



CS 225

Data Structures

September 24 – Iterators and Trees

G Carl Evans

stlList.cpp

```
1 #include <list>
2 #include <string>
3 #include <iostream>
4
5 struct Animal {
6     std::string name, food;
7     bool big;
8     Animal(std::string name = "blob", std::string food = "you", bool big = true) :
9         name(name), food(food), big(big) { /* nothing */ }
10 };
11
12 int main() {
13     Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
14     std::vector<Animal> zoo;
15
16     zoo.push_back(g);
17     zoo.push_back(p);    // std::vector's insertAtEnd
18     zoo.push_back(b);
19
20     for ( std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); it++ ) {
21         std::cout << (*it).name << " " << (*it).food << std::endl;
22     }
23
24     return 0;
25 }
```

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16     zoo.push_back(g);
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19
20     for ( auto it = zoo.begin(); it != zoo.end(); it++ ) {
21         std::cout << (*it).name << " " << (*it).food << std::endl;
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```

For Each and Iterators

```
for ( const TYPE & variable : collection ) {  
    // ...  
}
```

```
14 std::vector<Animal> zoo;  
   ...  
20 for ( const Animal & animal : zoo ) {  
21     std::cout << animal.name << " " << animal.food << std::endl;  
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For Each and Iterators

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for ( const TYPE & variable : collection ) {  
    // ...  
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14 std::vector<Animal> zoo;  
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20 for ( const Animal & animal : zoo ) {  
21     std::cout << animal.name << " " << animal.food << std::endl;  
22 }
```

```
... std::unordered_set<std::string, Animal> zoo;  
...  
20 for ( const Animal & animal : zoo ) {  
21     std::cout << animal.name << " " << animal.food << std::endl;  
22 }
```

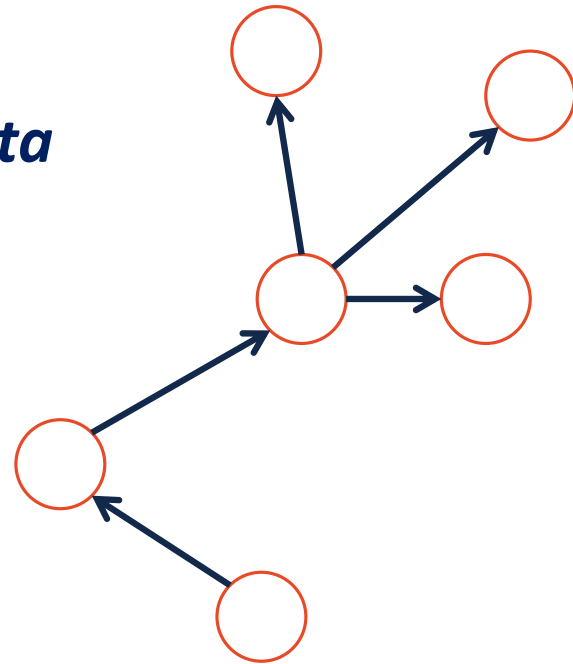
Trees

“The most important non-linear data structure in computer science.”

- David Knuth, The Art of Programming, Vol. 1

A tree is:

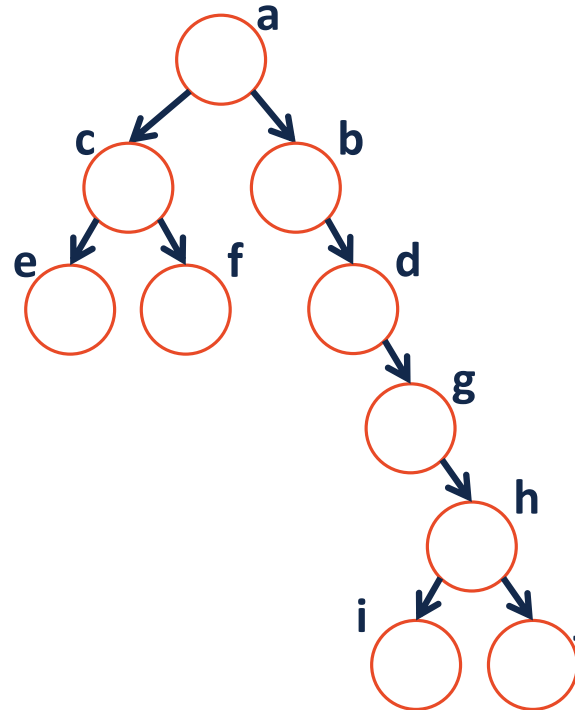
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More Specific Trees

We'll focus on **binary trees**:

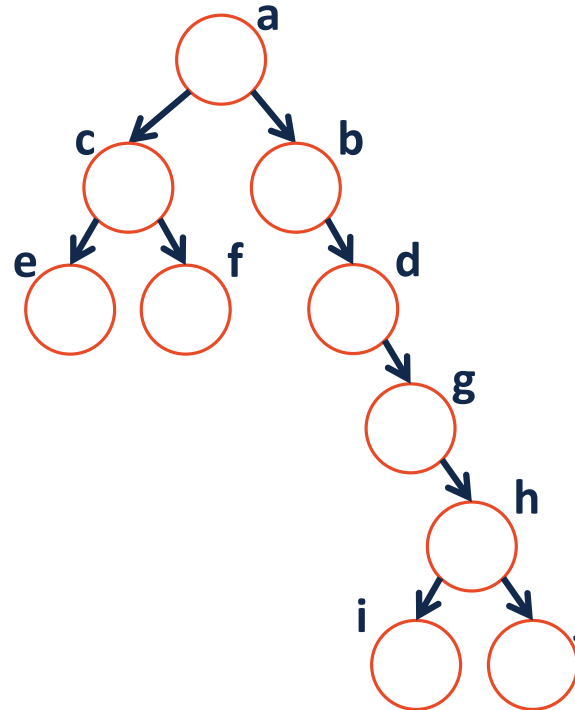
- A binary tree is **acyclic** – there are no cycles within the graph



