# CS 225

**Data Structures**

*October 14 – AVL Applications*

*G Carl Evans*

# AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is **2 × lg(n)** or **O( lg(n) )**:

**N(h)** := Minimum # of nodes in an AVL tree of height h

**N(h)** = 1 + **N(h-1)** + **N(h-2)**

$> 1 + 2^{h-1/2 + } 2^{h-2/2}$

$> 2 \times 2^{h-2/2 = } 2^{h-2/2+1 = } 2^{h/2}$

**Theorem #1:**

**Every AVL tree of height h has at least $2^{h/2}$ nodes.**

# AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is **$2 \times \lg(n)$** or **$O(\lg(n))$**:

$\# \text{ of nodes } (n) \geq N(h) > 2^{h/2}$

$n > 2^{h/2}$

$\lg(n) > h/2$

$2 \times \lg(n) > h$

$h < 2 \times \lg(n)$ , *for $h \geq 1$*

**Proved: The maximum number of nodes in an AVL tree of height h is less than $2 \times \lg(n)$.**

# Summary of Balanced BST

**AVL Trees**

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- Rotations:

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**AVL Trees**

- Max height: 1.44 * lg(n)
- Rotations:

  Zero rotations on find
  One rotation on insert
  O(**h**) == O(**lg(n)**) rotations on remove

**Red-Black Trees**

- Max height: 2 * lg(n)
- Constant number of rotations on insert (max 2), remove (max 3).

# Why AVL?

# Summary of Balanced BST

**Pros:**

- Running Time:

    - Improvement Over:

- Great for specific applications:

# Summary of Balanced BST

**Cons:**

- Running Time:

- In-memory Requirement:

# Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```
std::map<K, V> map;
```

# Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

# Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

```
std::map<K, V>::erase( const K & )
```

# Red-Black Trees in C++

```
iterator std::map<K, V>::lower_bound( const K & );
iterator std::map<K, V>::upper_bound( const K & );
```

# CS 225 -- Course Update

Over the next two days, your grades will be uploaded into Compass for our first “grade update” where we will calculate your current course grade for you.

We will discuss the grades for the course as a whole (ex: average, etc) in lecture on Wednesday.

# Iterators

## Why do we care?

```
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
  std::cout << (*it) << std::endl;
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```
ImageTraversal & traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

# Every Data Structure So Far

| | Unsorted Array | Sorted Array | Unsorted List | Sorted List | Binary Tree | BST | AVL |
|---|---|---|---|---|---|---|---|
| **Find** | | | | | | | |
| **Insert** | | | | | | | |
| **Remove** | | | | | | | |
| **Traverse** | | | | | | | |

# Range-based Searches

**Q:** Consider points in 1D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

…what points fall in [11, 42]?

**Tree construction:**

# Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

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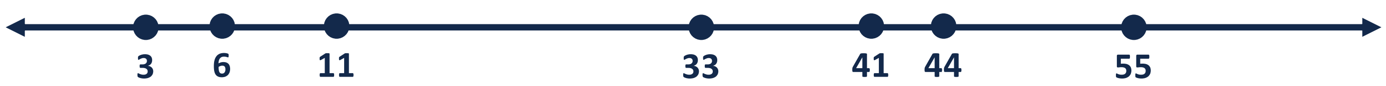
**Ex:**



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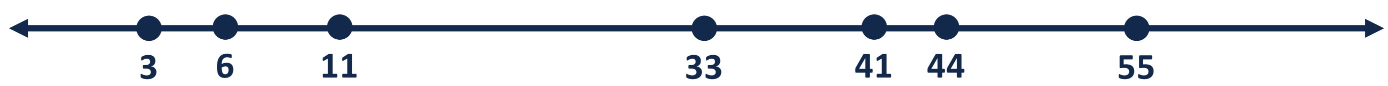

