# CS 225

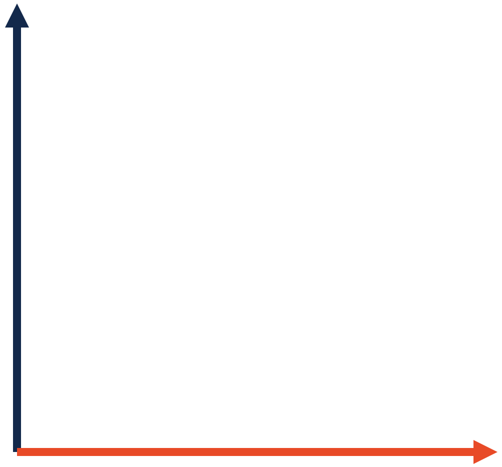
**Data Structures**

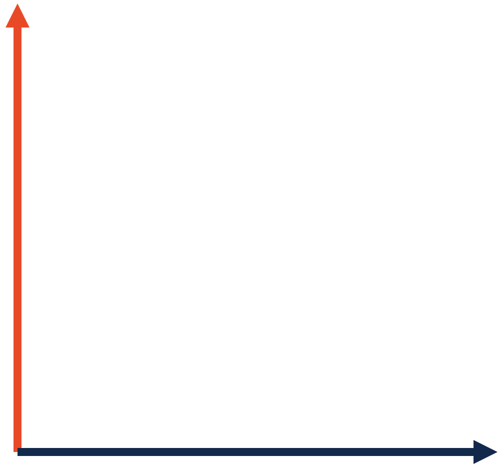
*Oct. 18 – AVL Runtime*

# AVL Tree Analysis

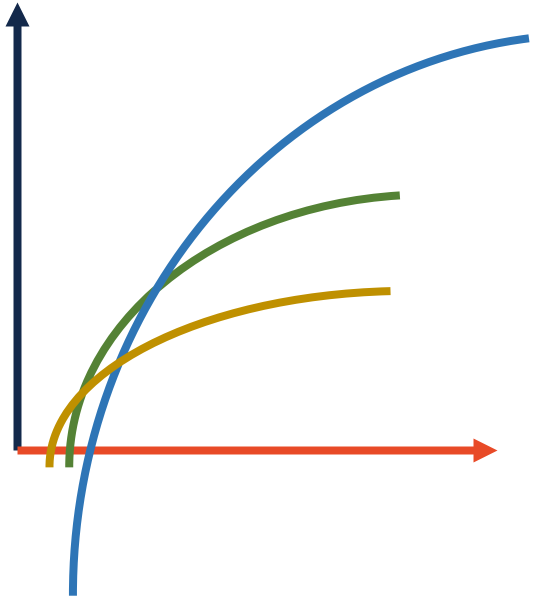
Definition of big-O:

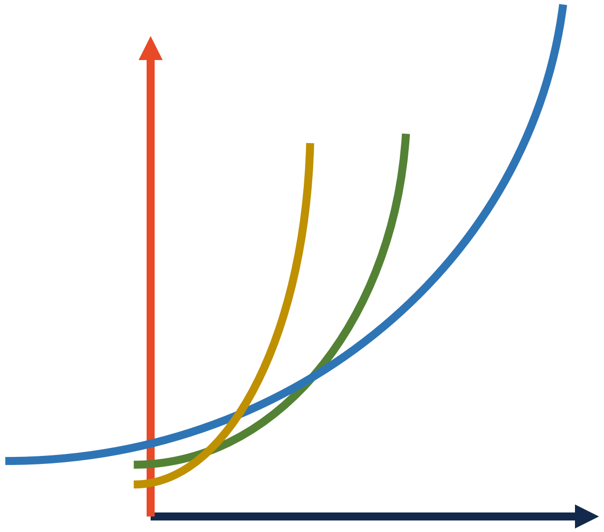
…or, with pictures:





# AVL Tree Analysis





An upper bound on the height **h** for a tree of **n** nodes

…is the same as…

A lower bound on the number of nodes **n** in a tree of height **h**

# Plan of Action

Begin by defining a function that defines the least number of nodes in an AVL tree of height **h**.

