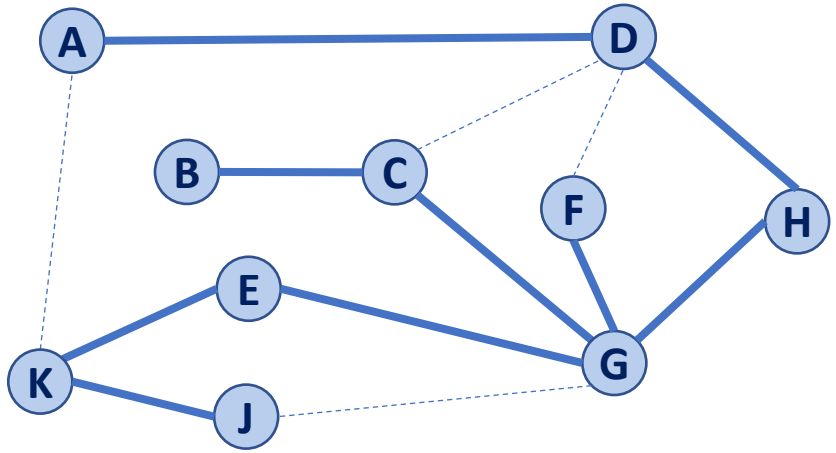
CS 225

Data Structures

Dec. 1 – Kruskal's Algorithm

Wade Fagen-Ulmschneider

Traversal: DFS



Discovery Edge

Back Edge

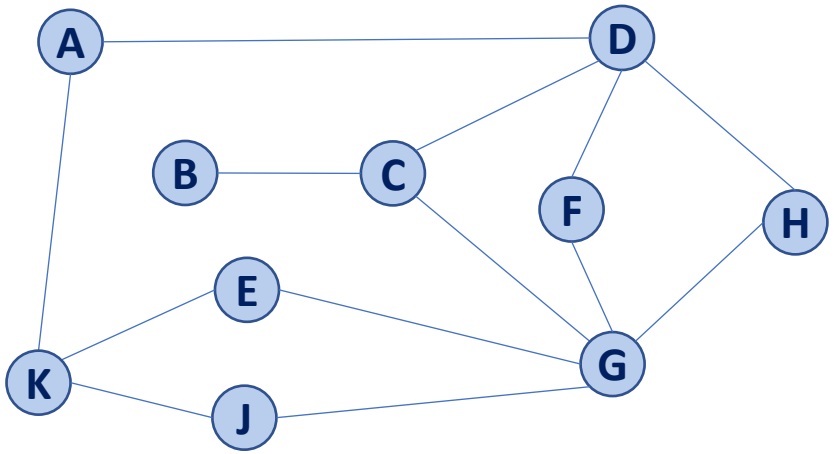
```
 1  BFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and cross edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12        BFS(G, v)
```

```
14  BFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23          setLabel(v, w, DISCOVERY)
24          setLabel(w, VISITED)
25          q.enqueue(w)
26        elseif getLabel(v, w) == UNEXPLORED:
27          setLabel(v, w, CROSS)
```

```
14  BFS(G, v):
15    Stack s
16    setLabel(v, VISITED)
17    s.push(v)
18
19    while !s.empty():
20      v = s.pop()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23           setLabel(v, w, DISCOVERY)
24           setLabel(w, VISITED)
25           s.push(w)
26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, CROSS)
```

```
14  DFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23           setLabel(v, w, DISCOVERY)
24           setLabel(w, VISITED)
25           DFS(G, w)
26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, BACK)
```

Running time of DFS



Labeling:

- Vertex:

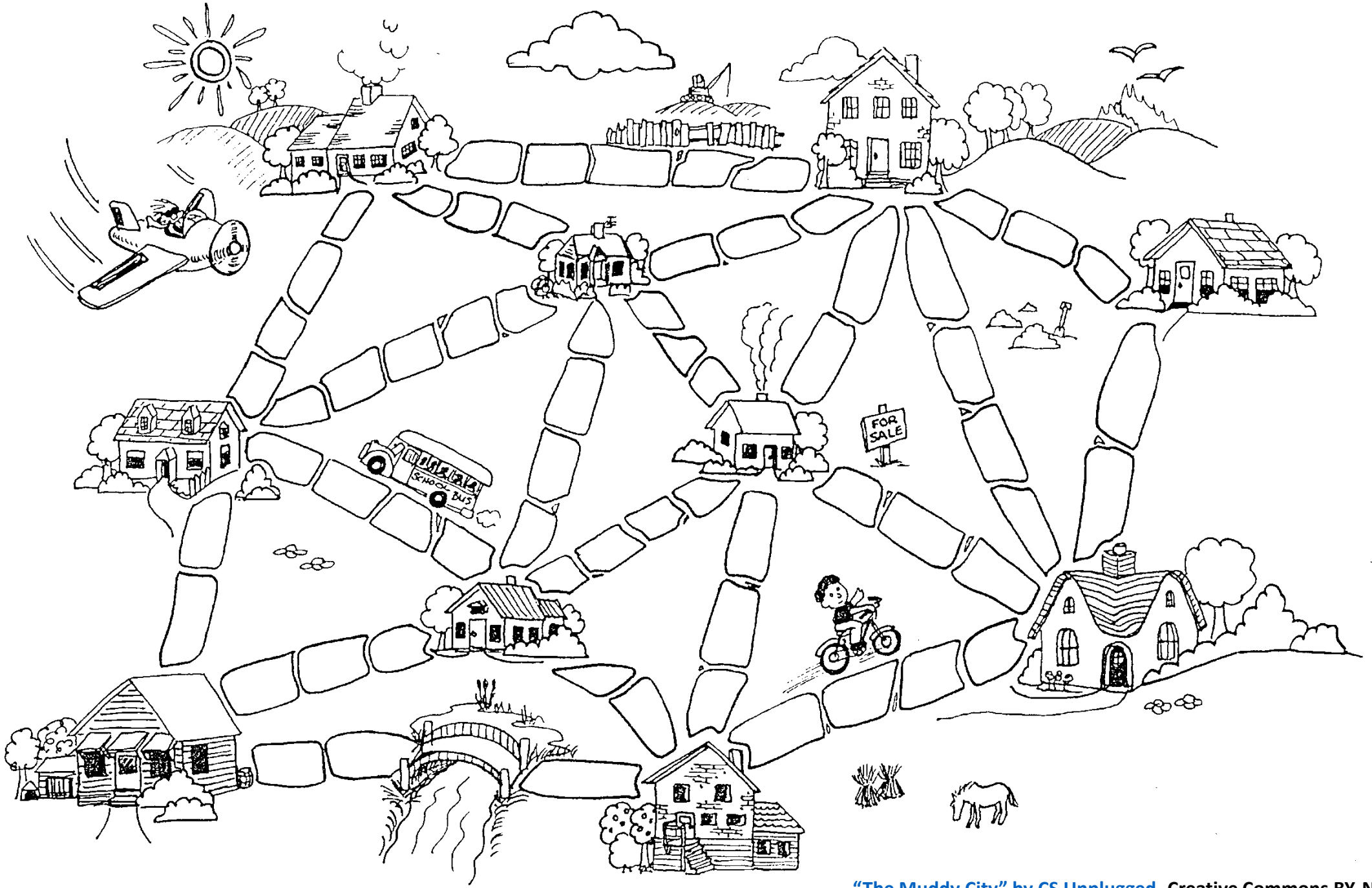
- Edge:

Queries:

- Vertex:

- Edge:

```
14  DFS(G, v):
15    setLabel(v, VISITED)
16    foreach (Vertex w : G.adjacent(v)):
17      if getLabel(w) == UNEXPLORED:
18        setLabel(v, w, DISCOVERY)
19        DFS(G, w)
20      elseif getLabel(v, w) == UNEXPLORED:
21        setLabel(v, w, BACK)
```