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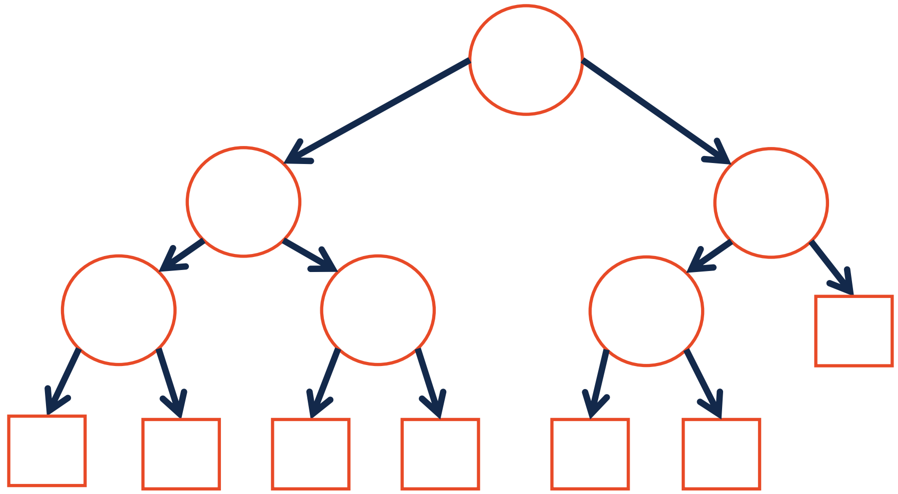

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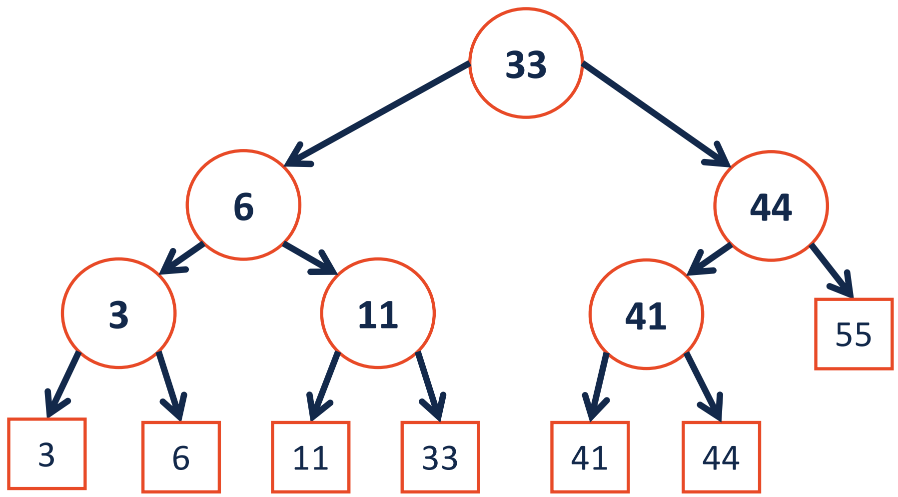

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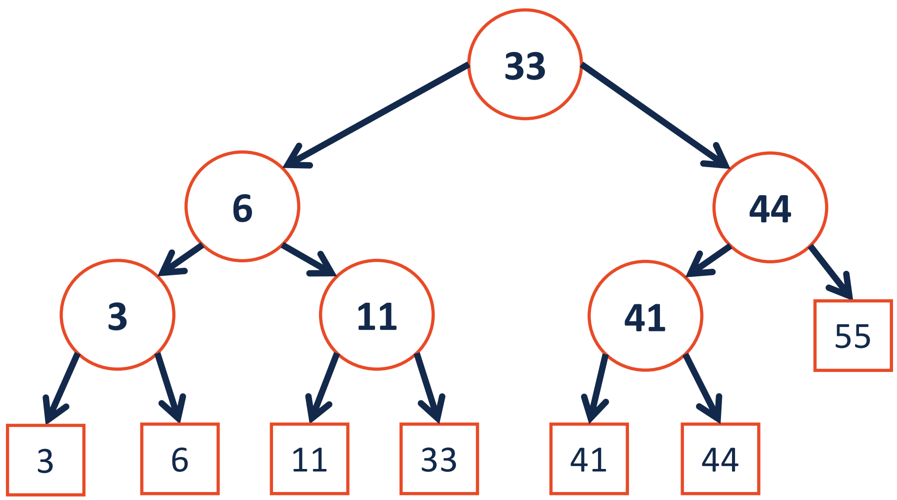

