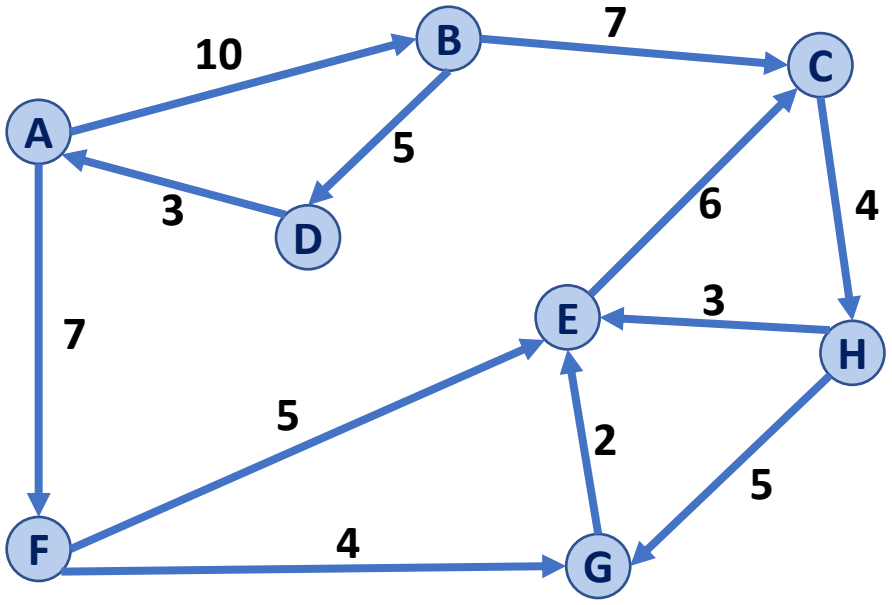
CS 225

Data Structures

Dec. 8 – Dijkstra's + Shortest Path

Wade Fagen-Ulmschneider

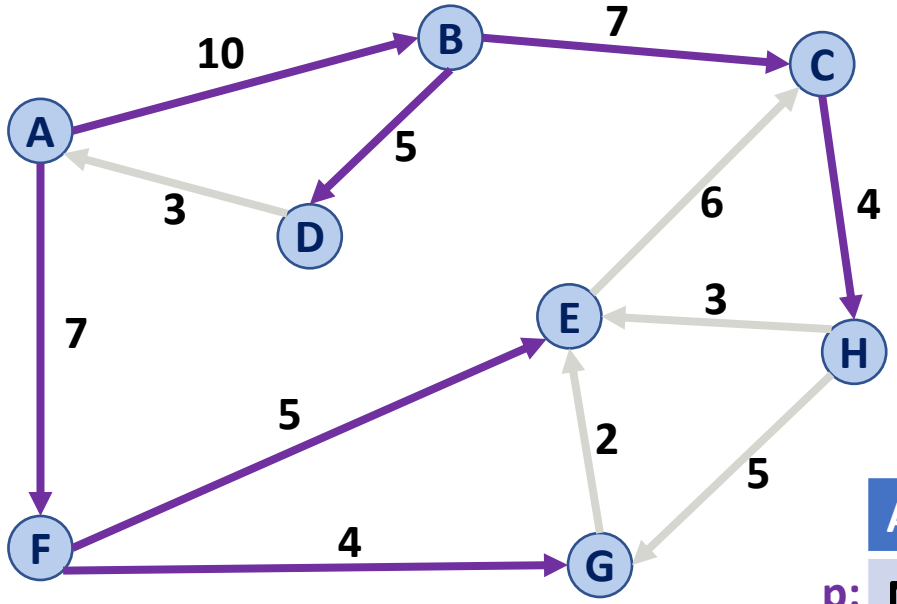
Dijkstra's Algorithm (SSSP)




```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13    Graph T          // "labeled set"
14
15    repeat n times:
16      Vertex u = Q.removeMin()
17      T.add(u)
18      foreach (Vertex v : neighbors of u not in T):
19        if cost(u, v) + d[u] < d[v]:
20          d[v] = cost(u, v) + d[u]
21          p[v] = m
```

