

#41: Floyd-Warshall's Algorithm

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From Friday:

- Graphs with a negative-weight **cycle** have no finite shortest path. (*We can always take the cycle one more time to get an even shorter path!*)
- Graphs with a negative-weight **edge without a negativeweight cycle** DO have a finite shortest path!

Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alternative to Dijkstra in the presence of negative-weight edges (but <u>not</u> negative weight cycles).

Algorithm Design:

- **Goal:** Find the shortest path from vertex **u** to **v**.
- **Setup:** Create an n×n matrix that maintains the best known path between every pair of vertices:
 - \circ Initialize (u, u) to 0.
 - Initialize all edges present on the graph to their edge weight.

A

B

3

• Initialize all other edges to +infinity.

	Α	В	С	D
Α				
В				
С				
D				

- For every vertex **k**, consider which of the following are shorter:
 - \circ path(u, v) or -
 - \circ path(u, k) + path(k, v)

Big Idea:

- Store intermediate results to improve build towards an optimal solution.
- Example application of memorization and **dynamic programming (DP)** more in CS 374!

Running Time:

Pseudocode for Floyd-Warshall's Algorithm				
1	FloydWarshall(G):			
2	Input: G, Graph;			
3	Output: d, an adjacency matrix of distances between			
4	All vertex pairs			
5				
6	Let d be an adj. matrix (2d array) initialized to +inf			
7	foreach (Vertex v : G):			
8	$\mathbf{d}[\mathbf{v}][\mathbf{v}] = 0$			
9	foreach (Edge $(u, v) : G$):			
10	d[u][v] = cost(u, v)			
11				
12	foreach (Vertex k : G):			
13	foreach (Vertex u : G):			
14	foreach (Vertex v : G):			
15	if $d[u, v] > d[u, k] + d[k, v]$:			
16	d[u, v] = d[u, k] + d[k, v]			
17				
18	return d			

Overview of Graphs:

Implementations

- Edge List
- Adjacency Matrix
- Adjacency List

Traversals

- Breadth First
- Depth First

Minimum Spanning Tree (MST)

- Kruskal's Algorithm
- Prim's Algorithm

Shortest Path

- Dijkstra's Algorithm *(Single Source)*
- Floyd-Warshall's Algorithm *(All Pairs)* Maximum Flow
 - Ford-Fulkerson (DFS paths) Algorithm
 - Edmonds-Karp (BFS paths) Algorithm

...and this is just the beginning. The journey continues to CS 374!

End of Semester :(

CS 225 Final Exam

- The final exam is a 3 hour CBTF exam, is a cumulative exam, and has the format of theory exam + programming exam
 - Expect ~25-30 theory questions
 - Expect ~2-3 programming problems
- Exam begins on Thursday (December 13th), last day of office hours is December 10th (*no office hours once finals begin*)
- In-lecture review w/ TAs on Wednesday (December 11th)

"Pre-Final" Grade Update

• As soon as possible after the MP7 deadline, I'll provide a "Pre-Final" grade update in Compass 2g with all grades except for your final exam.

End of Semester Grade Review

- Did we miss something that impacts your final grade? I want to be absolutely sure you get the grade you earned!
- After final grades are posted, I will provide a Google form that allows you to submit a **Grade Review** if you believe the grade review will change your final letter grade.
 - You will have the chance to justify why you received an incorrect grade and how it impacts your letter grade in the course.
 - $\circ~$ Instructions on Piazza at the same time as that the final grades are posted.

CS 225 – Things To Be Doing:

- **1.** Lab_flow due Sunday!
- **2.** MP7 due Tuesday
- 3. Review on Wednesday for Final in class.
- 4. Final Exam starts Reading Day