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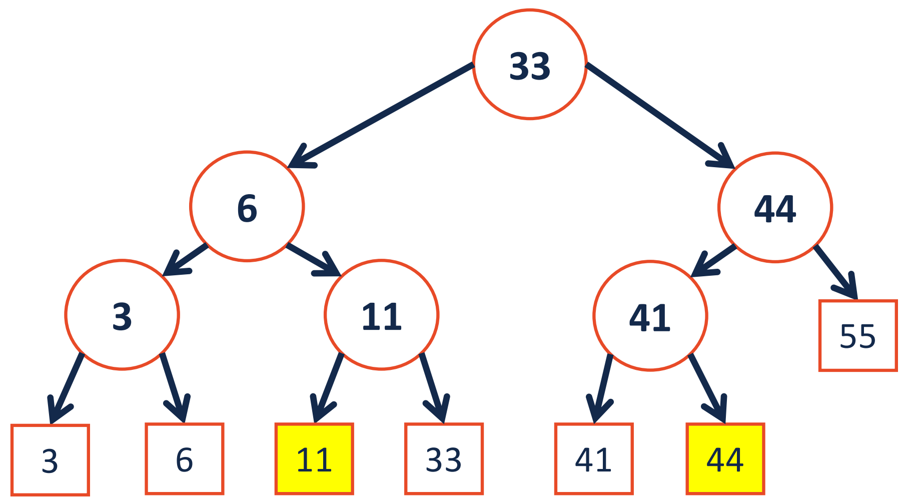
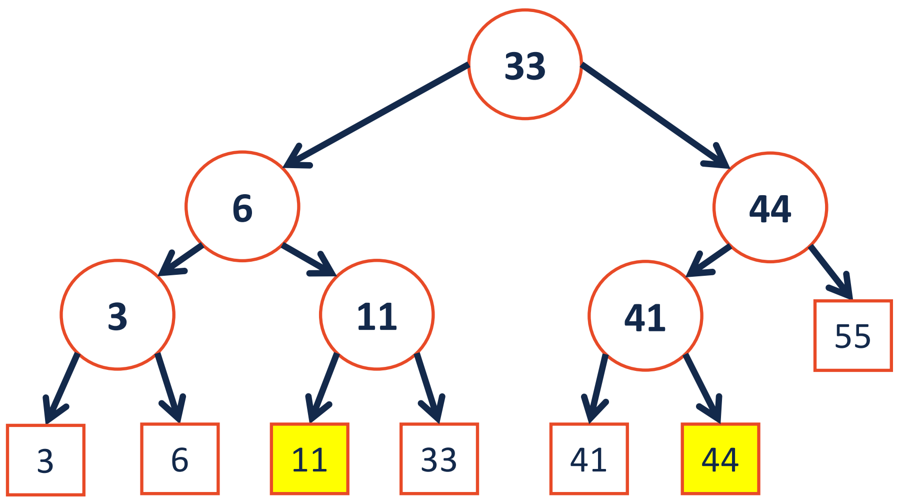


