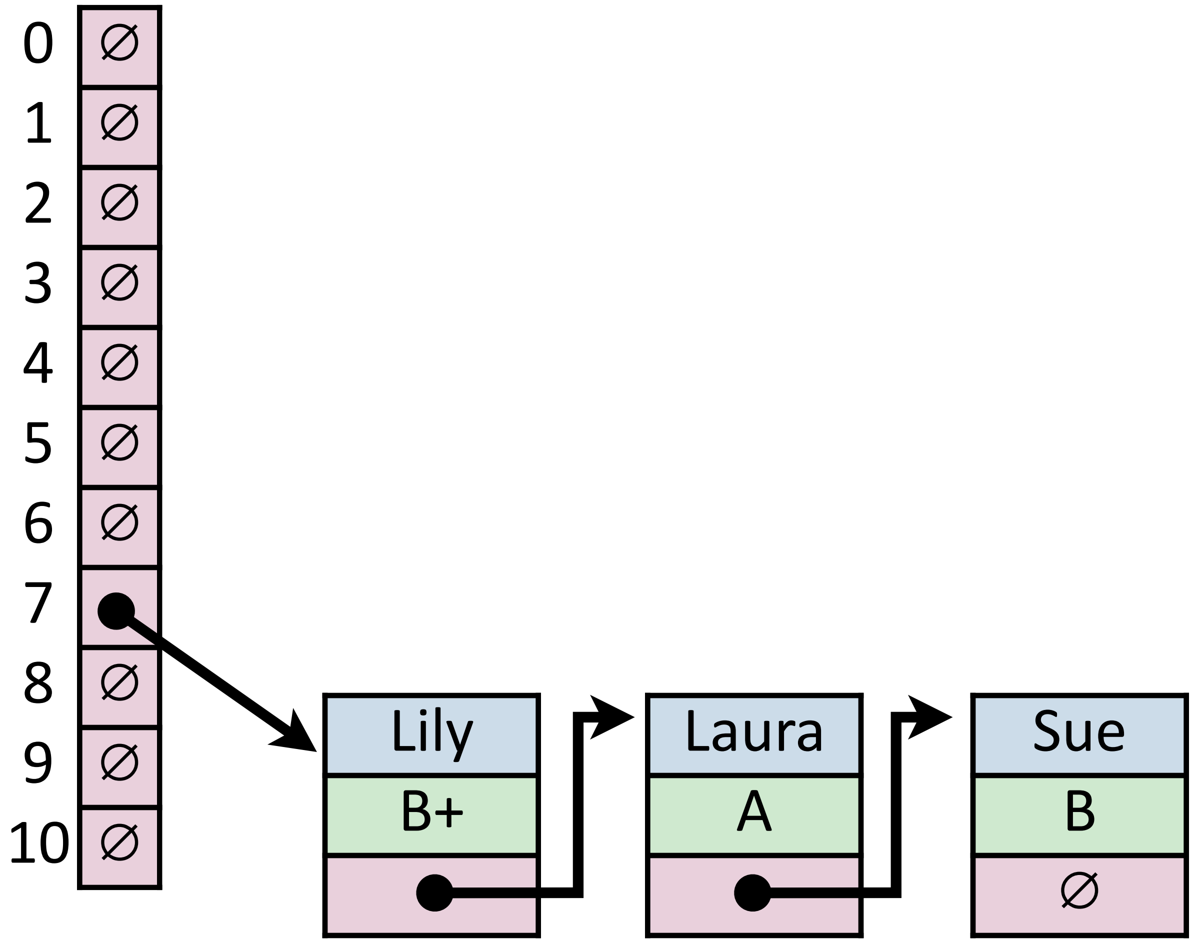
Key	Value	Hash
Bob	B+	2
Anna	A-	4
Alice	A+	4
Betty	B	2
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7



Find (Separate Chaining)