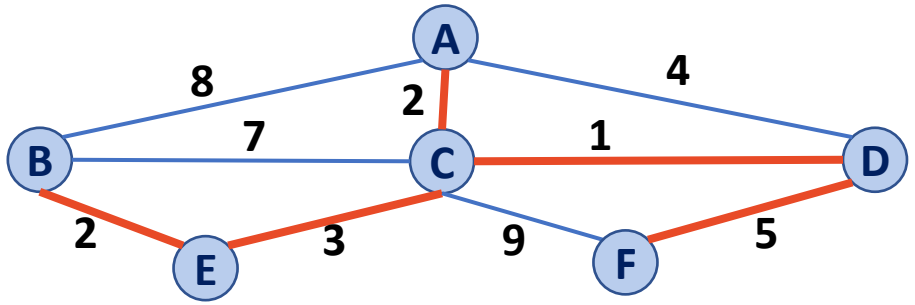


Minimum Spanning Tree Algorithms

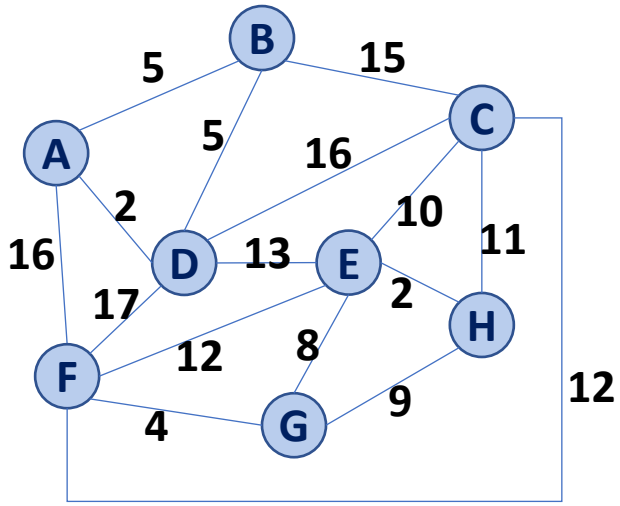
Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

Output: A graph G' with the following properties:

- G' is a spanning graph of G
- G' is a tree (connected, acyclic)
- G' has a minimal total weight among all spanning trees

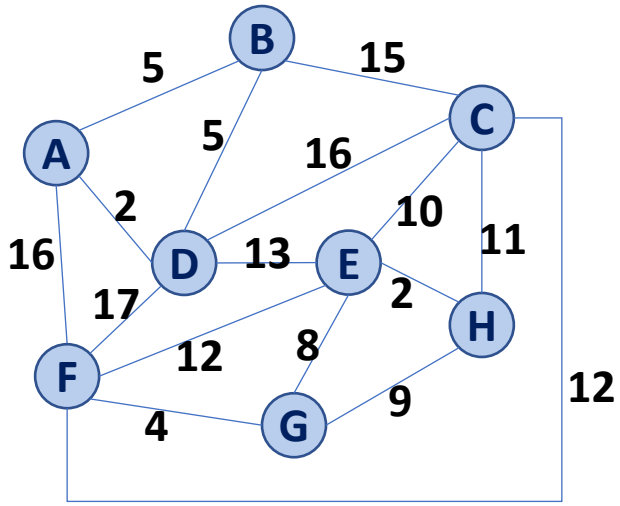


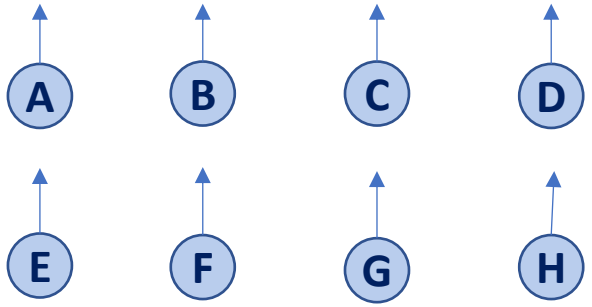
Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