More Specific Trees

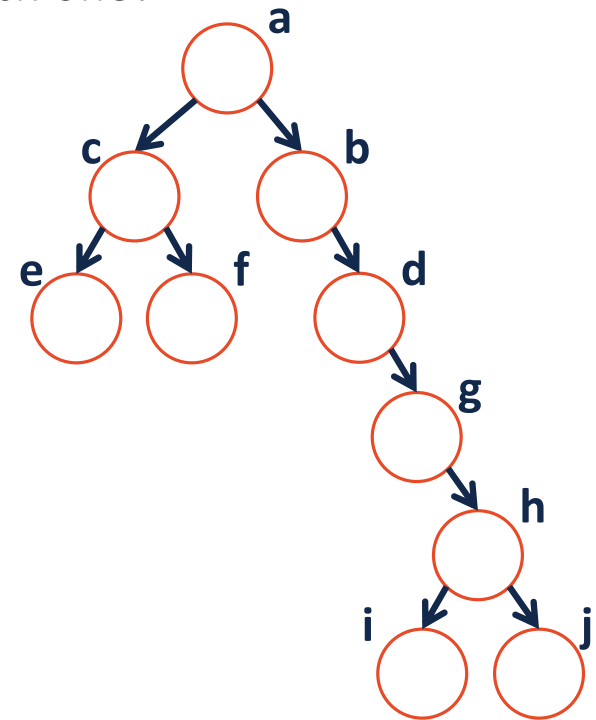
We'll focus on **binary trees**:

- A binary tree contains **two or fewer children** – where one is the “left child” and one is the “right child”:



Tree Terminology

- Find an **edge** that is not on the longest **path** in the tree. Give that edge a reasonable name.
- One of the vertices is called the **root** of the tree. Which one?
- How many parents does each vertex have?
- Which vertex has the fewest **children**?
- Which vertex has the most **ancestors**?
- Which vertex has the most **descendants**?
- List all the vertices in b's left **subtree**.
- List all the **leaves** in the tree.



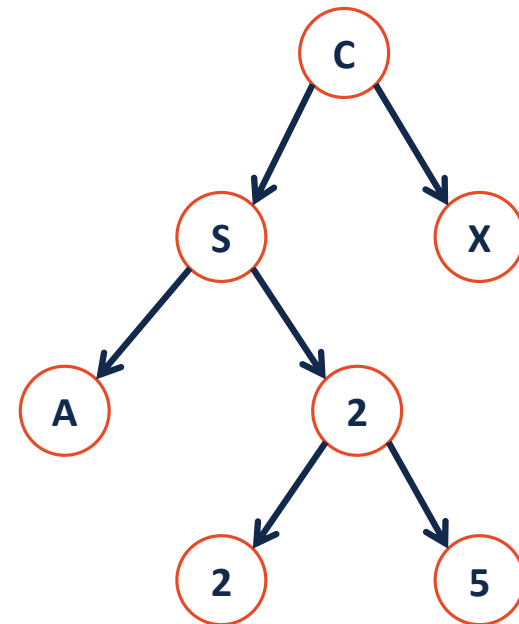
Binary Tree – Defined

A binary tree T is either:

-

OR

-

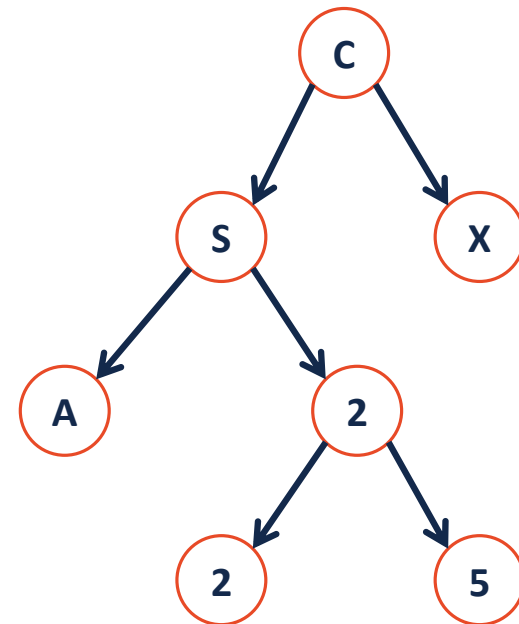


Tree Property: height

height(T): length of the longest path from the root to a leaf

Given a binary tree T:

