CS 225

Data Structures

Nov. 6 – Heap Operations

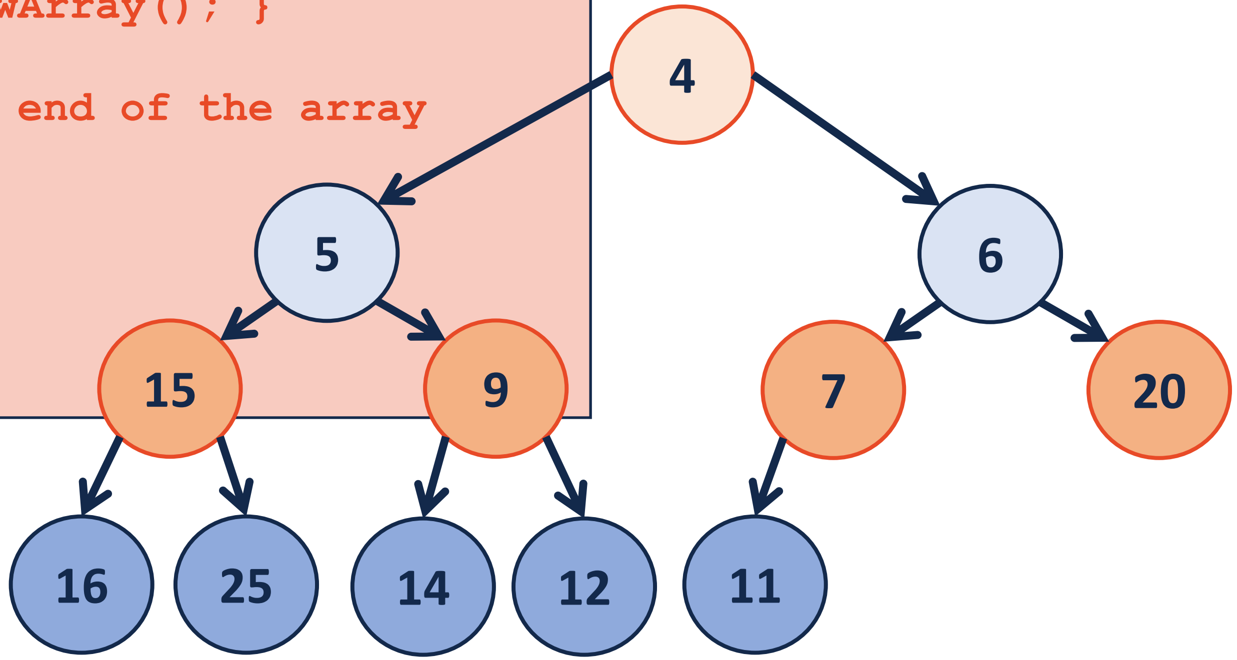
Wade Fagen-Ulmschneider

insert

```
template <class T>
void Heap<T>::_insert(const T & key) {
  // Check to ensure there's space to insert an element
  // ...if not, grow the array
  if ( size_ == capacity_ ) { _growArray(); }

  // Insert the new element at the end of the array
  item_[++size] = key;

  // Restore the heap property
  _heapifyUp(size);
}
```



	4	5	6	15	9	7	20	16	25	14	12	11			

Exam Updates

Exam 9 (theory) is live!

Exam 10 is a programming exam:

