CS 225

Data Structures

Nov. 15 – Graph Implementations

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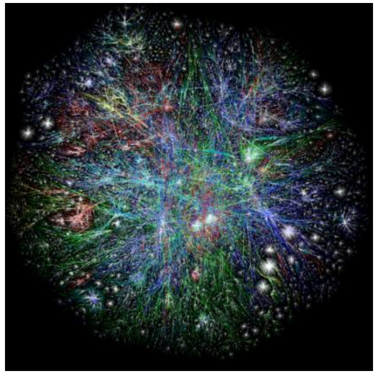
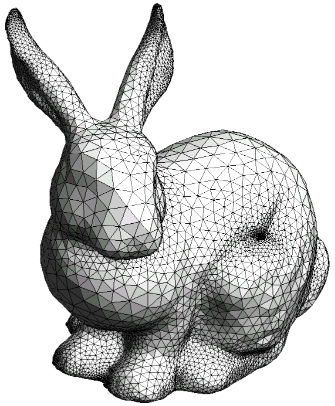
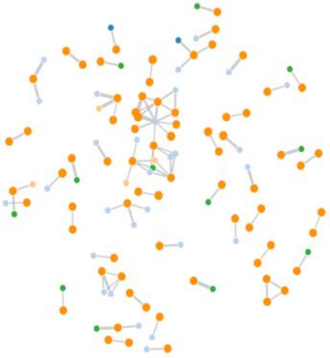
Graphs

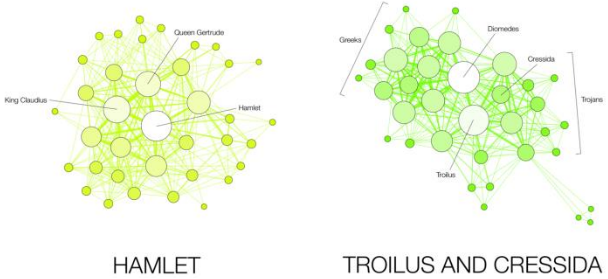


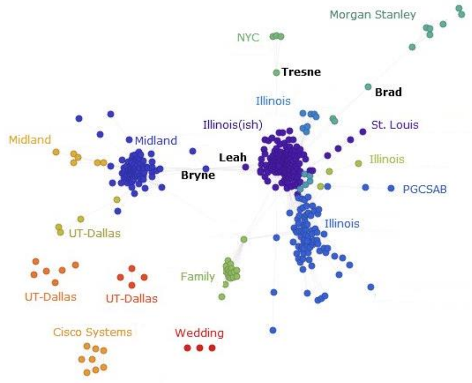


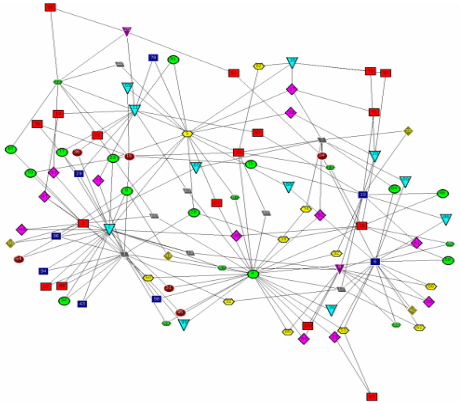
To study all of these structures:

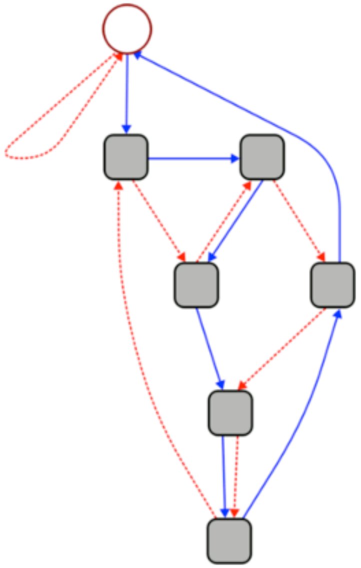
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