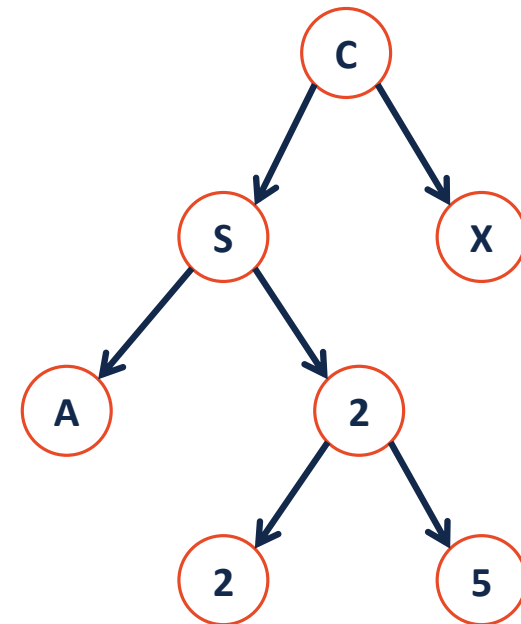
height(T) =



Tree Property: full

A tree F is **full** if and only if:

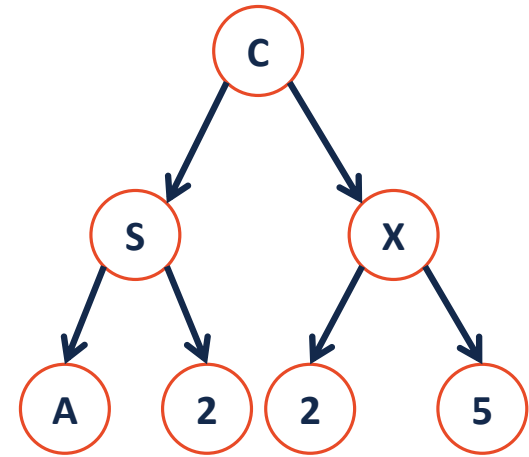
- 1.
- 2.



Tree Property: perfect

A **perfect** tree P is:

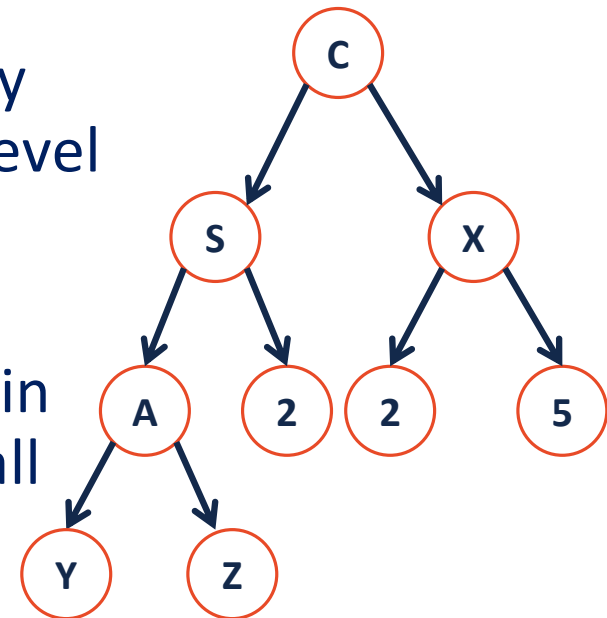
- 1.
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Tree Property: complete

Conceptually: A perfect tree for every level except the last, where the last level is “pushed to the left”.

Slightly more formal: For any level k in $[0, h-1]$, k has 2^k nodes. For level h , all nodes are “pushed to the left”.



Tree Property: complete

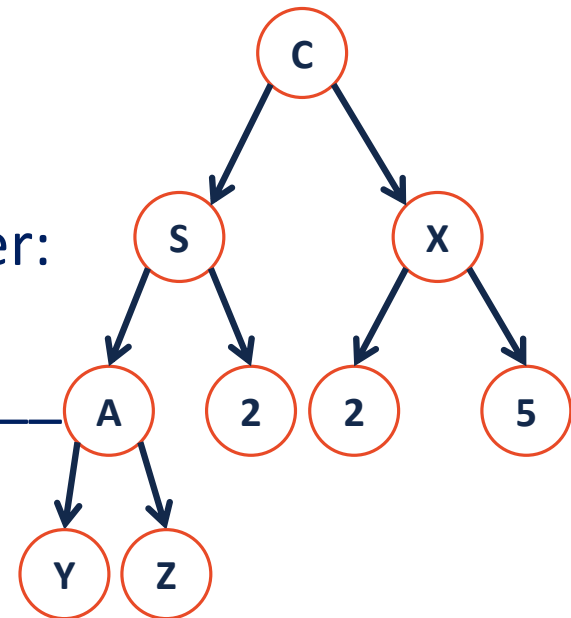
A **complete** tree C of height h , C_h :

1. $C_{-1} = \{\}$
2. C_h (where $h > 0$) = $\{r, T_L, T_R\}$ and either:

T_L is _____ and T_R is _____

OR

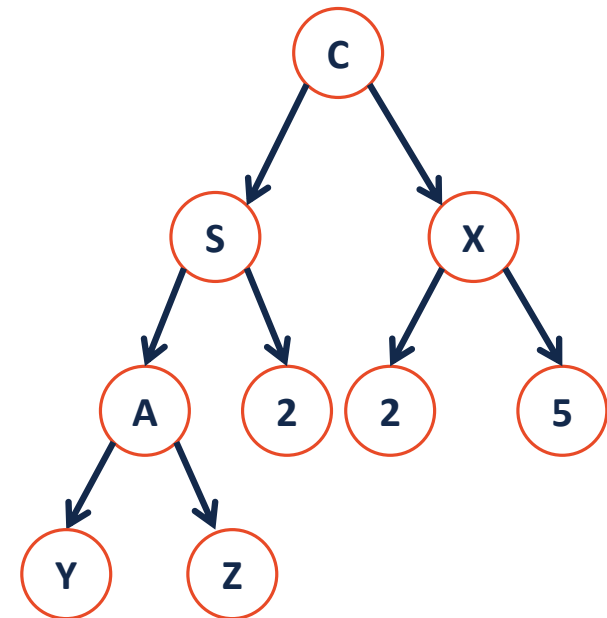
T_L is _____ and T_R is _____



Tree Property: complete

Is every **full** tree **complete**?

