



CS 225

Data Structures

Oct. 21 – BTree Analysis

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BTree Analysis

The height of the BTree determines maximum number of _____ possible in search data.

...and the height of the structure is: _____.

Therefore: The number of seeks is no more than _____.

...suppose we want to prove this!



BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given n) is the same as finding a lower bound on the nodes (given h).

We want to find a relationship for BTrees between the number of keys (n) and the height (h).



BTree Analysis

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.



BTree Analysis

The minimum number of **nodes** for a BTree of order m **at each level:**

root:

level 1:

level 2:

level 3:

...

level h :



BTree Analysis

The **total number of nodes** is the sum of all of the levels:



BTree Analysis

The **total number of keys:**



BTree Analysis

The **smallest total number of keys** is:

So an inequality about **n**, the total number of keys:

Solving for **h**, since **h** is the number of seek operations:



BTree Analysis

Given $m=101$, a tree of height $h=4$ has:

Minimum Keys:

Maximum Keys:



Hashing



Hashing

Goals:

We want to define a **keyspace**, a (mathematical) description of the keys for a set of data.

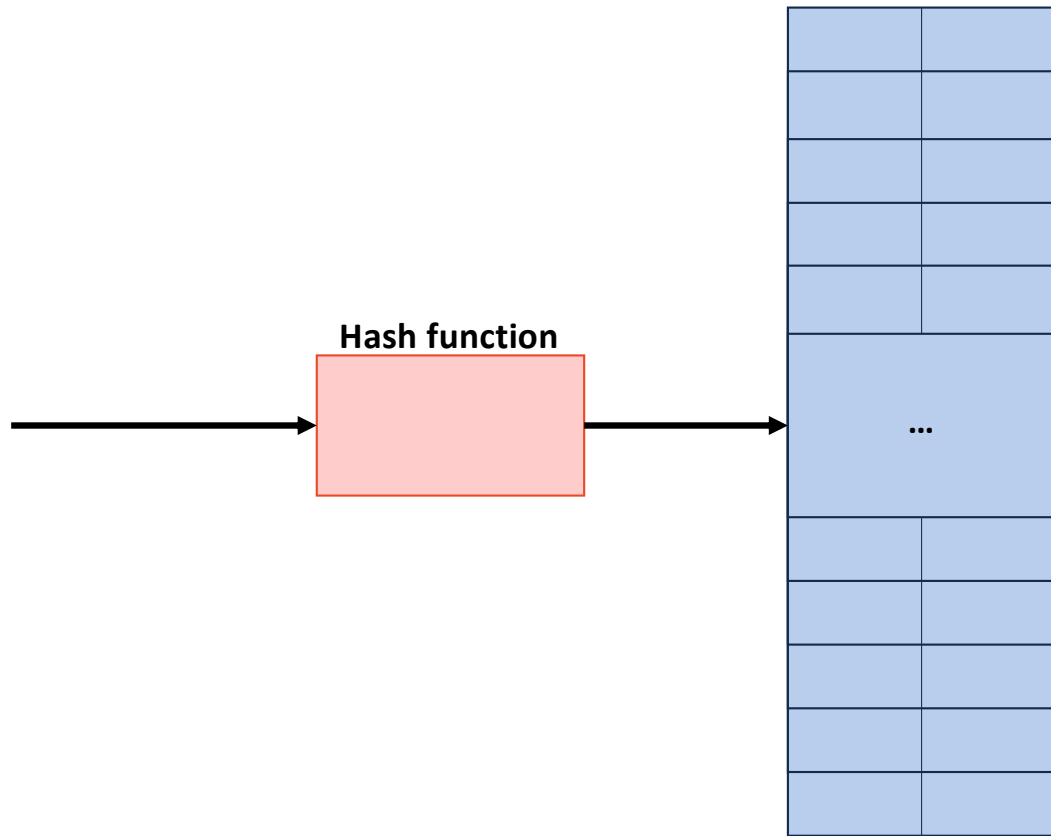
...use a function to map the **keyspace** into a small set of integers.



Hashing

Locker Number	Name
103	
92	
330	
46	
124	

Hashing



A Hash Table based Dictionary

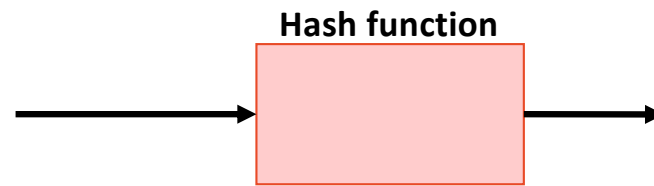
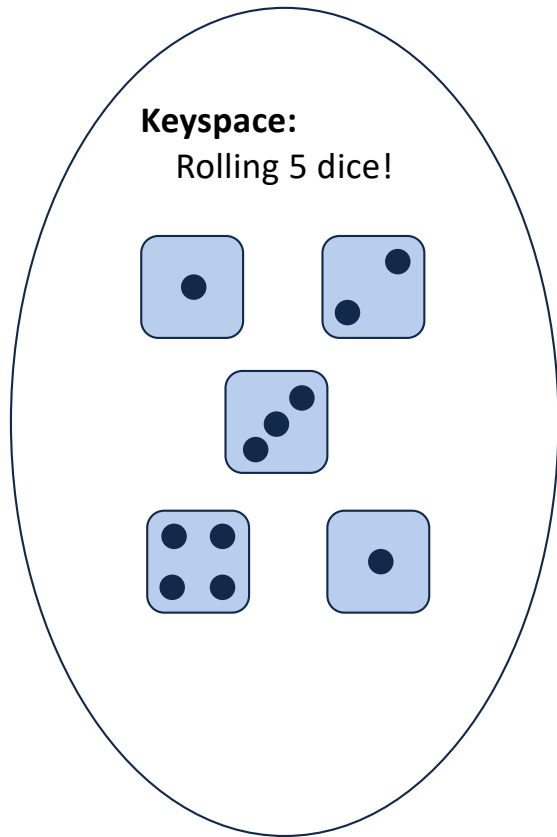
Client Code:

```
1 Dictionary<KeyType, ValueType> d;  
2 d[k] = v;
```

A **Hash Table** consists of three things:

- 1.
- 2.
- 3.

A Perfect Hash Function



Key	Value
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	