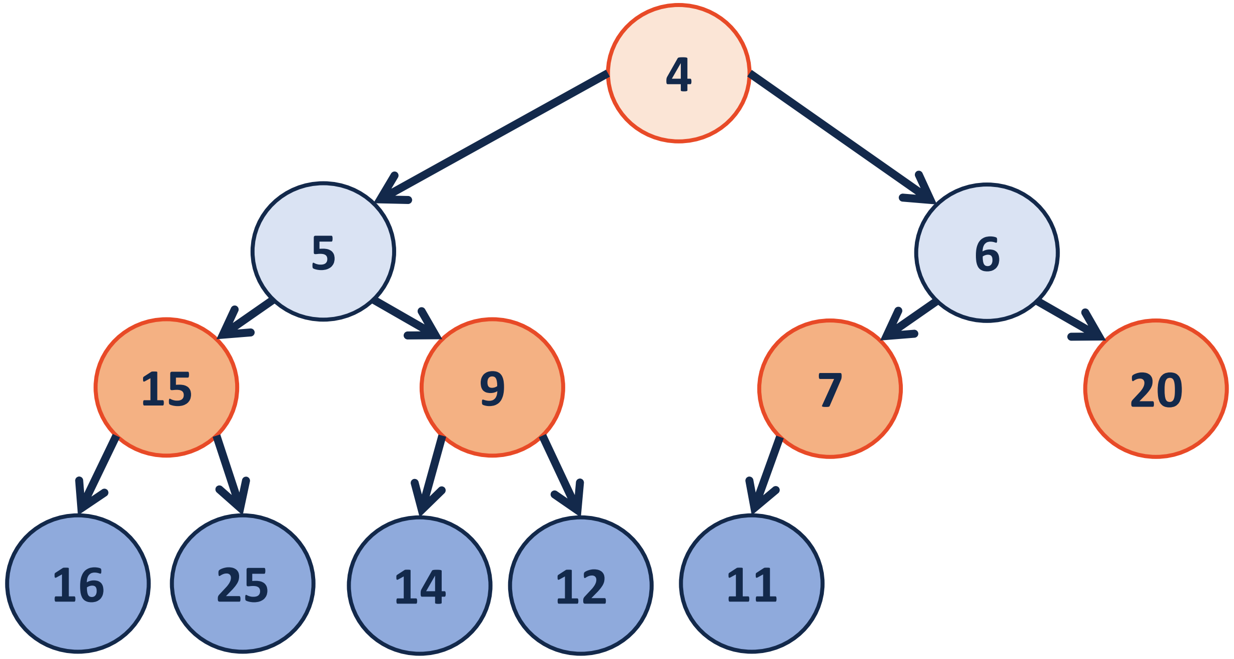
- **MPs**: mp5
- **Labs**: lab_btree, lab_hash
- **Lecture**: Hashing Implementation (eg: Double Hashing)
  Heap Implementation

ICPC Regionals

UIUC’s ICPC Team at Regionals:

- We took 5 teams: our top three places 1st, 3rd, and 6th
- It’s not too late to join IPL
  - **Mondays, 7pm – 9pm**