Dijkstra's Algorithm (SSSP)

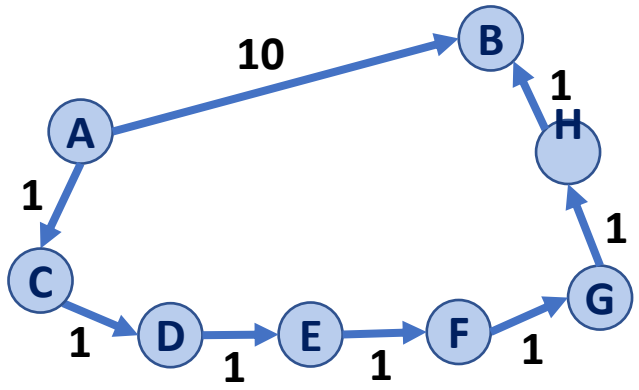
Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	NULL	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

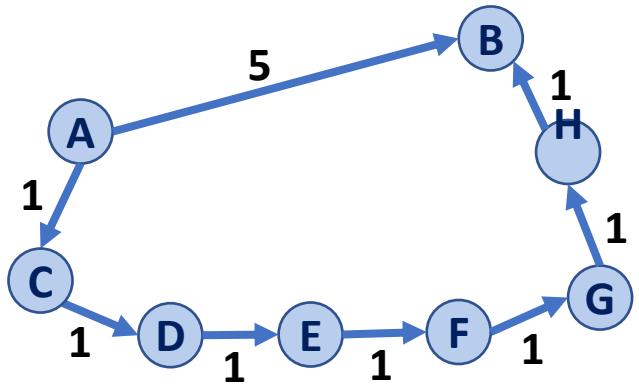
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



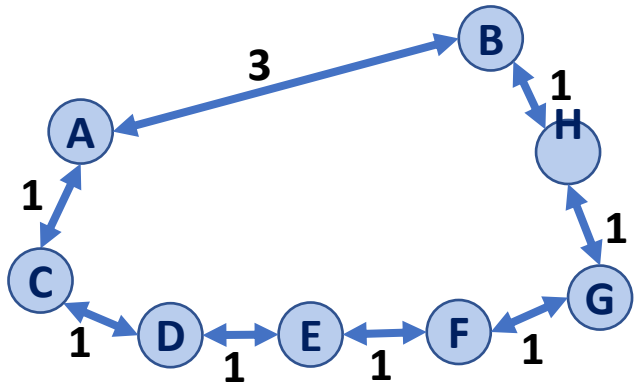
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



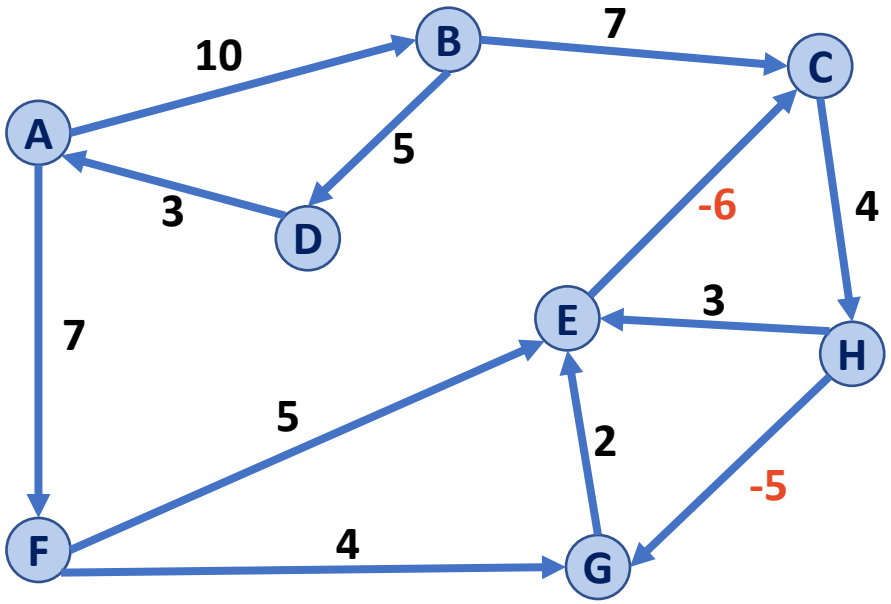
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



