CS 225

Data Structures

Dec. 4 – Prim's Algorithm

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Kruskal’s Algorithm

Priority Queue:	Total Running Time
Heap	$O(n + m) + O(m \lg(n))$
Sorted Array	$O(n + m \lg(n)) + O(m)$

```
KruskalMST(G):
  DisjointSets forest
  foreach (Vertex v : G):
    forest.makeSet(v)

  PriorityQueue Q    // min edge weight
  foreach (Edge e : G):
    Q.insert(e)

  Graph T = (V, {})

  while |T.edges()| < n-1:
    Vertex (u, v) = Q.removeMin()
    if forest.find(u) != forest.find(v):
       T.addEdge(u, v)
       forest.union( forest.find(u),
                     forest.find(v) )

  return T
```

Kruskal’s Algorithm

Which Priority Queue Implementation is better for running Kruskal’s Algorithm?

- Heap:

- Sorted Array:

Mattox Monday

Exam 12

- Programming Exam

Mattox Monday

- **Exam 13**
  - Second Chance!
  - You may pick **one** of the previous 12 exams to retake.
    - *(Let the exam you choose be denoted as **examN**)*
  - exam13score = max(avg(exam1..exam12), retake)
  - examNscore = max(examN, retake)

If you are happy with your exam average, you can stay home. ☺

CS Education Week: Hour of Code

This week is the 111th birthday of Grace Hopper, a pioneer of the field of Computer Science.

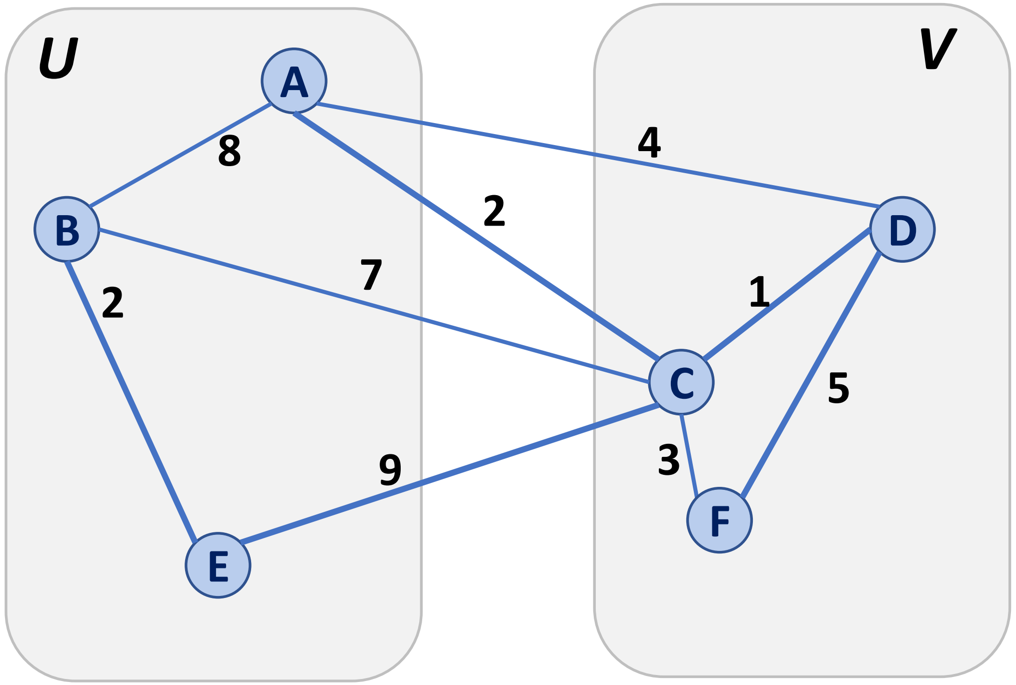
Tonight, CS@Illinois + Women in Computer Science is hosting an **Hour of Code**

Volunteer: Help others program their first line of code!

Tonight, Dec. 4, 6:00pm – 8:00pm
Basement of Siebel

Partition Property

Consider an arbitrary partition of the vertices on **G** into two subsets **U** and **V**.