If every **complete** tree **full**?



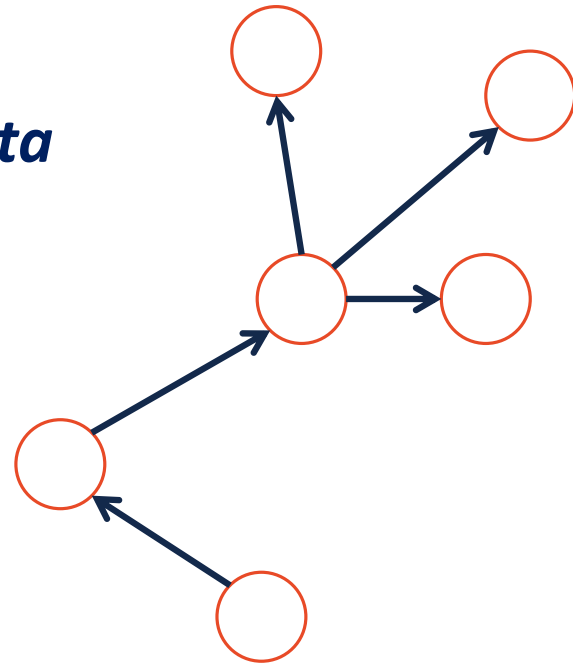
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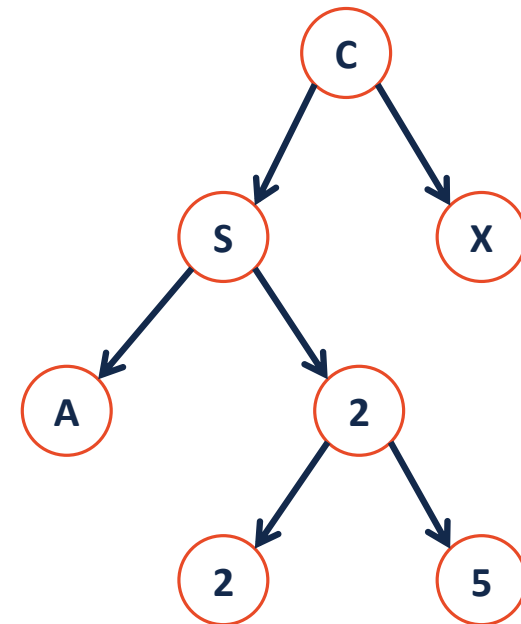
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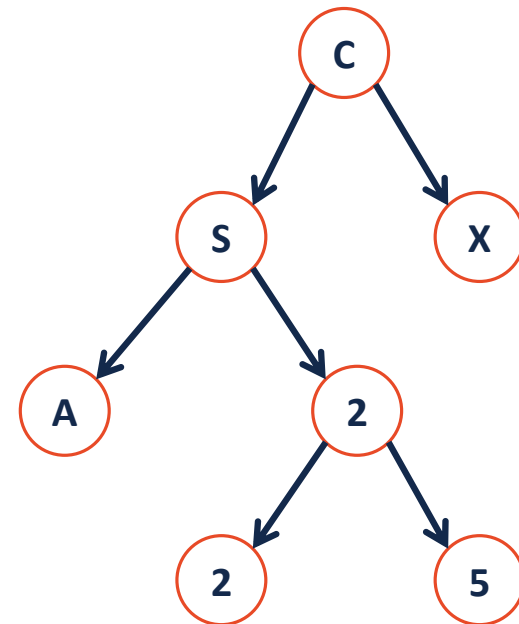


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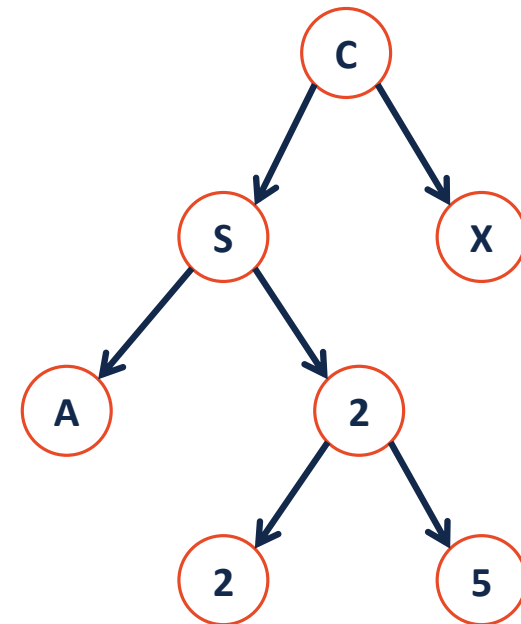
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A tree F is **full** if and only if:

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- 2.