**N(h)**: The least number of nodes in an AVL tree of height **h**.

# Simplify the Recurrence

$$\mathbf{N(h)} = 1 + N(h - 1) + N(h - 2)$$

# State a Theorem

**Theorem:** An AVL tree of height h has at least ___________.

**Proof:**

I. Consider an AVL tree and let **h** denote its height.

II. Case: ________________

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

III. Case: ________________

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

IV. Case: ________________

By an Inductive Hypothesis (IH):

We will show that:

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

V. Using a proof by induction, we have shown that:

…and inverting:

# Summary of Balanced BST

**Red-Black Trees**

- Max height: 2 * lg(n)
- Constant number of rotations on insert, remove, and find

**AVL Trees**

- Max height: 1.44 * lg(n)
- Rotations:

# Summary of Balanced BST

**Pros:**

- Running Time:

  - Improvement Over:

- Great for specific applications:

# Summary of Balanced BST

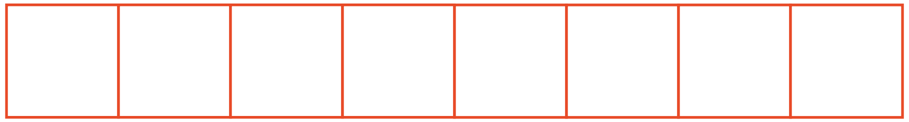
**Cons:**

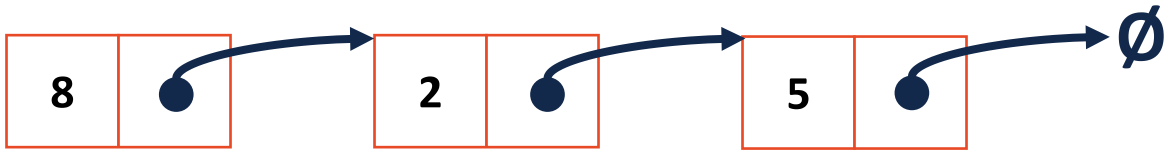
- Running Time:

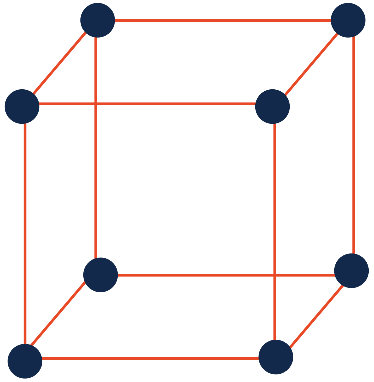
- In-memory Requirement:

# Iterators

Iterators give client code access to traverse the data!







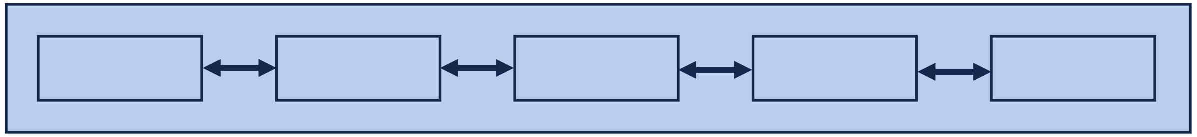
Operators:

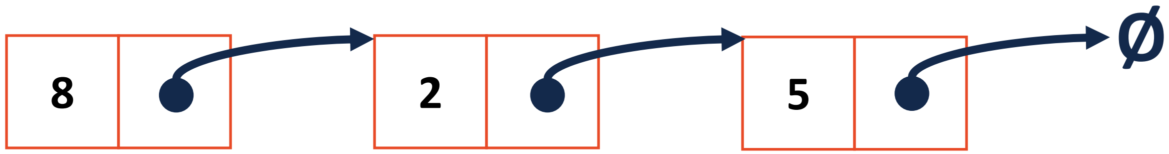
- operator++
- operator==
- operator!=
- operator=
- operator*