Partition Property

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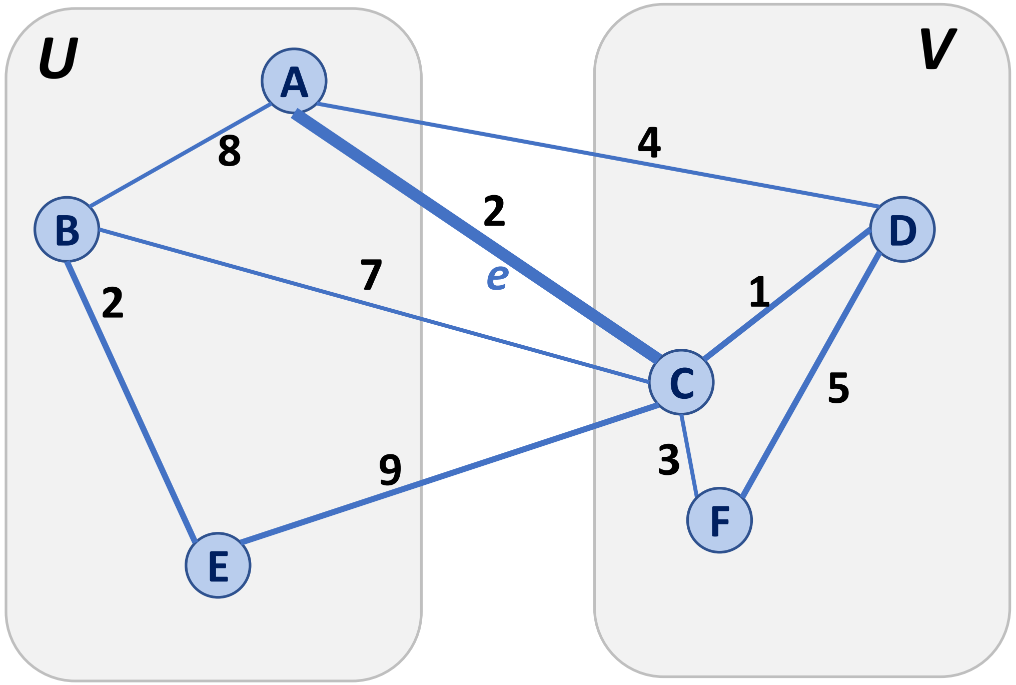
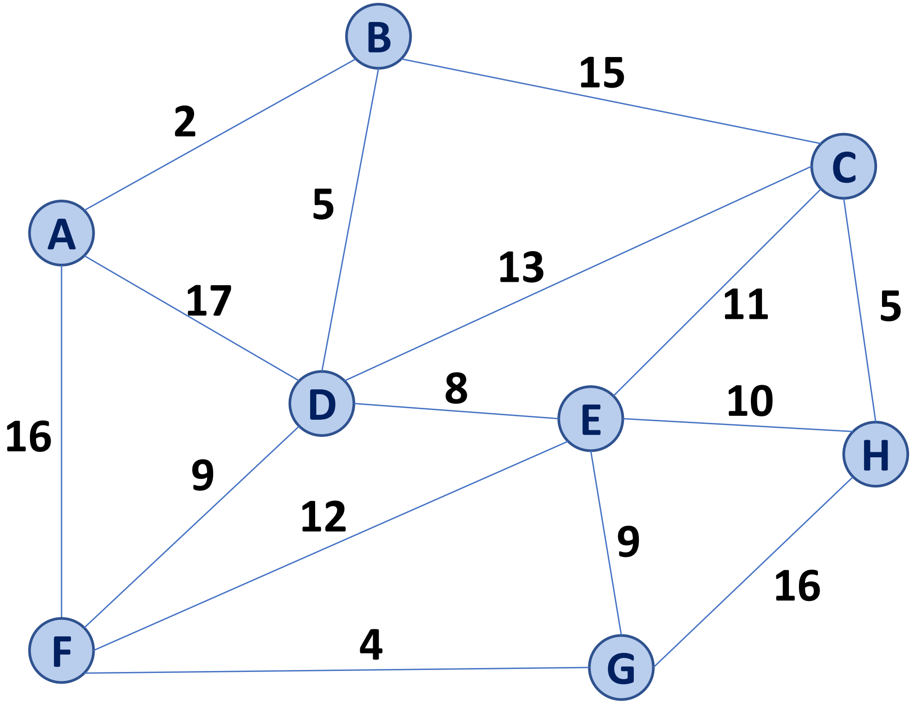


Let **e** be an edge of minimum weight across the partition.