Kruskal's Algorithm

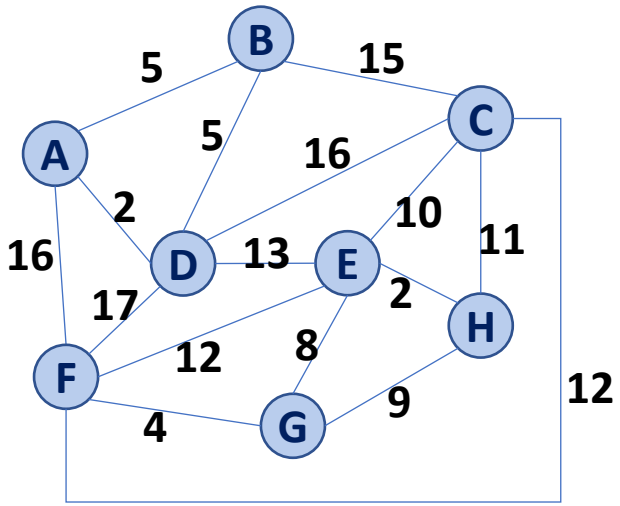


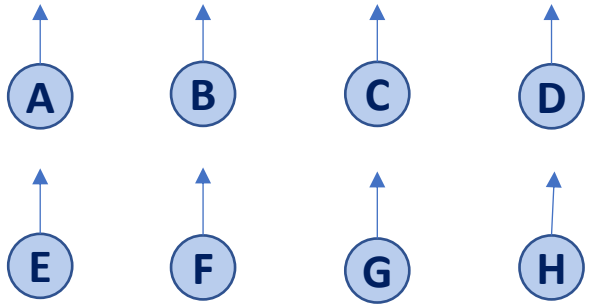


(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

Kruskal's Algorithm

(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)






```
1   KruskalMST(G):
2     DisjointSets forest
3     foreach (Vertex v : G):
4       forest.makeSet(v)
5
6     PriorityQueue Q     // min edge weight
7     foreach (Edge e : G):
8       Q.insert(e)
9
10    Graph T = (V, {})
11
12    while |T.edges()| < n-1:
13      Vertex (u, v) = Q.removeMin()
14      if forest.find(u) == forest.find(v):
15         T.addEdge(u, v)
16         forest.union( forest.find(u),
17                       forest.find(v) )
18
19    return T
```

Kruskal's Algorithm

Priority Queue:	Heap	Sorted Array
Building :7-9		
Each removeMin :13		

```
 1  KruskalMST(G):
 2    DisjointSets forest
 3    foreach (Vertex v : G):
 4      forest.makeSet(v)
 5
 6    PriorityQueue Q     // min edge weight
 7    foreach (Edge e : G):
 8      Q.insert(e)
 9
10    Graph T = (V, {})
11
12    while |T.edges()| < n-1:
13      Vertex (u, v) = Q.removeMin()
14      if forest.find(u) == forest.find(v):
15         T.addEdge(u, v)
16         forest.union( forest.find(u),
17                       forest.find(v) )
18
19    return T
```

Kruskal's Algorithm

Priority Queue:	Total Running Time
Heap	
Sorted Array	

```
1  KruskalMST(G):
2    DisjointSets forest
3    foreach (Vertex v : G):
4      forest.makeSet(v)
5
6    PriorityQueue Q     // min edge weight
7    foreach (Edge e : G):
8      Q.insert(e)
9
10   Graph T = (V, {})
11
12   while |T.edges()| < n-1:
13     Vertex (u, v) = Q.removeMin()
14     if forest.find(u) == forest.find(v):
15        T.addEdge(u, v)
16        forest.union( forest.find(u),
17                      forest.find(v) )
18
19   return T
```

CS 225 – Things To Be Doing

Exam 12 (programming) starts Monday, last programming exam before the final!

More Info: https://courses.engr.illinois.edu/cs225/fa2017/exams/

MP7: The final MP!

Extra Credit (+14): Monday, Dec. 4 at 11:59pm
Due: Monday, Dec. 11 at 11:59pm

Lab: lab_graphs due Sunday

lab_graphs: Due Sunday @ 11:59pm

New POTDs every M/W/F

Worth +1 Extra Credit /problem (up to +40 total)