# Running Time



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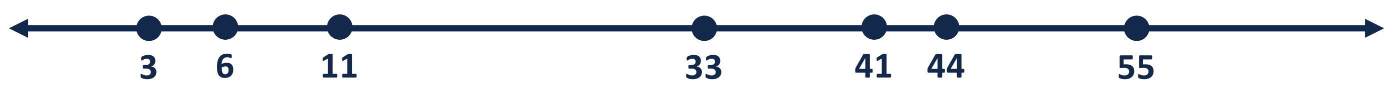
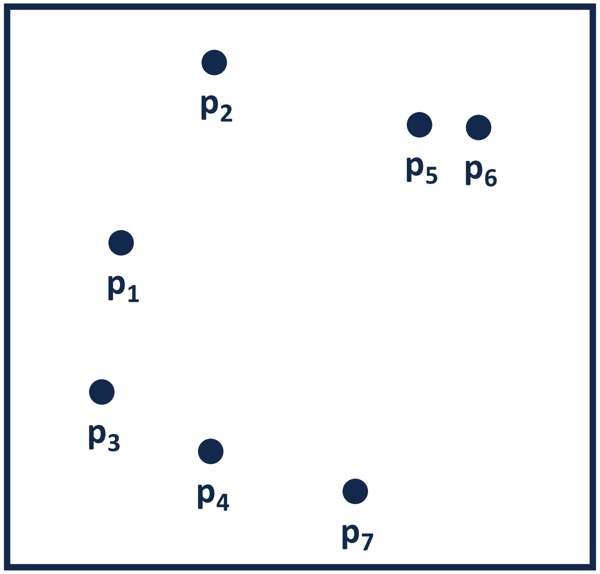


# Range-based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

**Q:** What points are in the rectangle:
$\mathbf{[\ (x_1, y_1),\ (x_2, y_2)\ ]}$?

**Q:** What is the nearest point to $\mathbf{(x_1, y_1)}$?



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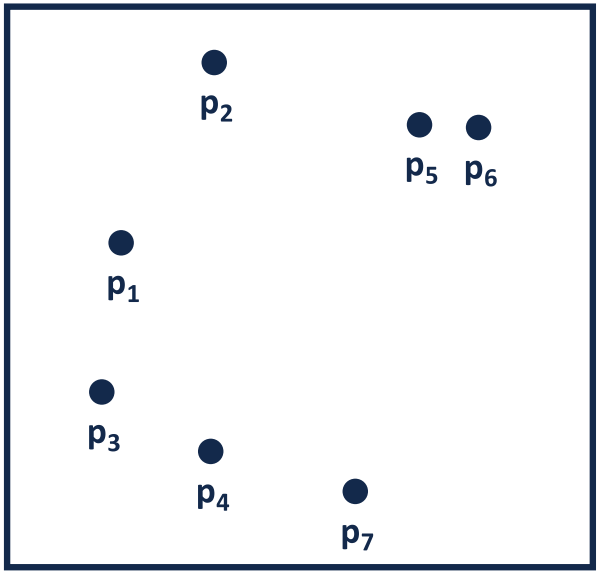
**Tree construction:**



