

CS 225

Data Structures

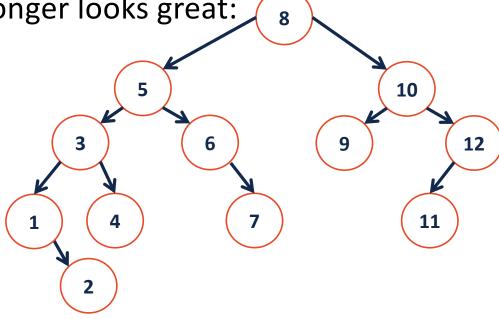
October 18 – BTrees
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B-Tree Motivation

In Big-O we have assumed uniform time for all operations, but this isn't always true.

However, seeking data from the cloud may take 40ms+.

...an O(lg(n)) AVL tree no longer looks great:



Real Application

Imagine storing Instagram profiles for everyone in the US:

How many records?

How much data in total?

How deep is the AVL tree?

BTree Motivations

Knowing that we have large seek times for data, we want to:

BTree (of order m)



```
Goal: Minimize the number of reads!

Build a tree that uses

[1 network packet]

[1 disk block]
```

BTree Insertion

A **BTrees** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** nodes.

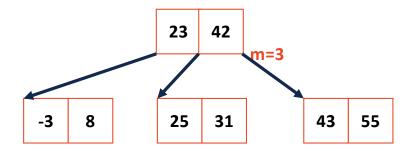


BTree Insertion

When a BTree node reaches **m** keys:

_____ m=!

BTree Recursive Insert



BTree Recursive Insert

23 42 m=3

-3 8

25 31

43 55

BTree Visualization/Tool

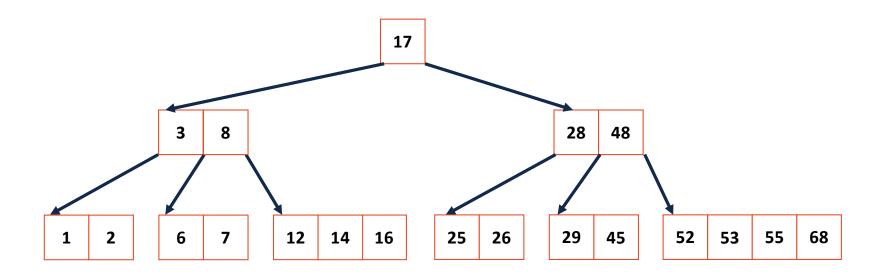
https://www.cs.usfca.edu/~galles/visualization/BTree.html

Btree Properties

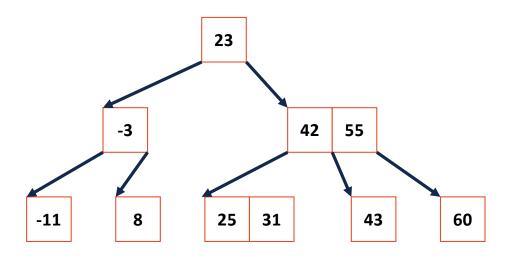
A **BTrees** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** nodes.
- All internal nodes have exactly one more key than children
- Root nodes can be a leaf or have [2, m] children.
- All non-root, internal nodes have [ceil(m/2), m] children.
- All leaves are on the same level

BTree



BTree Search



BTree Search

```
bool Btree:: exists(BTreeNode & node, const K & key) {
 2
 3
    unsigned i;
    for ( i = 0; i < node.keys ct && key < node.keys [i]; i++) { }</pre>
     if ( i < node.keys ct && key == node.keys [i] ) {</pre>
       return true;
 8
 9
10
     if ( node.isLeaf() ) {
       return false;
11
12
     } else {
13
       BTreeNode nextChild = node. fetchChild(i);
                                                                 23
       return exists(nextChild, key);
14
15
16
                                                       -3
                                                                          42
                                                                              55
                                               -11
                                                         8
                                                                 25
                                                                      31
                                                                                43
                                                                                          60
```

The height of the BTree determines maximum numl	oer	of
possible in search data.		

...and the height of the structure is: _____.

Therefore: The number of seeks is no more than _____.

...suppose we want to prove this!

In our AVL Analysis, we saw finding an upper bound on the height (given **n**) is the same as finding a lower bound on the nodes (given **h**).

We want to find a relationship for BTrees between the number of keys (n) and the height (h).

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (n).

The minimum number of nodes will tell us the largest possible height (h), allowing us to find an upper-bound on height.

The minimum number of **nodes** for a BTree of order m **at each level**:

root:

level 1:

level 2:

level 3:

...

level h:

The **total number of nodes** is the sum of all of the levels:

The total number of keys:

The smallest total number of keys is:

So an inequality about **n**, the total number of keys:

Solving for **h**, since **h** is the number of seek operations:

Given m=101, a tree of height h=4 has:

Minimum Keys:

Maximum Keys: