#35: DFS and Minimum Spanning Trees (MST)

November 12, 2021 · *G Carl Evans*

Graph Traversal – BFS

Big Ideas: Utility of a BFS Traversal

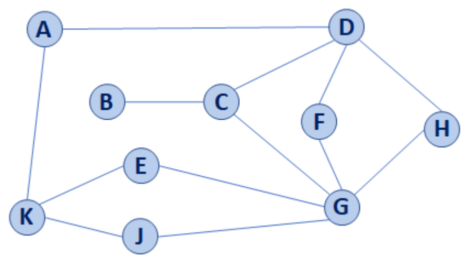
Obs. 1: Traversals can be used to count components.
Obs. 2: Traversals can be used to detect cycles.
Obs. 3: In BFS, **d** provides the shortest distance to every vertex.
Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, d, by more than 1: $|\mathbf{d(u)} - \mathbf{d(v)}| = 1$

Modifying BFS to create DFS

```
BFS(G):
  Input: Graph, G
  Output: A labeling of the edges on
      G as discovery and cross edges

  foreach (Vertex v : G.vertices()):
    setLabel(v, UNEXPLORED)
  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)
  foreach (Vertex v : G.vertices()):
    if getLabel(v) == UNEXPLORED:
       BFS(G, v)

BFS(G, v):
  Queue q
  setLabel(v, VISITED)
  q.enqueue(v)

  while !q.empty():
    v = q.dequeue()
    foreach (Vertex w : G.adjacent(v)):
      if getLabel(w) == UNEXPLORED:
         setLabel(v, w, DISCOVERY)
         setLabel(w, VISITED)
         q.enqueue(w)
      elseif getLabel(v, w) == UNEXPLORED:
         setLabel(v, w, CROSS)
```

DFS Graph Traversal

Idea: Traverse deep into the graph quickly, visiting more distant nodes before neighbors.

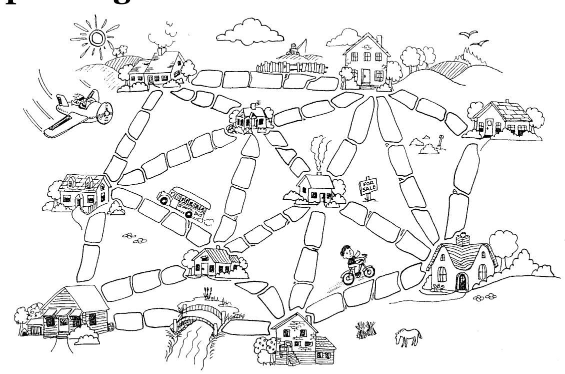


Two types of edges:

-

-

Minimum Spanning Tree



A **Spanning Tree** on a connected graph **G** is a subgraph, **G'**, such that:

1. Every vertex is G is in G' and
2. G' is connected with the minimum number of edges

This construction will always create a new graph that is a __________ (connected, acyclic graph) that spans G.

A **Minimum Spanning Tree** is a spanning tree with the **minimal total edge weights** among all spanning trees.

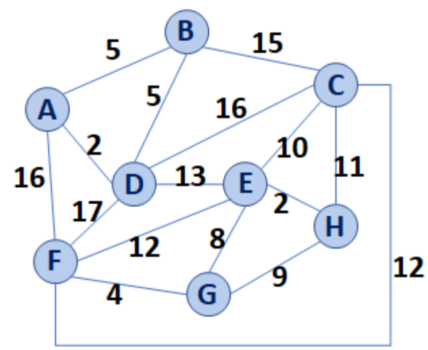
- Every edge must have a weight
  - The weights are unconstrained, except they must be additive *(eg: can be negative, can be non-integers)*
- Output of a MST algorithm produces G':
  - G' is a spanning graph of G
  - G' is a tree

G' has a minimal total weight among all spanning trees. There may be multiple minimum spanning trees, but they will have the same total weight.

Pseudocode for Kruskal's MST Algorithm

```
KruskalMST(G):
  DisjointSets forest
  foreach (Vertex v : G):
    forest.makeSet(v)

  PriorityQueue Q    // min edge weight
  foreach (Edge e : G):
    Q.insert(e)

  Graph T = (V, {})

  while |T.edges()| < n-1:
    Vertex (u, v) = Q.removeMin()
    if forest.find(u) == forest.find(v):
       T.addEdge(u, v)
       forest.union( forest.find(u),
                     forest.find(v) )

  return T
```

Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

CS 225 – Things To Be Doing:
1. lab_dict due Sunday **2.** mp_mazes due Monday **3.** Daily POTDs are ongoing for +1 point /problem