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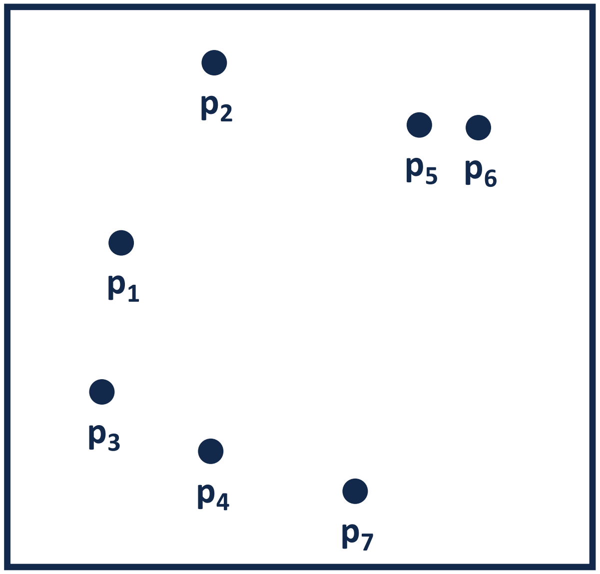
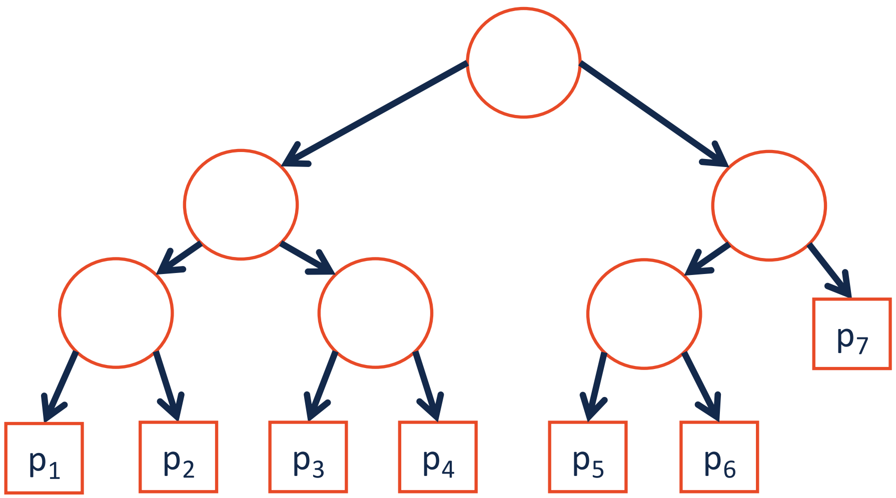
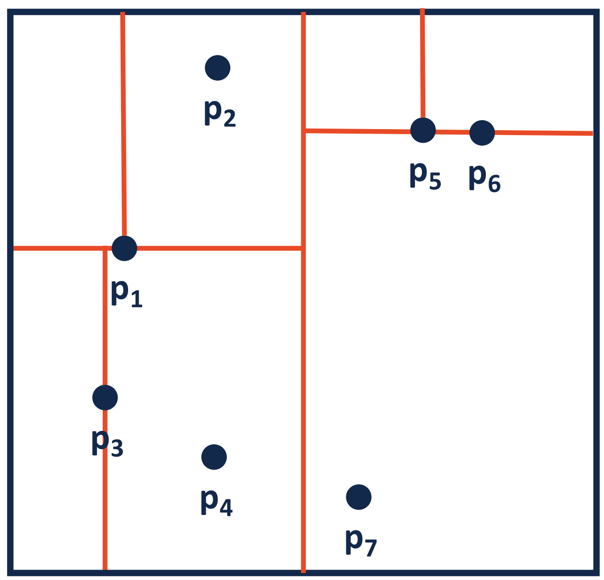
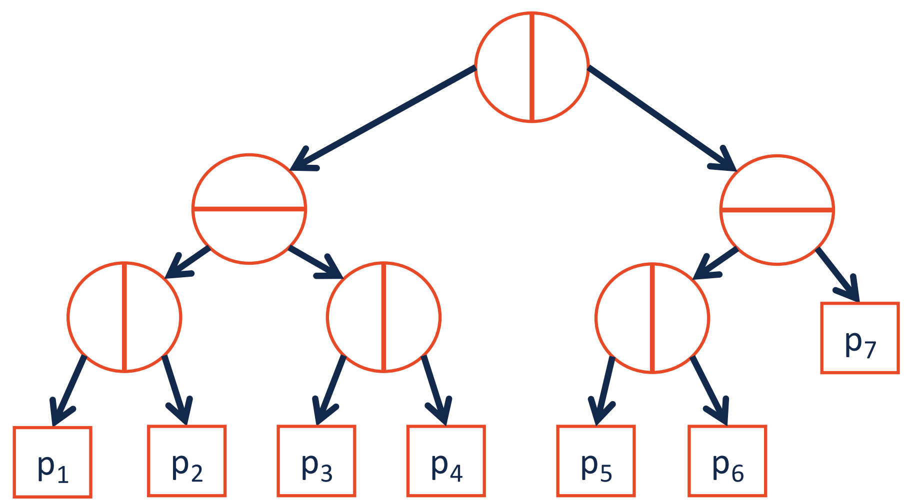


# kD-Trees



# kD-Trees