`_find("Sue")`

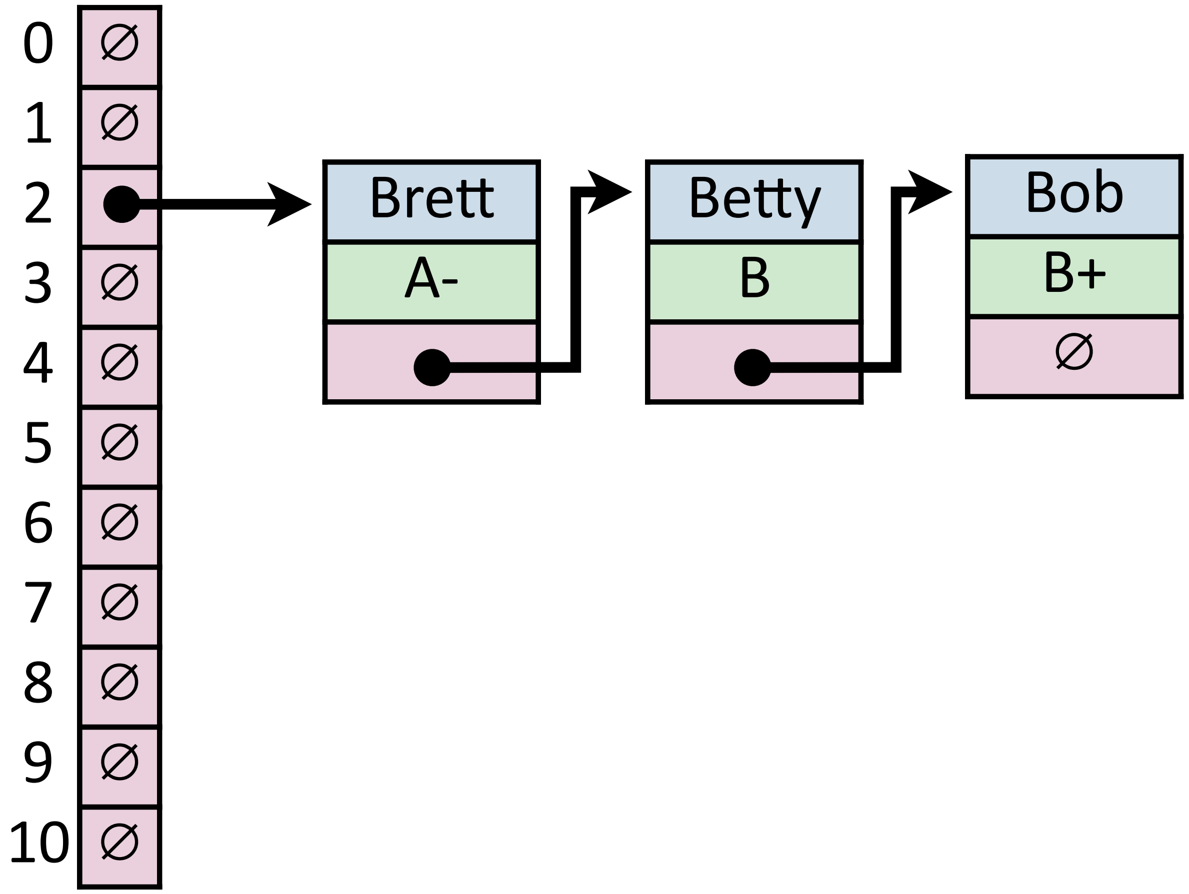
Key	Hash
Sue	7



Remove (Separate Chaining)

`_remove("Betty")`

Key	Hash
Betty	2



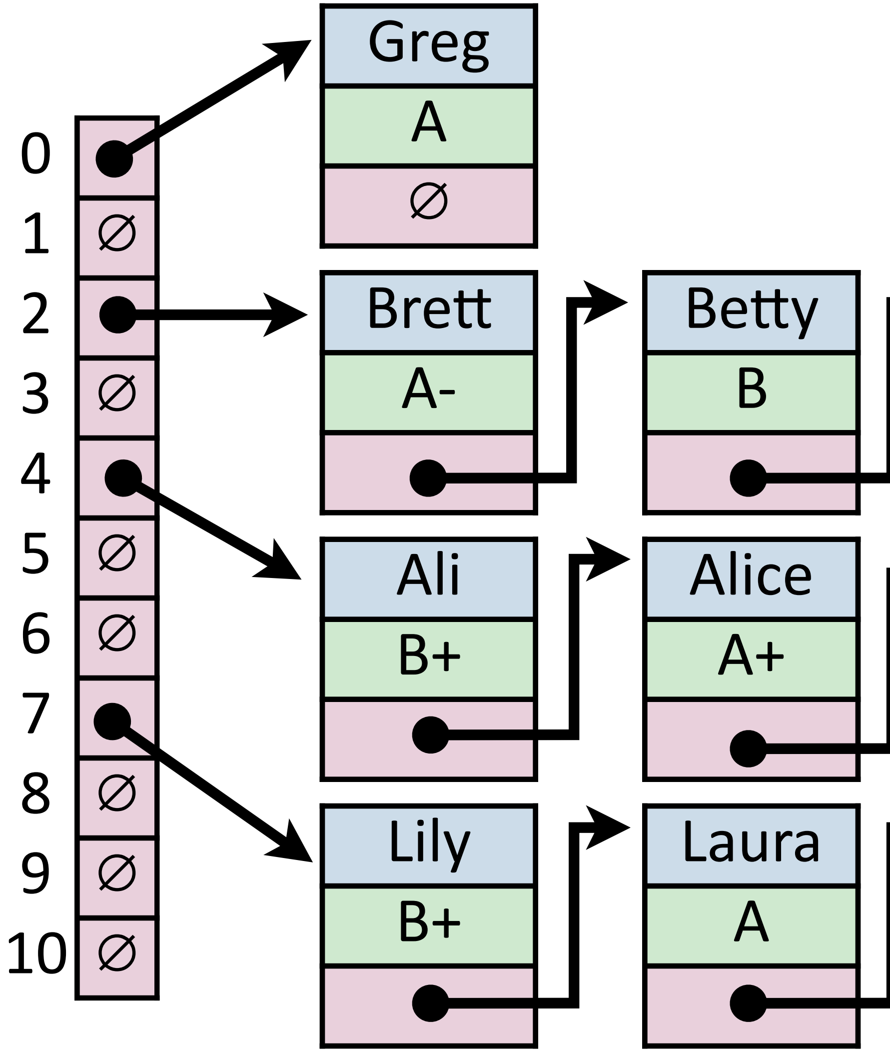
Hash Table (Separate Chaining)