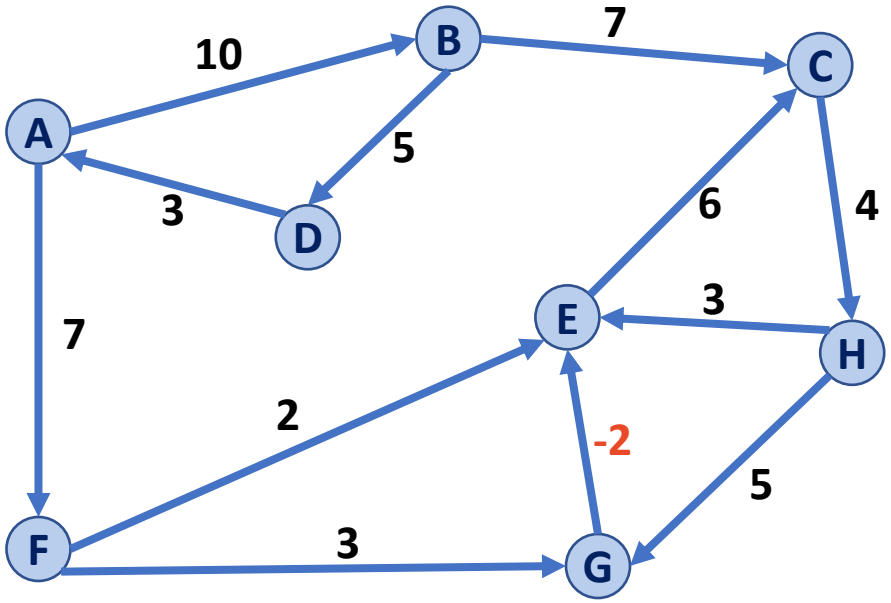
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges, without a negative weight cycle?



Dijkstra's Algorithm (SSSP)

What is Dijkstra's running time?

```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13    Graph T          // "labeled set"
14
15    repeat n times:
16      Vertex u = Q.removeMin()
17      T.add(u)
18      foreach (Vertex v : neighbors of u not in T):
19        if cost(u, v) + d[u] < d[v]:
20          d[v] = cost(u, v) + d[u]
21          p[v] = m
22
23    return T
```

CS 225 – Office Hours

Open Lab Hours:

Approx. ~40 hours of open lab hours left this semester!

Last day of office hours: Wednesday, Dec. 13

Professor Office Hours (DYB):

Mattox: Today from 12:15pm – 1:45pm, 3034 ECEB

Wade: Next Wednesday from 12:15pm – 1:45pm, 3034 ECEB

Open Door Hours *(no CS225 content)*:

Wade: Today from 3:30pm – 5pm+, 2215 Siebel Center

CS 225 – Course Assistants for Spring 2018!

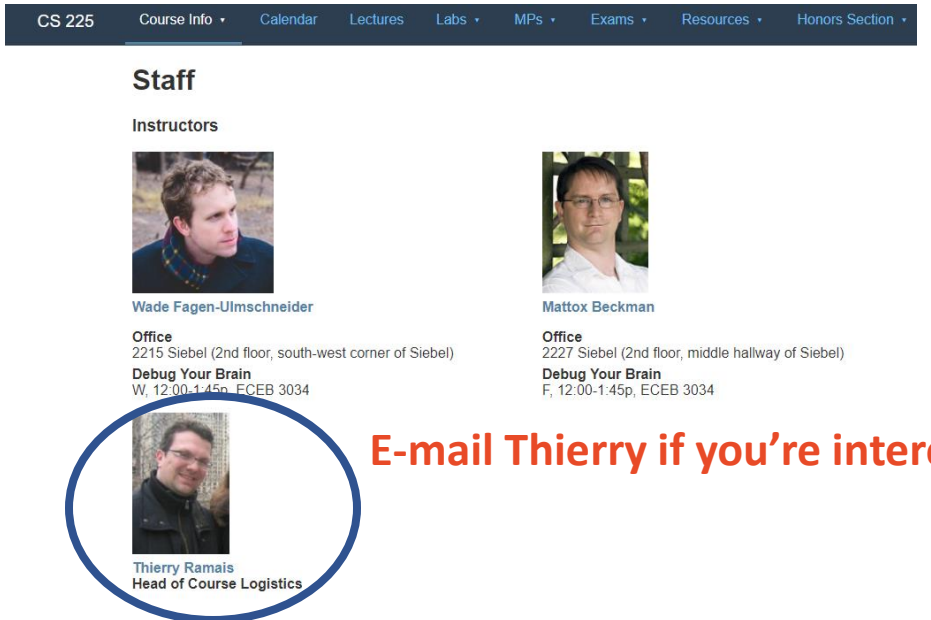
CS 225 will be expanding our CA team!