removeMin



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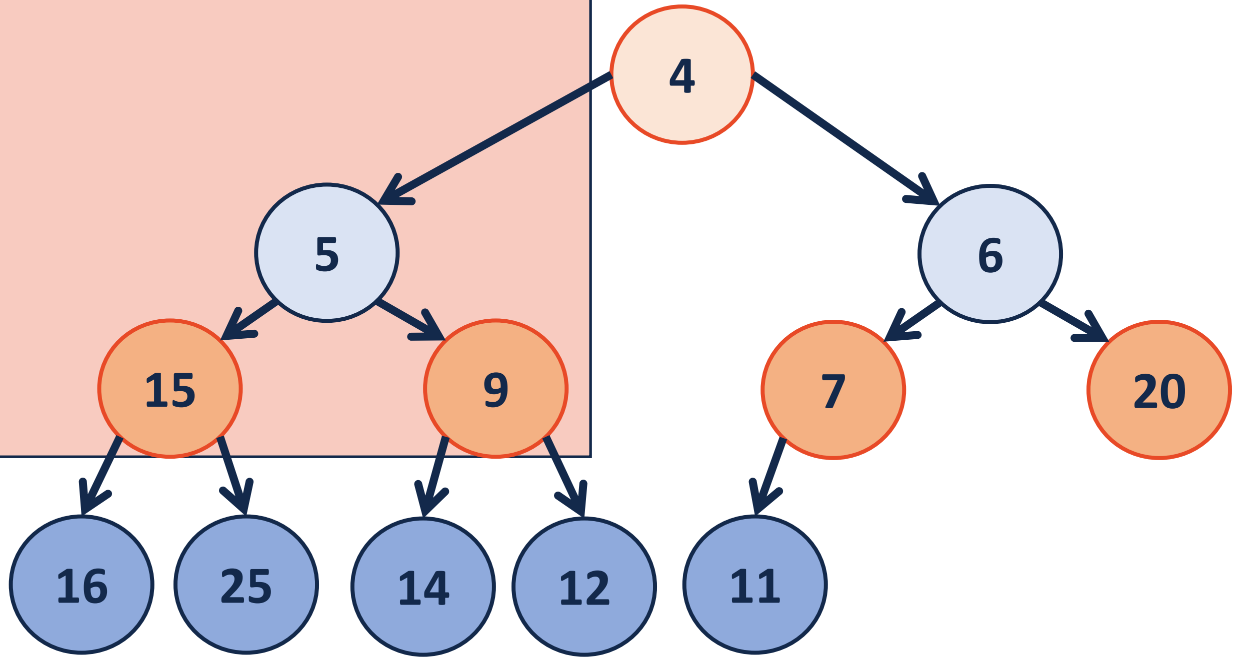
removeMin

```
template <class T>
void Heap<T>::_removeMin() {
  // Swap with the last value
  T minValue = item_[1];
  item_[1] = item_[size_];
  size--;

  // Restore the heap property
  heapifyDown();

  // Return the minimum value
  return minValue;
}
```