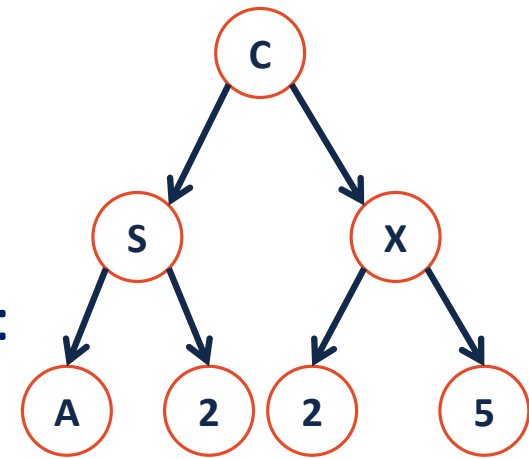
Tree Property: perfect

A **perfect** tree P is defined in terms of the tree's height.

Let P_h be a perfect tree of height h , and:

1.

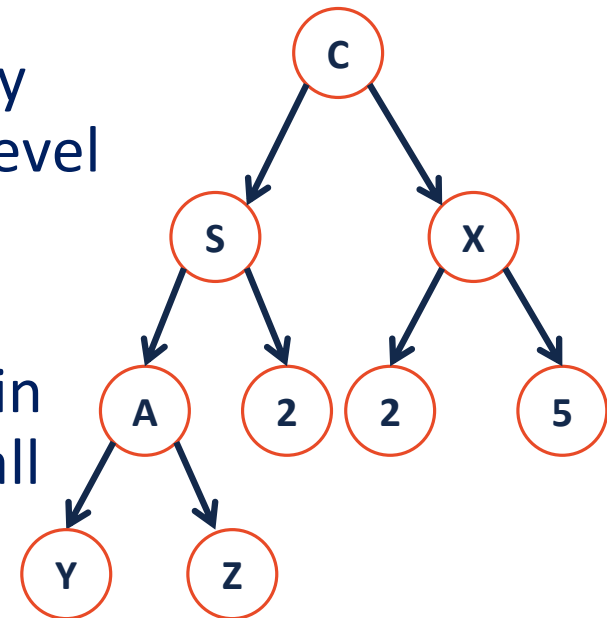
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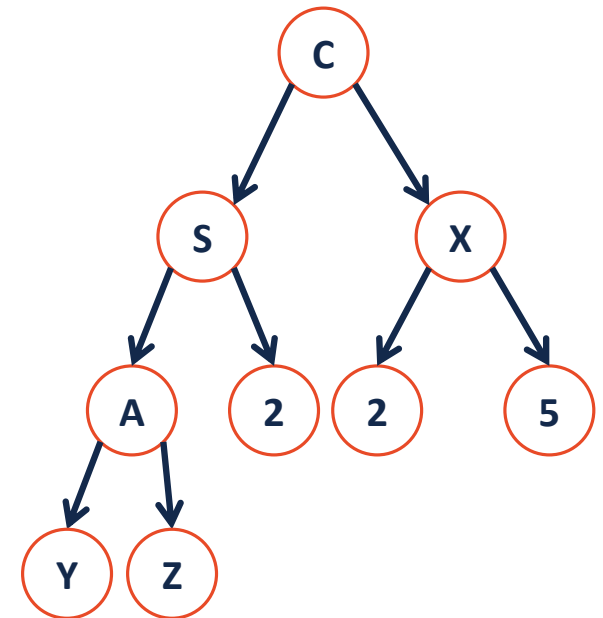
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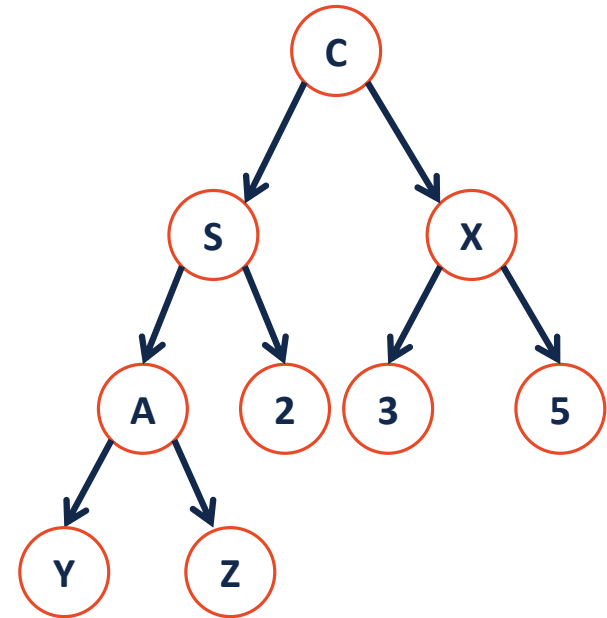
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Tree Property: complete

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If every **complete** tree **full**?





Tree ADT



Tree ADT

insert, inserts an element to the tree.