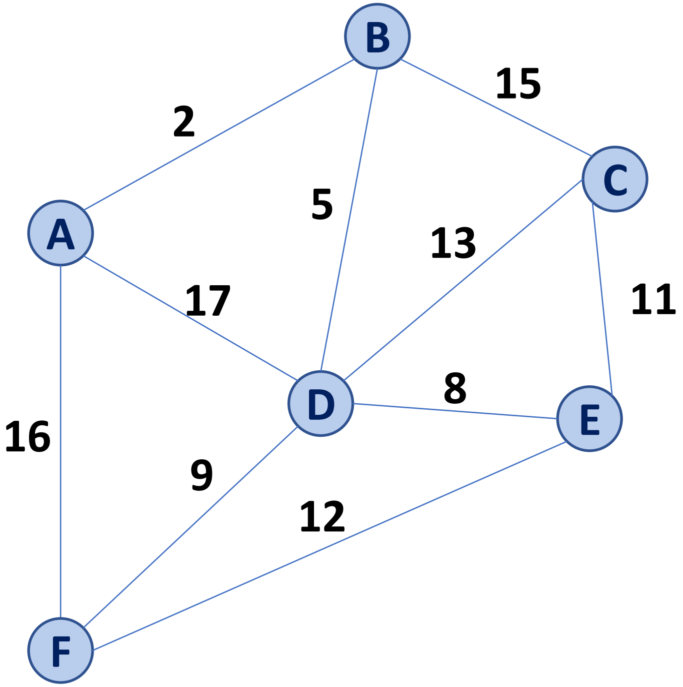
Then **e** is part of some minimum spanning tree.

Partition Property

The partition property suggests an algorithm:



Prim’s Algorithm



```
PrimMST(G, s):
  Input: G, Graph;
         s, vertex in G, starting vertex
  Output: T, a minimum spanning tree (MST) of G

  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q   // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T           // "labeled set"

  repeat n times:
    Vertex m = Q.removeMin()
    T.add(m)
    foreach (Vertex v : neighbors of m not in T):
      if cost(v, m) < d[v]:
        d[v] = cost(v, m)
        p[v] = m

  return T
```

Prim’s Algorithm

```
PrimMST(G, s):
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  PriorityQueue Q // min distance, defined by d[v]
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```

	Adj. Matrix	Adj. List
Heap		
Unsorted Array		

Prim’s Algorithm

Sparse Graph:

Dense Graph:

```
PrimMST(G, s):
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      if cost(v, m) < d[v]:
        d[v] = cost(v, m)
        p[v] = m
```

	Adj. Matrix	Adj. List
Heap	$O(n^2 + m \lg(n))$	$O(n \lg(n) + m \lg(n))$
Unsorted Array	$O(n^2)$	$O(n^2)$

MST Algorithm Runtime:

- Kruskal's Algorithm:

  O(n + m lg(n))

- Prim's Algorithm:

  O(n lg(n) + m lg(n))

- What must be true about the connectivity of a graph when running an MST algorithm?

  - How does n and m relate?

MST Algorithm Runtime:

- Kruskal's Algorithm:

  O(n + m lg(n))

- Prim's Algorithm:

  O(n lg(n) + m lg(n))

CS 225 – Things To Be Doing

Exam 12 (programming) starts Monday, last programming exam before the final!

More Info: https://courses.engr.illinois.edu/cs225/fa2017/exams/

MP7: The final MP!

Extra Credit (+14): Monday, Dec. 4 at 11:59pm
Due: Monday, Dec. 11 at 11:59pm

Lab: lab_graphs due Sunday

lab_graphs: Due Sunday @ 11:59pm

New POTDs every M/W/F

Worth +1 Extra Credit /problem (up to +40 total)