Help out in lab, office hours, piazza.

Help out on developing CS 225 resources

(including: infrastructure, guidebook, MPs, labs, etc.)

Reach out to us for an application:



E-mail Thierry if you're interested!

Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alterative to Dijkstra in the presence of negative-weight edges (but not negative weight cycles).

```
    FloydWarshall(G):
 6    Let d be a adj. matrix initialized to +inf
 7    foreach (Vertex v : G):
 8      d[v][v] = 0
 9    foreach (Edge (u, v) : G):
10      d[u][v] = cost(u, v)
11
12    foreach (Vertex u : G):
13      foreach (Vertex v : G):
14        foreach (Vertex w : G):
15          if d[u, v] > d[u, w] + d[w, v]:
16            d[u, v] = d[u, w] + d[w, v]
```

Floyd-Warshall Algorithm

```
    FloydWarshall(G):
 6    Let d be a adj. matrix initialized to +inf
 7    foreach (Vertex v : G):
 8      d[v][v] = 0
 9    foreach (Edge (u, v) : G):
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14        foreach (Vertex w : G):
15          if d[u, v] > d[u, w] + d[w, v]:
16            d[u, v] = d[u, w] + d[w, v]
```
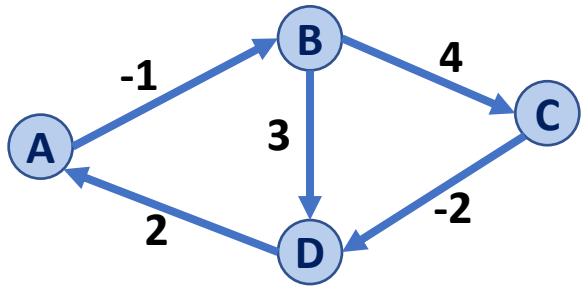

	A	B	C	D
A				
B				
C				
D				



Dijkstra's Algorithm (SSSP)

What is Floyd-Warshall's running time?

```
    FloydWarshall(G):
 6    Let d be a adj. matrix initialized to +inf
 7    foreach (Vertex v : G):
 8      d[v][v] = 0
 9    foreach (Edge (u, v) : G):
10      d[u][v] = cost(u, v)
11
12    foreach (Vertex u : G):
13      foreach (Vertex v : G):
14        foreach (Vertex w : G):
15          if d[u, v] > d[u, w] + d[w, v]:
16            d[u, v] = d[u, w] + d[w, v]
```

MST Algorithm Runtime:

- Dijkstra's Algorithm:
  O(m + n lg(n))

All Pairs Shortest Path:

Dense Graphs:

Sparse Graphs:

- Floyd-Warshall:
  $O(n^3)$

CS 225 – Things To Be Doing

Exam 13: Makeup Exam starts Monday

More Info: https://courses.engr.illinois.edu/cs225/fa2017/exams/

MP7: The final MP!

Due: Monday, Dec. 11 at 11:59pm

Lab: The final lab, lab_ml, is due Sunday

lab_ml: Due Sunday @ 11:59pm

New POTDs every M/W/F

Worth +1 Extra Credit /problem (up to +40 total)