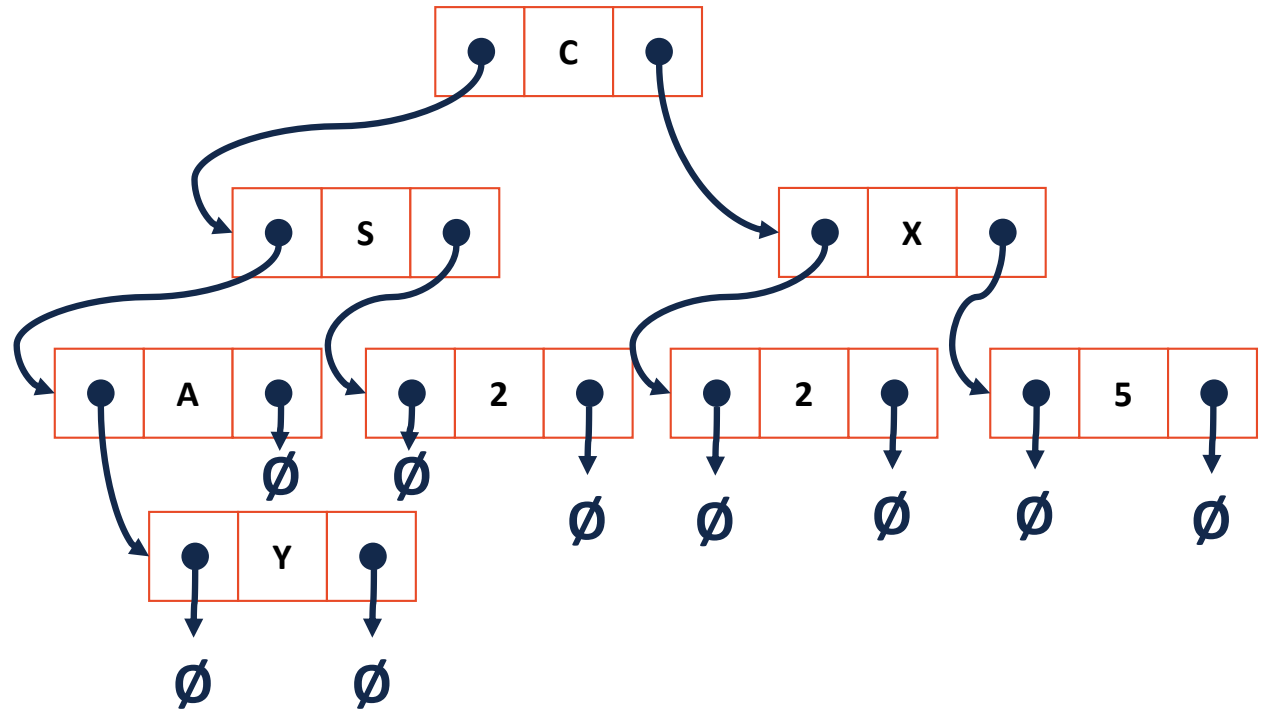
remove, removes an element from the tree.

traverse,

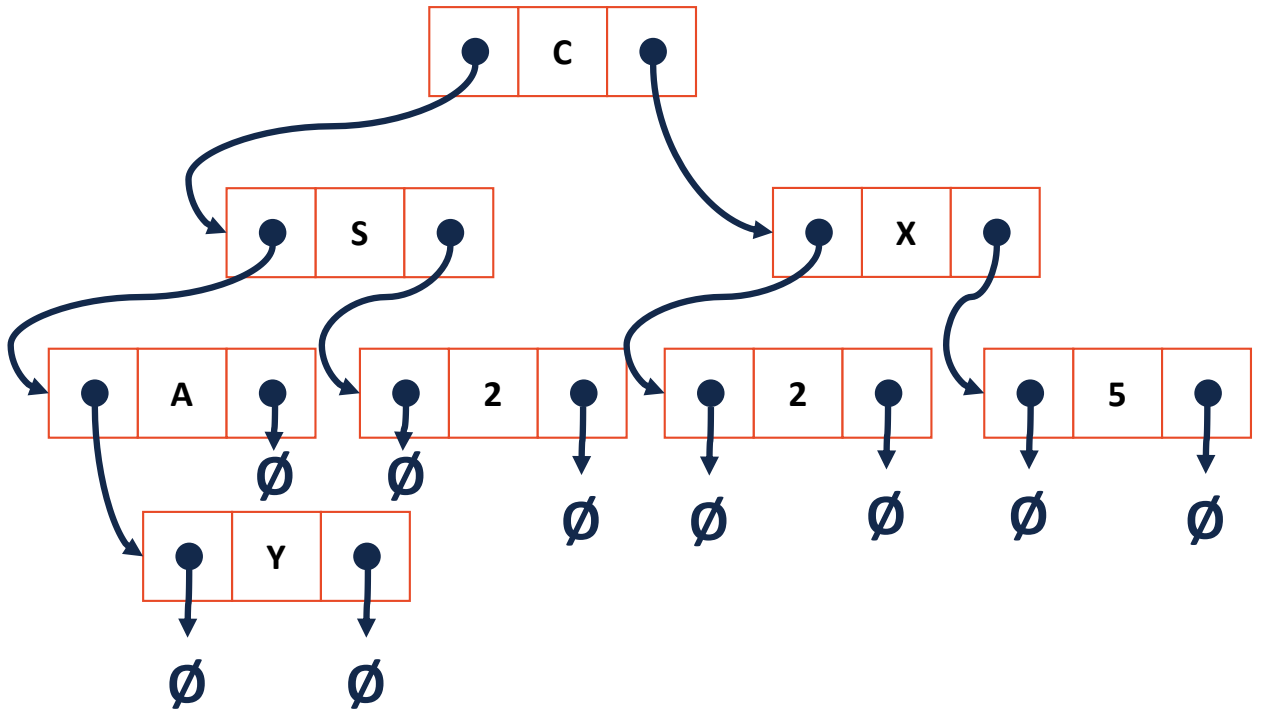
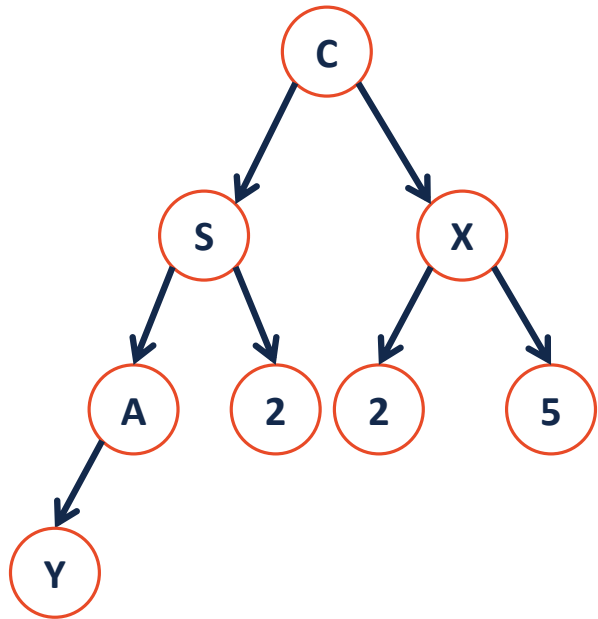
BinaryTree.h

```
1 #pragma once
2
3 template <class T>
4 class BinaryTree {
5     public:
6         /* ... */
7
8     private:
9
10
11
12
13
14
15
16
17
18
19 };
```