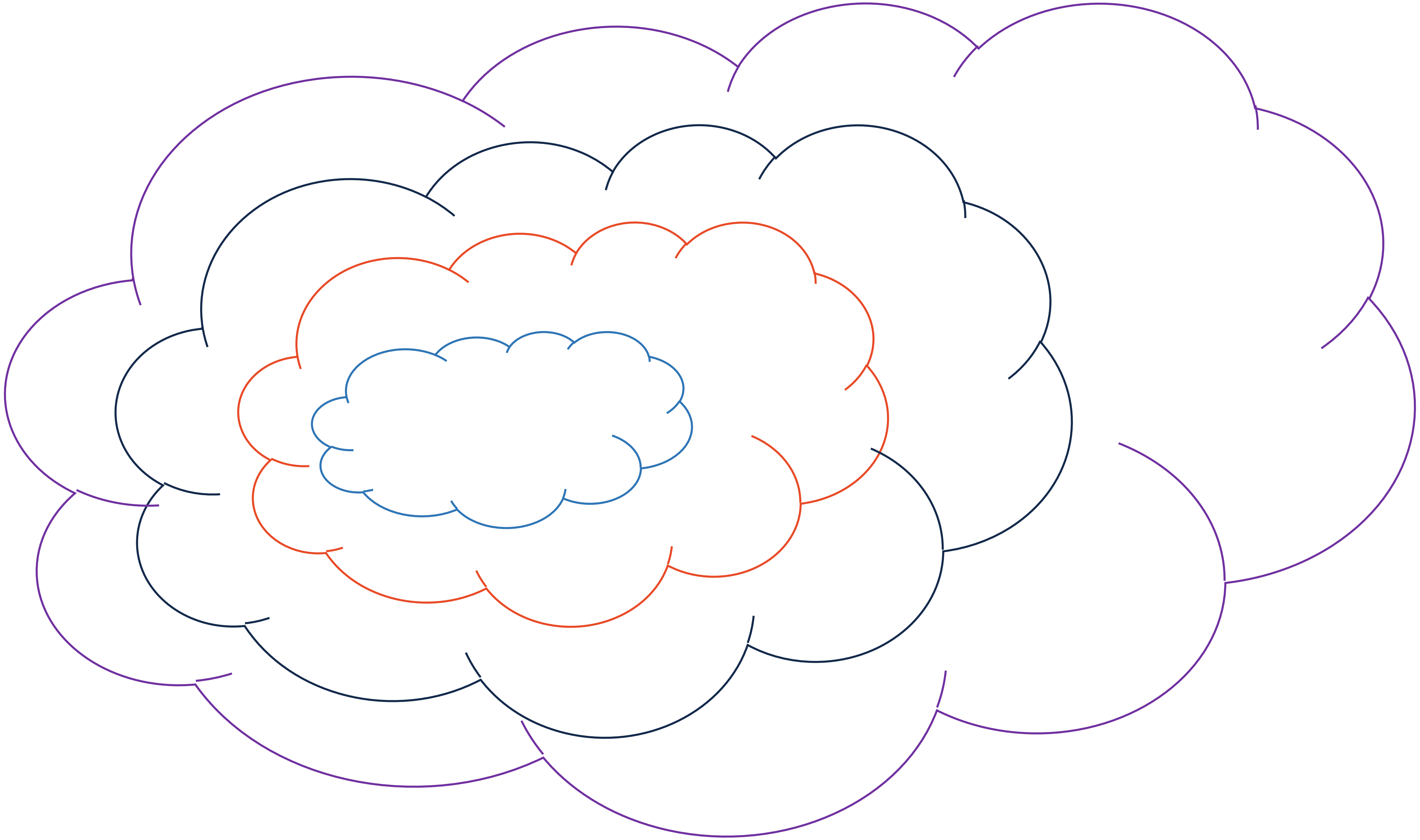
	4	5	6	15	9	7	20	16	25	14	12	11			

insert - heapifyUp

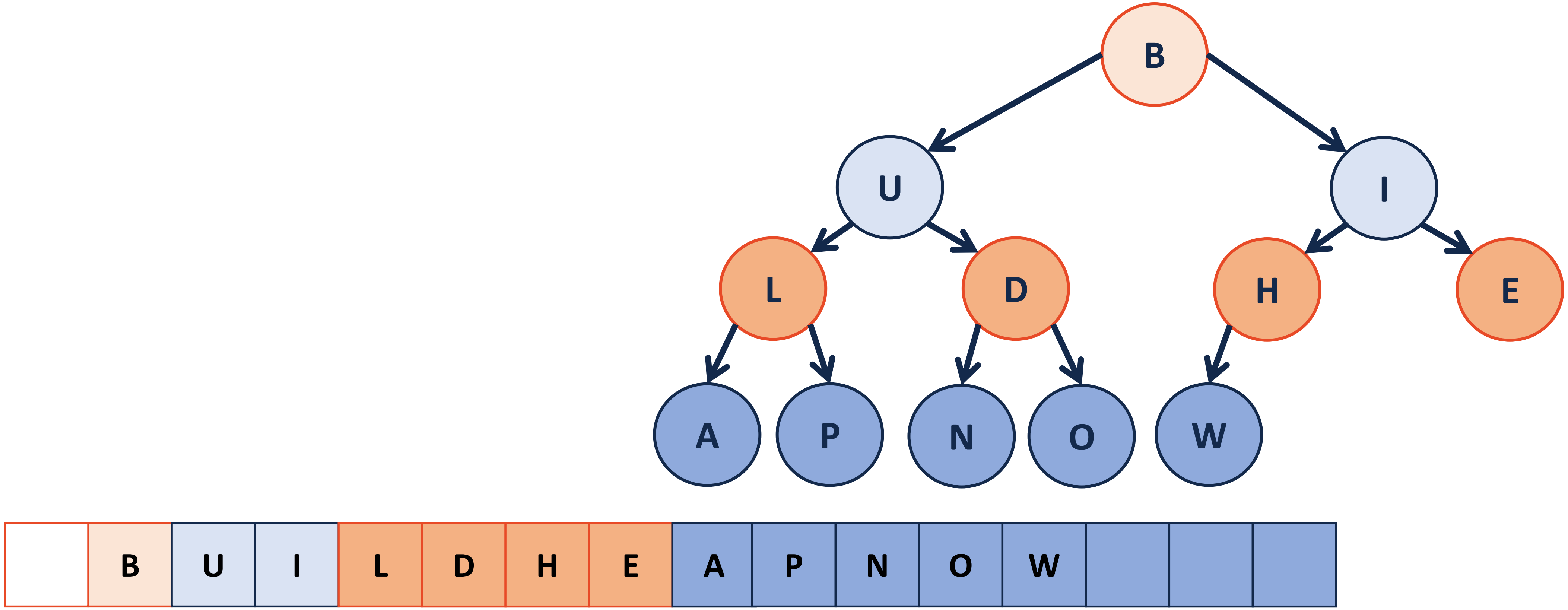
```
template <class T>
void Heap<T>::_removeMin() {
  // Swap with the last value
  T minValue = item_[1];
  item_[1] = item_[size_];
  size--;

  // Restore the heap property
  _heapifyDown();

  // Return the minimum value
  return minValue;
}
```

```
template <class T>
void Heap<T>::_heapifyDown(int index) {
  if ( !_isLeaf(index) ) {
    T minChildIndex = _minChild(index);
    if ( item_[index] ___ item_[minChildIndex] ) {
      std::swap( item_[index], item_[minChildIndex] );
      _heapifyDown( ________________ );
    }
  }
}
```