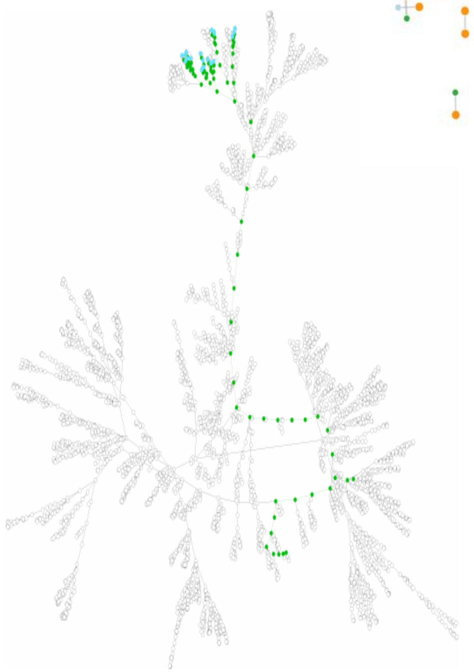






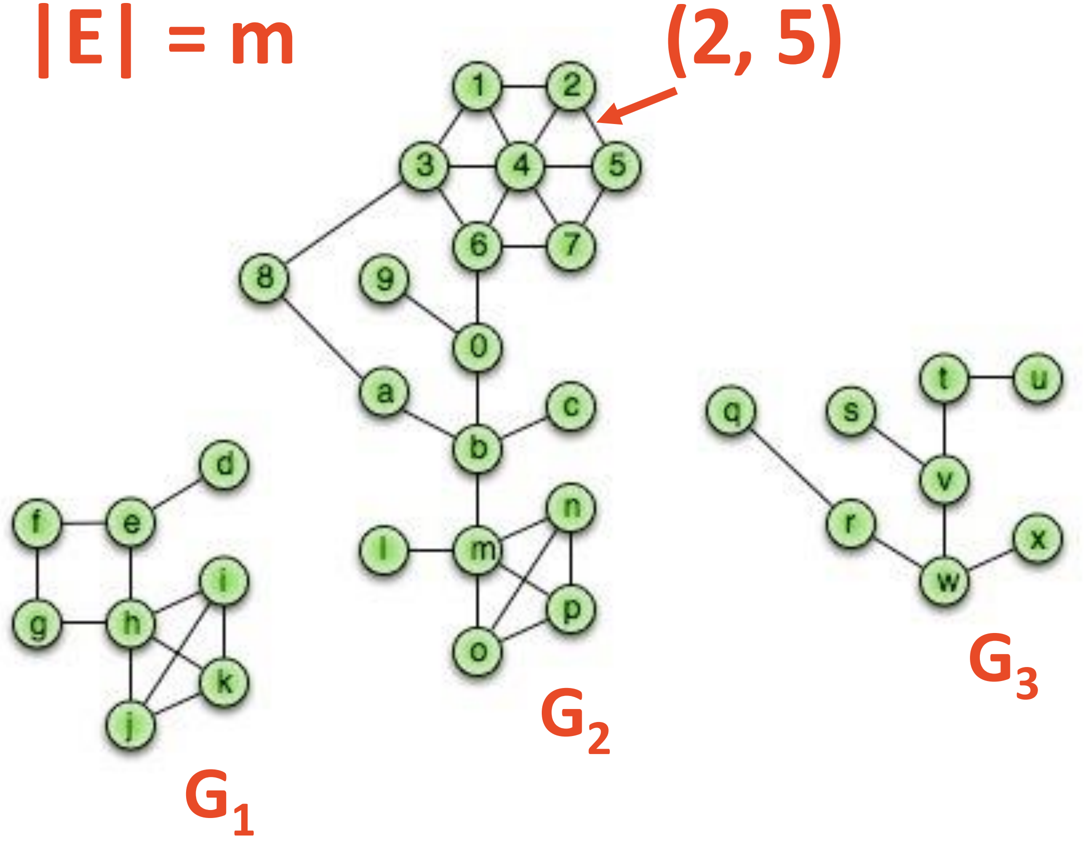






Graph Vocabulary

G = (V, E)
|V| = n
|E| = m



Incident Edges:
I(v) = { (x, v) in E }

Degree(v): |I|

Adjacent Vertices:
A(v) = { x : (x, v) in E }

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

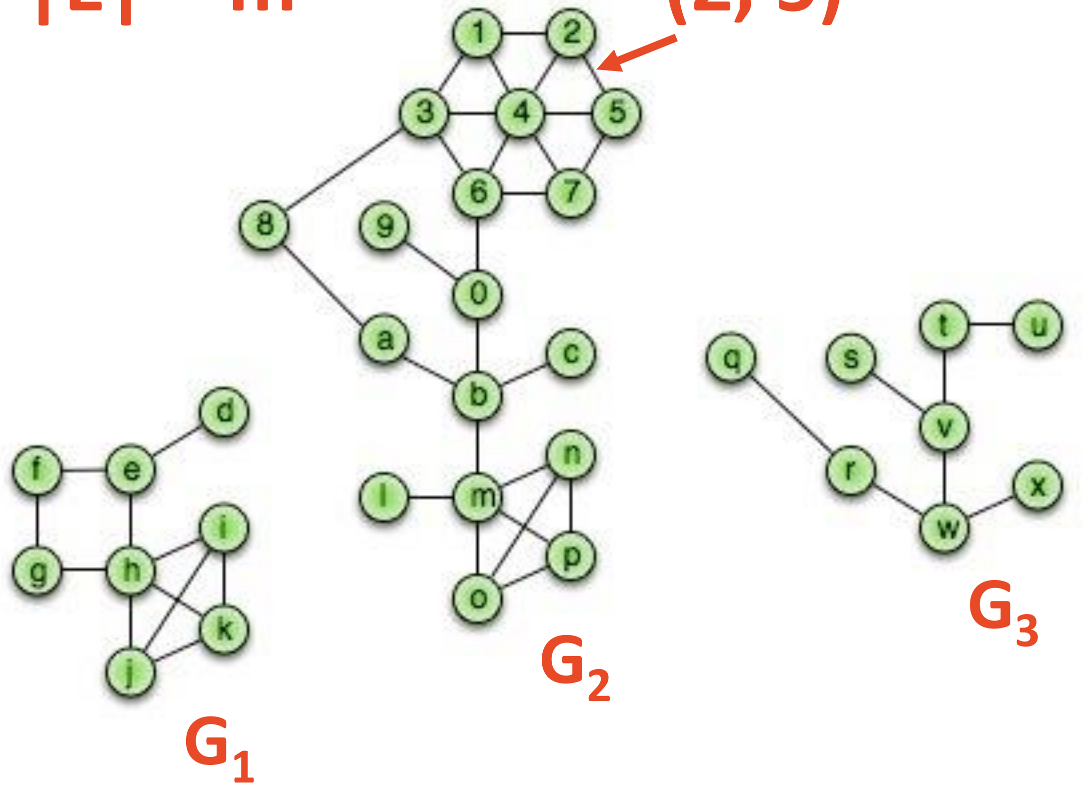
Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

$G = (V, E)$

$|V| = n$