Trees aren't new:



Trees aren't new:





How many NULLs?

Theorem: If there are n data items in our representation of a binary tree, then there are _____ NULL pointers.



How many NULLs?

Base Cases:

$n = 0$:

$n = 1$:

$n = 2$:



How many NULLs?

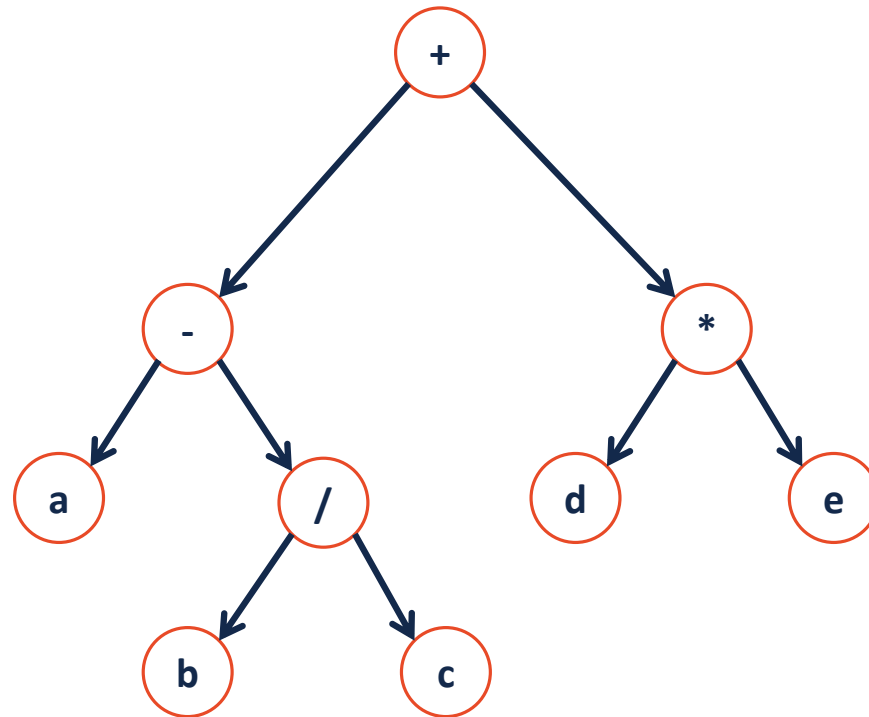
Induction Hypothesis:



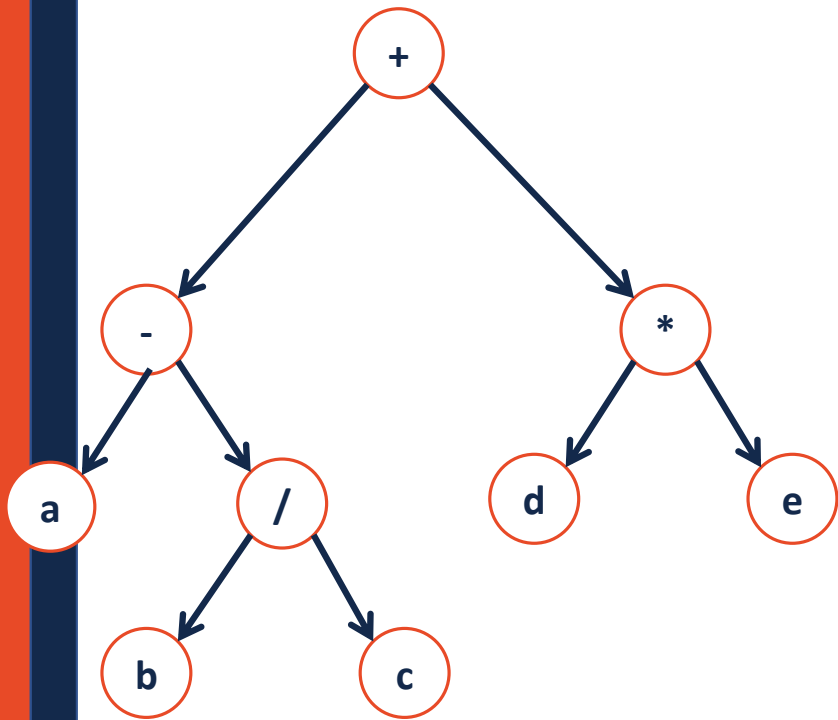
How many NULLs?