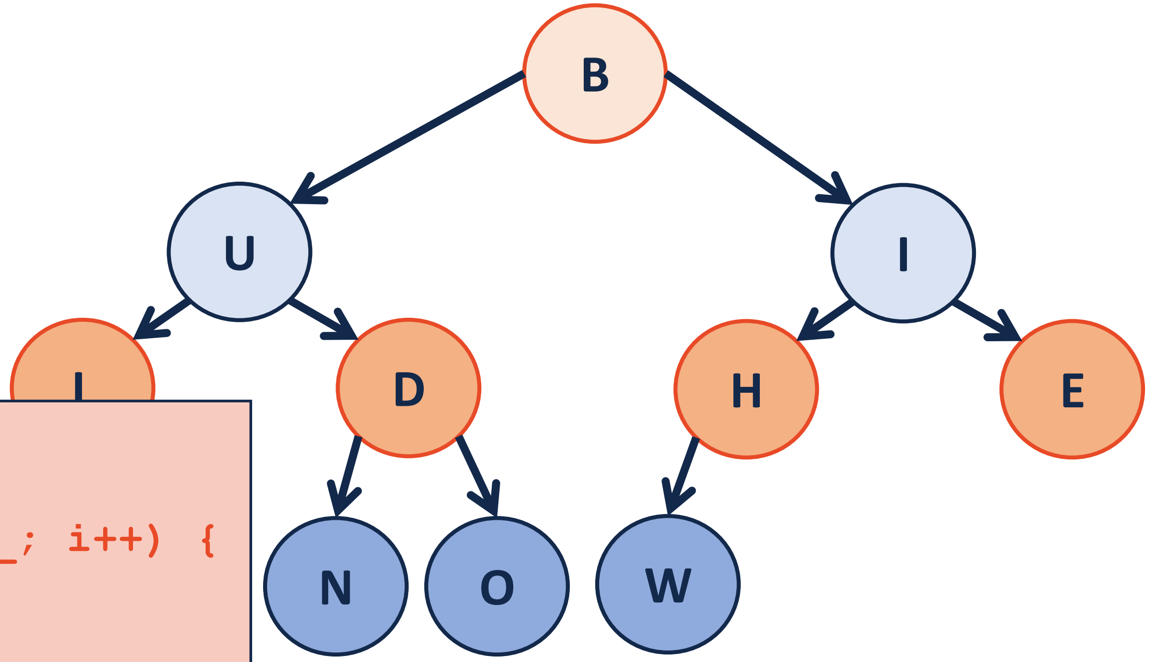
Array Abstractions



buildHeap



buildHeap



1. Sort the array – it's a heap!

2.

```
template <class T>
void Heap<T>::buildHeap() {
  for (unsigned i = 0; i <= size_; i++) {
    heapifyUp(i);
  }
}
```

3.

```
template <class T>
void Heap<T>::buildHeap() {
  for (unsigned i = parent(size); i > 0; i--) {
    heapifyDown(i);
  }
}
```

	B	U	I	L	D	H	E	A	P	N	O	W			

Proving buildHeap Running Time

Theorem: The running time of buildHeap on array of size **n** is: ___________.

Strategy:

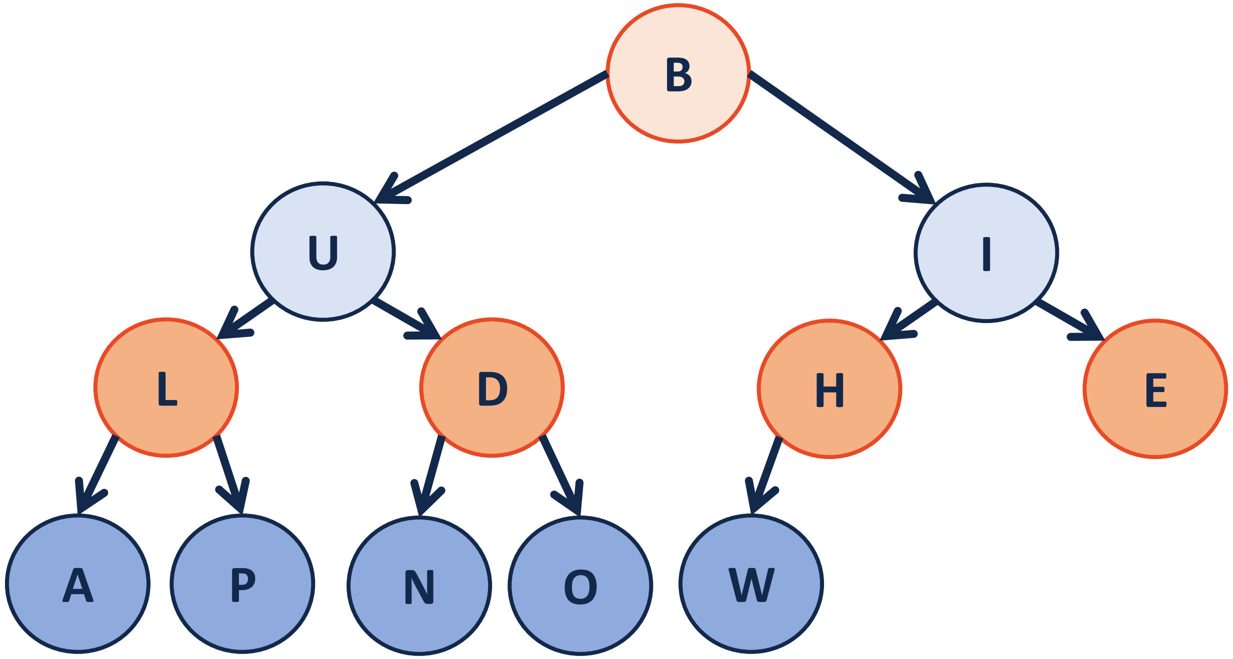
- We know that constant work is done based on the distance a node is away from the root (eg: it's height).

- Therefore, the running time is proportional to the sum of the heights of the heights of all the nodes.

- We will work towards creating a proof around the sum of the heights of all the nodes.

Proving buildHeap Running Time

S(h): Sum of the heights of all nodes in a complete tree of height **h**.