$|E| = m$



Subgraph(G):

$G' = (V', E')$:

$V' \in V$, $E' \in E$, and

$(u, v) \in E \rightarrow u \in V', v \in V'$

Complete subgraph(G)

Connected subgraph(G)

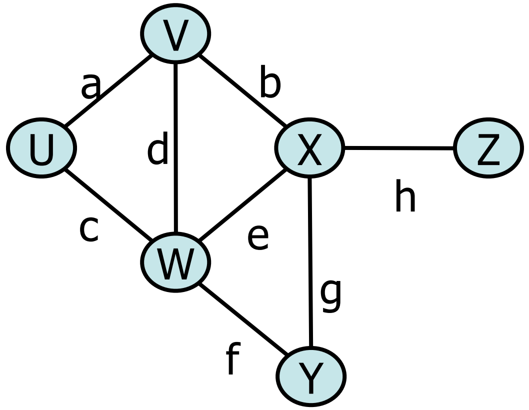
Connected component(G)

Acyclic subgraph(G)

Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges?



Minimum edges:

Not Connected:

Connected:

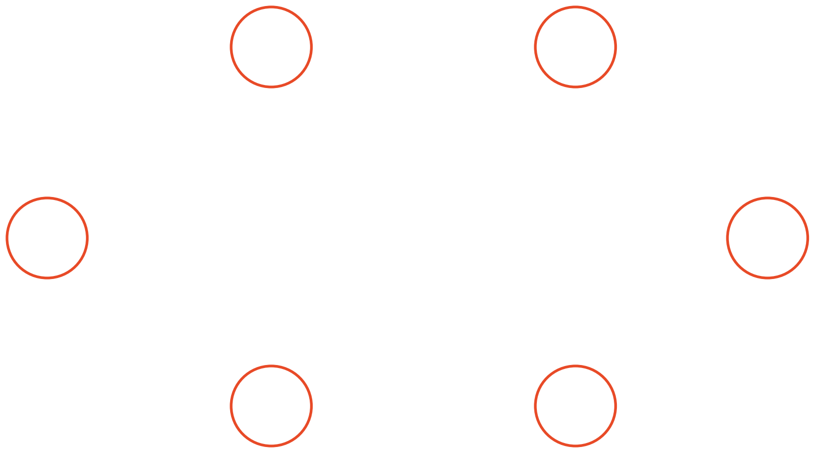
Maximum edges:

Simple:

Not simple:

$$\sum_{v \in V} \deg(v) =$$

UpTree



Proving the size of a minimally connected graph

Theorem:
Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Thm: Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Proof: Consider an arbitrary, minimally connected graph **G=(V, E)**.

Lemma 1: Every connected subgraph of **G** is minimally connected.
(Easy proof by contradiction left for you.)

Inductive Hypothesis: For any **j < |V|**, any minimally connected graph of **j** vertices has **j-1** edges.

Suppose |V| = 1:

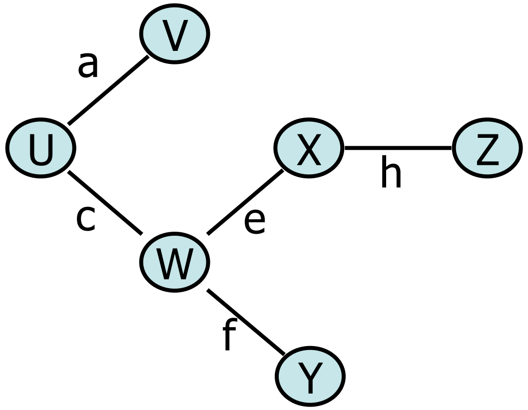
Definition: A minimally connected graph of 1 vertex has 0 edges.

Theorem: |V|-1 edges → 1-1 = 0.

Suppose |V| > 1:

Choose any vertex **u** and let **d** denote the degree of **u**.

Remove the incident edges of **u**, partitioning the graph into ___ components: $\mathbf{C_0 = (V_0, E_0)}$, …, $\mathbf{C_d = (V_d, E_d)}$.



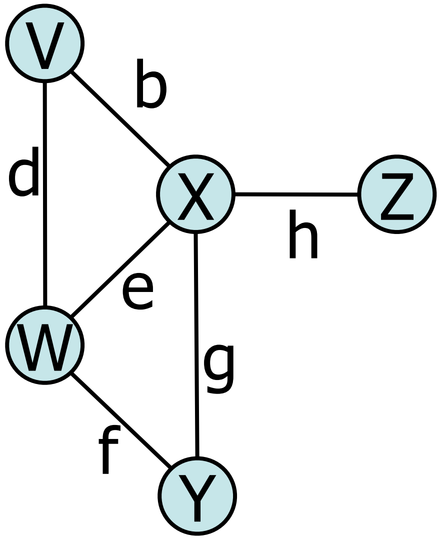
By Lemma 1, every component $\mathbf{C_k}$ is a minimally connected subgraph of **G**.

By our ________: ____________________.

Finally, we count edges:

Graph ADT

Data:
- Vertices
- Edges
- Some data structure maintaining the structure between vertices and edges.



