Every hash table contains three pieces:

1. A hash function, $\mathbf{f}(\mathbf{k})$ : keyspace $\rightarrow$ integer
2. A data storage structure. (Usually an array)
3. A method of handling hash collisions.

Dealing with hashing depends on which type of storage structure you are using.

## Open Hashing:

## Closed Hashing:

## Collision Handling Strategy \#1: Linear Probing

Example: $\mathbf{S}=\{\mathbf{1 6}, \mathbf{8}, \mathbf{4}, \mathbf{1 3}, \mathbf{2 9}, \mathbf{1 1}, \mathbf{2 2}\},|\mathbf{S}|=\mathbf{n}$

$$
\mathbf{h}(\mathbf{k})=\mathbf{k} \% 7
$$

$\mid$ Array $\mid=\mathbf{N}$

| [0] |  |
| ---: | ---: |
| $[1]$ |  |
| $[2]$ |  |
| $[3]$ |  |
| $[4]$ |  |
| $[5]$ |  |
| $[6]$ |  |
| $[7]$ |  |

## Linear Probing:

Try $h(k)=(k+o) \% 7$, if full...
$\operatorname{Try} h(k)=(k+1) \% 7$, if full...
$\operatorname{Try} h(k)=(k+2) \% 7$, if full...

## What problem occurs?

Collision Handling Strategy \#2: Quadratic Probing
Example: $\mathbf{S}=\{\mathbf{1 6}, \mathbf{8}, \mathbf{4}, \mathbf{1 3}, \mathbf{2 9}, \mathbf{1 1}, 22\},|\mathbf{S}|=\mathbf{n}$

$$
\mathbf{h}(\mathbf{k})=\mathbf{k} \% 7
$$

$\mid$ Array $\mid=\mathbf{N}$

|  |  |
| ---: | ---: |
| $[0]$ |  |
| $[1]$ |  |
| $[2]$ |  |
| $[3]$ |  |
| $[4]$ |  |
| $[5]$ |  |
| $[6]$ |  |
| $[7]$ |  |

## Quadratic Probing:

Try $h(k)=(k+0) \% 7$, if full...
$\operatorname{Try} h(k)=\left(k+1^{*} 1\right) \% 7$, if full...
Try $h(k)=\left(k+2^{*} 2\right) \% 7$, if full...

## What problem occurs?

Collision Handling Strategy \#3: Double Hashing:
Example: $S=\{\mathbf{1 6}, \mathbf{8}, \mathbf{4}, \mathbf{1 3}, \mathbf{2 9}, \mathbf{1 1}, \mathbf{2 2}\},|\mathbf{S}|=\mathbf{n}$

$$
\mathbf{h}_{1}(k)=\mathbf{k} \% 7, \mathbf{h}_{2}(k)=5-(k \% 5),|A r r a y|=\mathbf{N}
$$

| [0] |  |
| :---: | :---: |
| [1] |  |
| [2] |  |
| [3] |  |
| [4] |  |
| [5] |  |
| [6] |  |
| [7] |  |

## Double Hashing:

Try $h(k)=\left(k++\mathrm{o}^{*} \mathrm{~h}_{2}(\mathrm{k})\right) \% 7$, if full...
Try $h(k)=\left(k++1^{*} h_{2}(k)\right) \% 7$, if full... Try $h(k)=\left(k++2^{*} h_{2}(k)\right) \% 7$, if full...
$h(k, i)=\left(h_{1}(k)+i^{*} h_{2}(k)\right) \% 7$

## Running Time:

Linear Probing:

- Successful: $\mathbf{1 / 2 ( 1 + 1 / ( 1 - \alpha ) )}$
- Unsuccessful: $1 / 2(\mathbf{1}+\mathbf{1} /(\mathbf{1 - \alpha}))^{\mathbf{2}}$

Double Hashing:

- Successful: $\mathbf{1} / \alpha^{*} \ln (\mathbf{1} /(\mathbf{1}-\alpha))$
- Unsuccessful: $\mathbf{1} /(\mathbf{1}-\boldsymbol{\alpha})$

Separate Chaining:

- Successful: $\mathbf{1}+\mathbf{\alpha} / \mathbf{2}$
- Unsuccessful: $\mathbf{1 + \boldsymbol { \alpha }}$


## Running Time Observations:

1. As $\boldsymbol{\alpha}$ increases:
2. If $\boldsymbol{\alpha}$ is held constant:

## Running Time Observations:



## Linear Probing:

Successful: $\mathbf{1 / 2 ( 1 + 1 / ( 1 - \alpha ) )}$
Unsuccessful: $1 / 2(\mathbf{1}+\mathbf{1} /(\mathbf{1 - \alpha}))^{\mathbf{2}}$


## Double Hashing:

Successful: $\mathbf{1} / \alpha$ * $\ln (\mathbf{1} /(\mathbf{1}-\alpha))$
Unsuccessful: 1/(1-a)

## Which collision resolution strategy is better?

- Big Records:
- Structure Speed:

What structure do hash tables replace?

What constraint exists on hashing that doesn't exist with BSTs?

## Why talk about BSTs at all?

Analysis of Dictionary-based Data Structures

|  | Hash Table |  | AVL | List |
| :--- | :--- | :--- | :--- | :--- |
|  | Amortized | Worst Case |  |  |
| Find |  |  |  |  |
| Insert |  |  |  |  |
| Storage <br> Space |  |  |  |  |

## ReHashing:

When do we want to resize?

How do we resize?

Algorithm:

