CS 225

#37: MSTs: Kruskal + Prim's Algorithm
November 15, 2021 · *G Carl Evans*

Kruskal's Algorithm

```
Pseudocode for Kruskal's MST Algorithm
1  KruskalMST(G):
2    DisjointSets forest
3    foreach (Vertex v : G):
4      forest.makeSet(v)
5
6    PriorityQueue Q    // min edge weight
7    foreach (Edge e : G):
8      Q.insert(e)
9
10   Graph T = (V, {})
11
12   while |T.edges()| < n-1:
13     Vertex (u, v) = Q.removeMin()
14     if forest.find(u) != forest.find(v):
15        T.addEdge(u, v)
16        forest.union( forest.find(u),
17                      forest.find(v) )
18
19   return T
```

Kruskal's Running Time Analysis
We have multiple choices on which underlying data structure to use to build the Priority Queue used in Kruskal's Algorithm:

Priority Queue Implementations:	Heap	Sorted Array
Building :6-8		
Each removeMin :13		

Based on our algorithm choice:

Priority Queue Implementation:	Total Running Time
Heap	
Sorted Array	

Reflections
Why would we prefer a Heap?

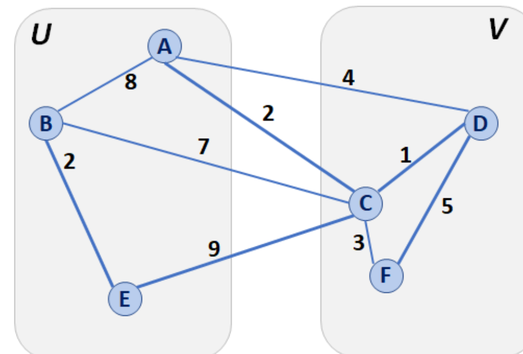
Why would be prefer a Sorted Array?

Partition Property
Consider an arbitrary partition of the vertices on **G** into two subsets **U** and **V**.

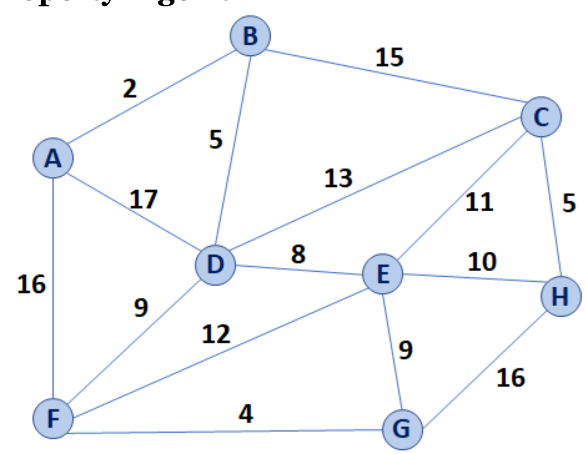
Let **e** be an edge of minimum weight across the partition.

Then **e** is part of some minimum spanning tree.

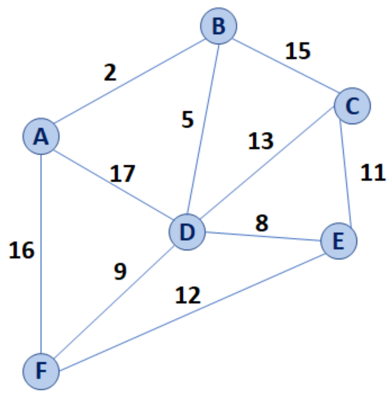
Proof in CS 374!



Partition Property Algorithm



Prim's Minimum Spanning Tree Algorithm



Pseudocode for Prim's MST Algorithm

```
PrimMST(G, s):
  Input: G, Graph;
         s, vertex in G, starting vertex of algorithm
  Output: T, a minimum spanning tree (MST) of G

  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q   // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T           // "labeled set"

  repeat n times:
    Vertex m = Q.removeMin()
    T.add(m)
    foreach (Vertex v : neighbors of m not in T):
      if cost(v, m) < d[v]:
        d[v] = cost(v, m)
        p[v] = m

  return T
```

	Adj. Matrix	**Adj. List**
Heap		
Unsorted Array		

Running Time of MST Algorithms

Kruskal's Algorithm:

Prim's Algorithm:

Q: What must be true about the connectivity of a graph when running an MST algorithm?

...what does this imply about the relationship between **n** and **m**?

Kruskal's MST	**Prim's MST**

Q: Suppose we built a new heap that optimized the decrease-key operation, where decreasing the value of a key in a heap updates the heap in amortized constant time, or O(1)*. How does that change Prim's Algorithm runtime?

Final big-O Running Times of classical MST algorithms:

Kruskal's MST	**Prim's MST**

CS 225 – Things To Be Doing:

1. **mp_mazes due today!**
2. If your final project has not been approved get it revised.
3. Daily POTDs are ongoing for +1 point /problem but pausing over break