For hash table of size *m* and *n* elements:

Find runs in: ______________________

Insert runs in: ______________________

Remove runs in: ______________________



Hash Table

Worst-Case behavior is bad — but what about randomness?

1) **Fix *h***, our hash, and assume it is good for ***all keys***:

2) Create a ***universal hash function family:***

Simple Uniform Hashing Assumption

Given table of size m, a simple uniform hash, h, implies

$$\forall k_1, k_2 \in U \text{ where } k_1 \neq k_2 \text{ , } Pr(h[k_1] = h[k_2]) = \frac{1}{m}$$

Uniform:

Independent:

Separate Chaining Under SUHA

Given table of size m and n inserted objects

Claim: Under SUHA, expected length of chain is $\frac{n}{m}$

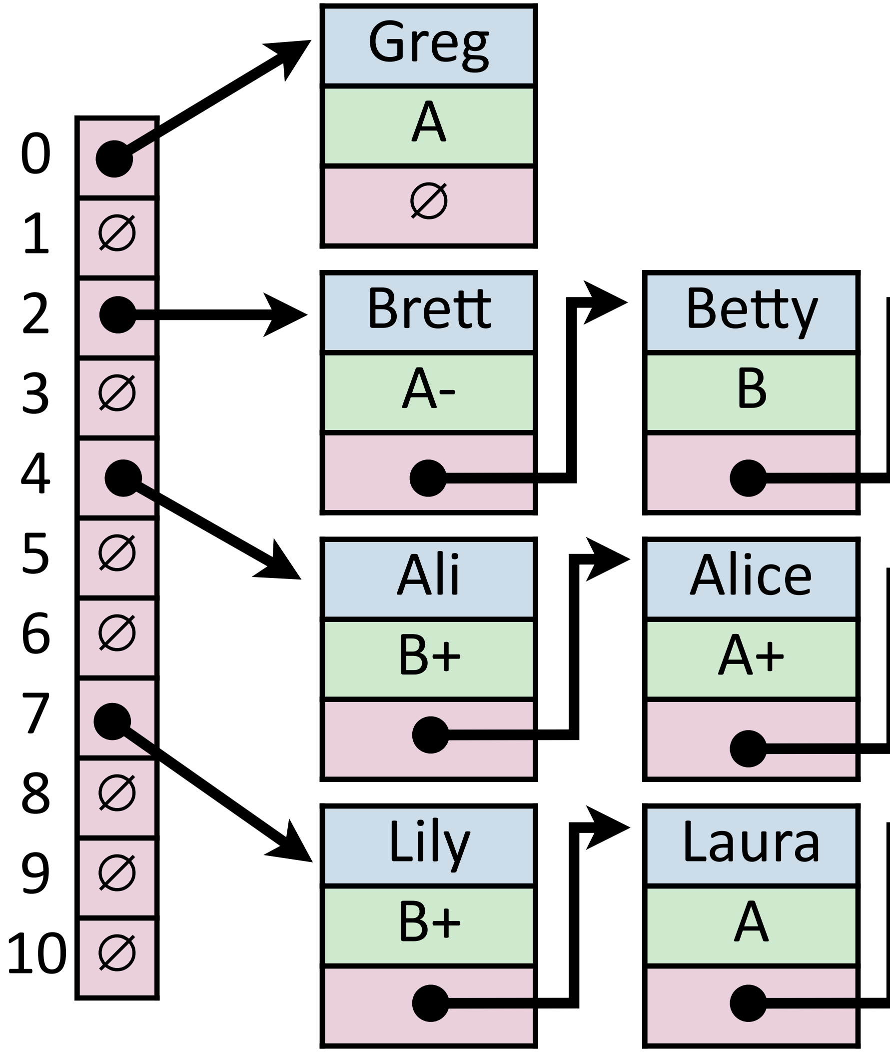
Hash Table (Separate Chaining w/ SUHA)

For hash table of size *m* and *n* elements:

Find runs in: ____________________

Insert runs in: ____________________

Remove runs in: ____________________



Separate Chaining Under SUHA

Pros:

Cons:

Next time: Closed Hashing

Closed Hashing: store *k,v* pairs in the hash table

S = { 1, 8 , 15}

h(k) = k % 7

0	
1	
2	
3	
4	
5	
6	