S(0) =

S(1) =

S(h) =

Proving buildHeap Running Time

Proof the recurrence:

Base Case:

General Case:

Proving buildHeap Running Time

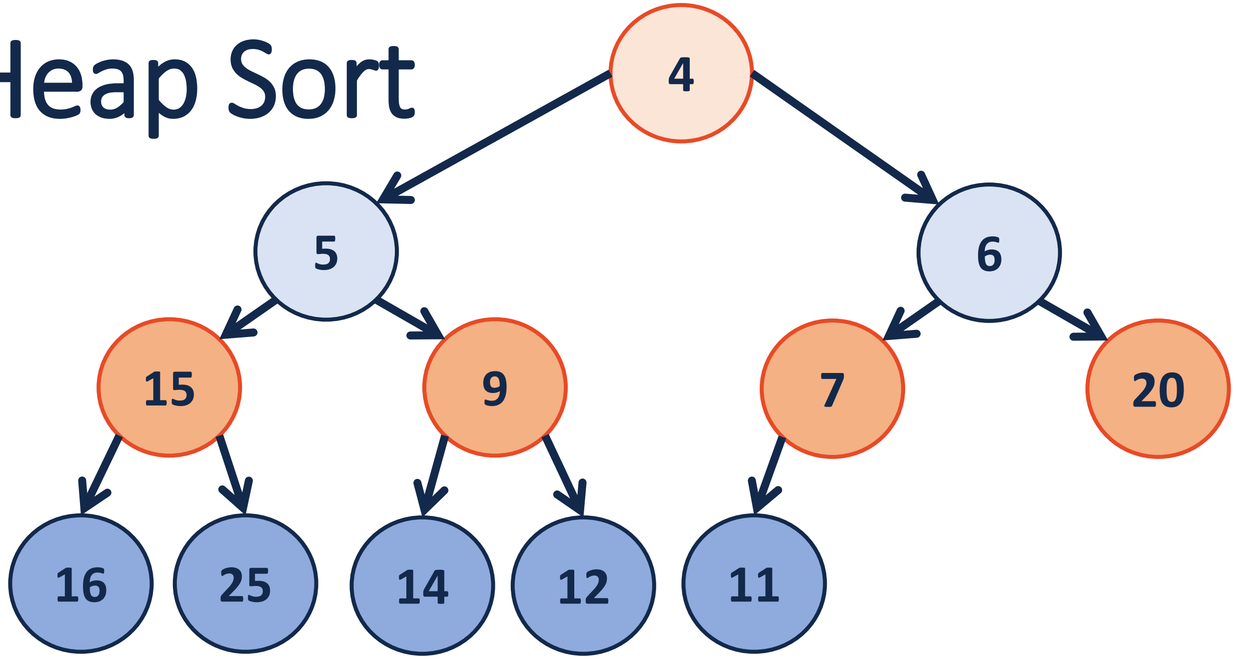
No one cares about things in terms of height:

$S(h)$:

Since $h \leq \lg(n)$:

RunningTime$(n) \leq$

Heap Sort



1.

2.

3.

	4	5	6	15	9	7	20	16	25	14	12	11			

Running Time?

Why do we care about another sort?

A(nother) throwback to CS 173…

Let **R** be an equivalence relation on *us* where **(s, t) ∈ R** if **s** and **t** have the same favorite among:

{ ____, ____, _____, ____, _____, }

CS 225 – Things To Be Doing

Register for CS 225's Final Exam!

Exam 9 (theory) is live!

More Info: https://courses.engr.illinois.edu/cs225/fa2017/exams/

MP5 is due tonight (grace period through tomorrow)

Due Monday, Nov. 6 at 11:59pm

New lab on Wednesday!

Due Sunday, Nov. 12 at 11:59pm

POTD

Every Monday-Friday – *Worth +1 Extra Credit /problem (up to +40 total)*