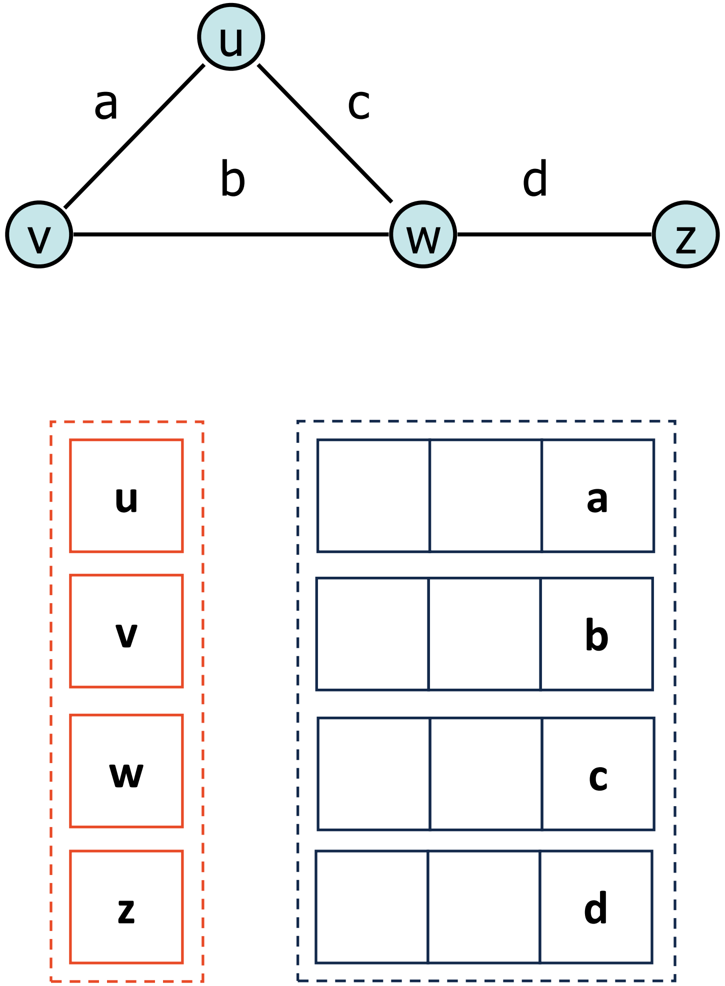
Functions:
- insertVertex(K key);
- insertEdge(Vertex v1, Vertex v2, K key);

- removeVertex(Vertex v);
- removeEdge(Vertex v1, Vertex v2);

- incidentEdges(Vertex v);
- areAdjacent(Vertex v1, Vertex v2);

- origin(Edge e);
- destination(Edge e);

Graph Implementation: Edge List



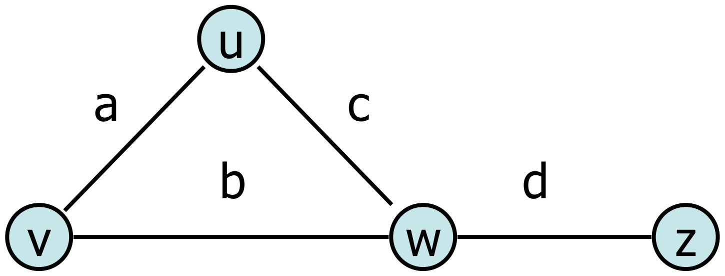
insertVertex(K key);

removeVertex(Vertex v);

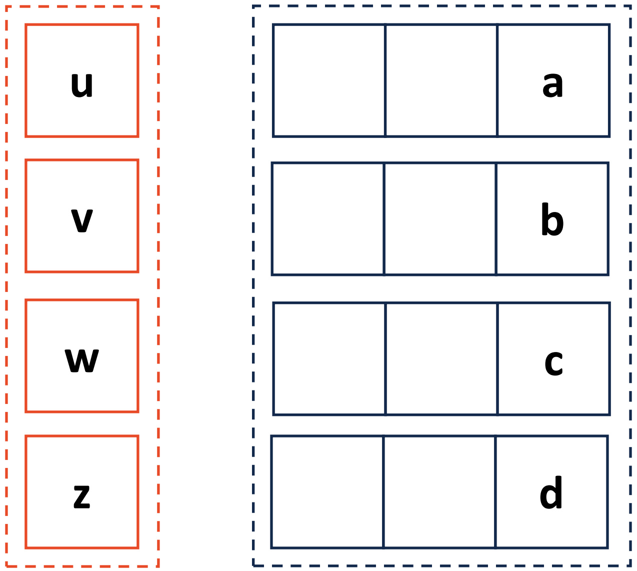
areAdjacent(Vertex v1, Vertex v2);

incidentEdges(Vertex v);

Graph Implementation: Adjacency Matrix



insertVertex(K key);
removeVertex(Vertex v);
areAdjacent(Vertex v1, Vertex v2);
incidentEdges(Vertex v);



	u	v	w	z
u				
v				
w				
z				

CS 225 – Things To Be Doing

Exam 10 (programming) is ongoing!

More Info: https://courses.engr.illinois.edu/cs225/fa2017/exams/

MP6: A one week reflection MP!

Due: Friday, Nov. 17 at 11:59pm

Lab: lab_dict released on Wednesday

Due: Wednesday, Nov. 29 @ 7pm (Before the first lab after break!)

POTDs

Worth +1 Extra Credit /problem (up to +40 total)