Consider an arbitrary tree **T** containing **n** data elements:

Traversals

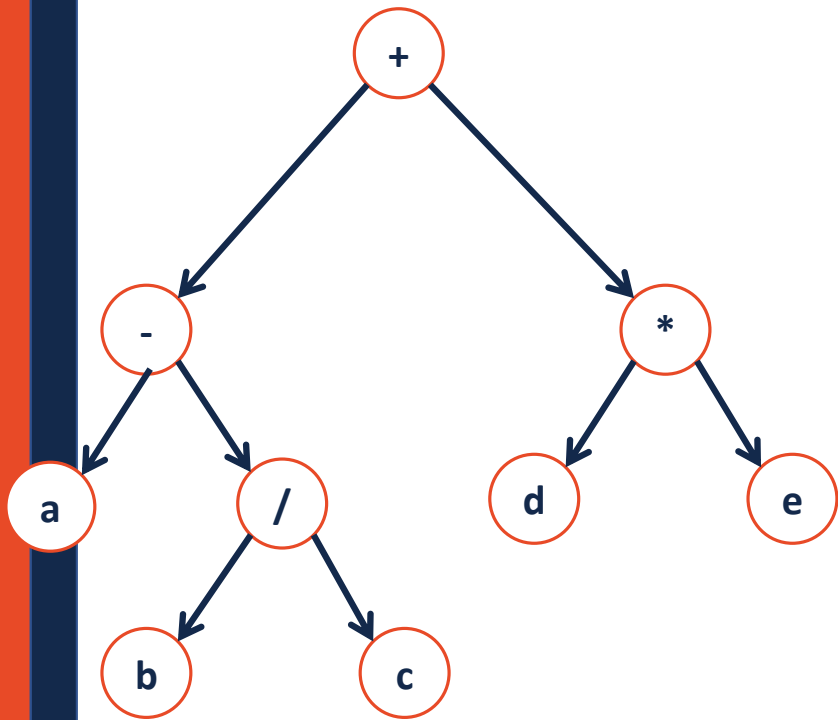


Traversals



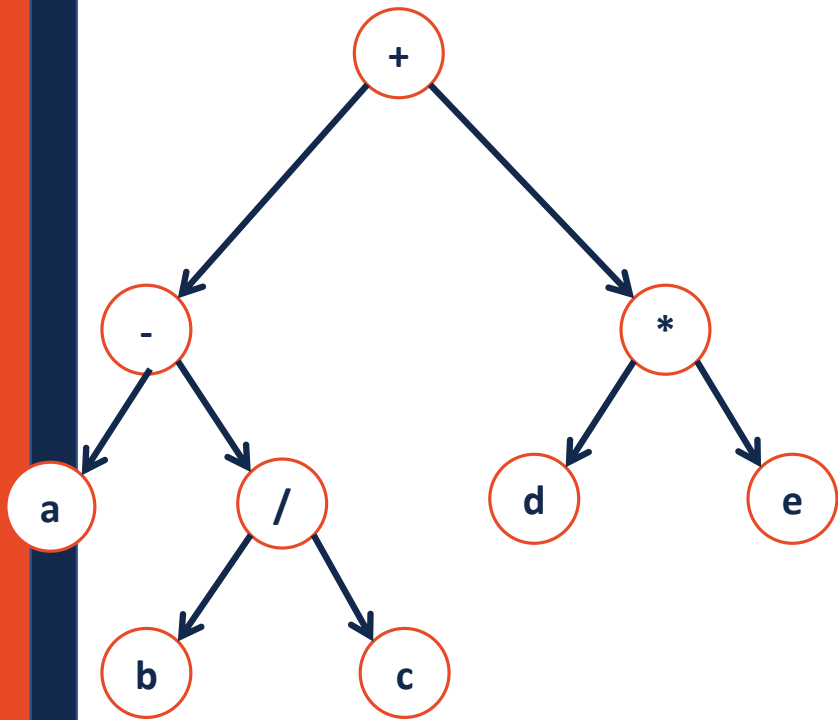
```
1  template<class T>
2  void BinaryTree<T>::__Order(TreeNode * root)
3  {
4      if (root != NULL) {
5          _____;
6          _____Order(root->left);
7          _____;
8          _____Order(root->right);
9          _____;
10         _____;
11         _____;
12         _____Order(root->right);
13         _____;
14         _____;
15         _____;
16     }
17 }
```

Traversals



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Traversals



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15         _____;
16     }
17 }
```