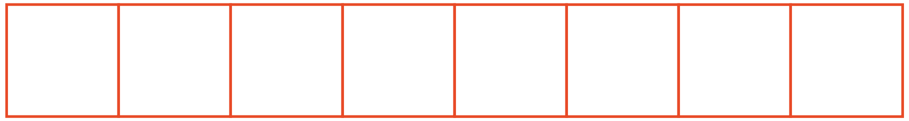
Types of iterators:

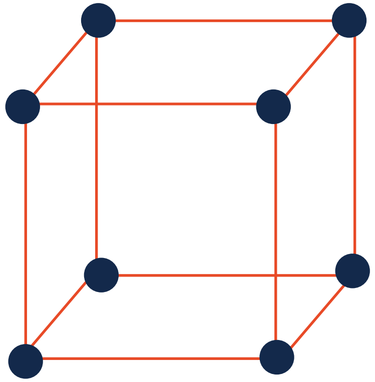
- Forward
- Backward
- Bidirectional

# Iterators encapsulate access to our data:





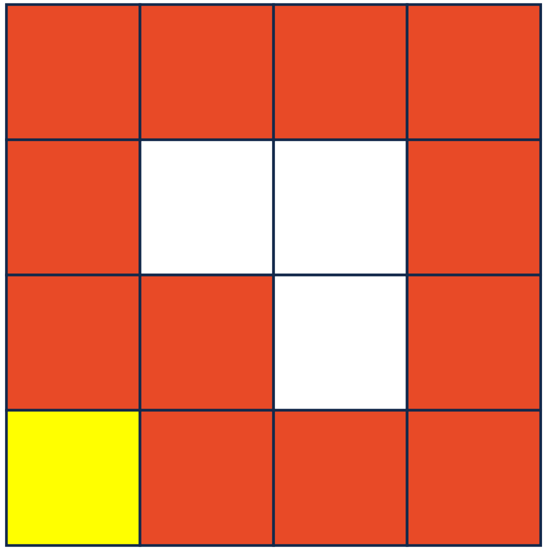




| private var | ++ | * |
|---|---|---|
| | | |
| | | |
| | | |

# MP4

```
const PNG & png;
```



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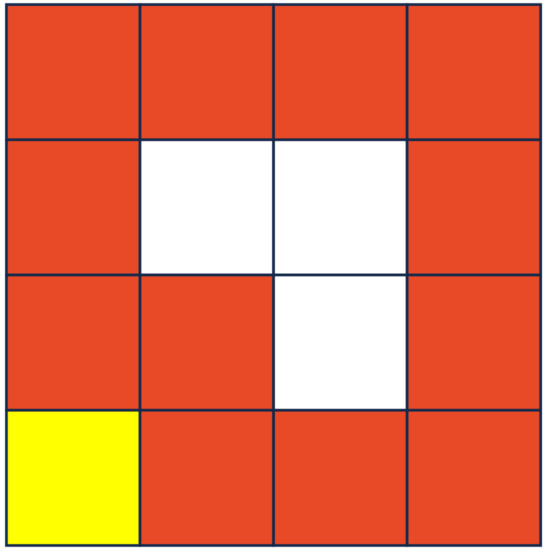
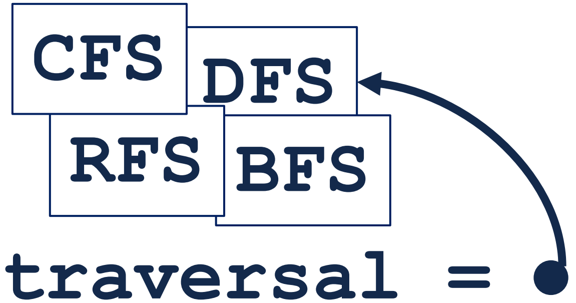


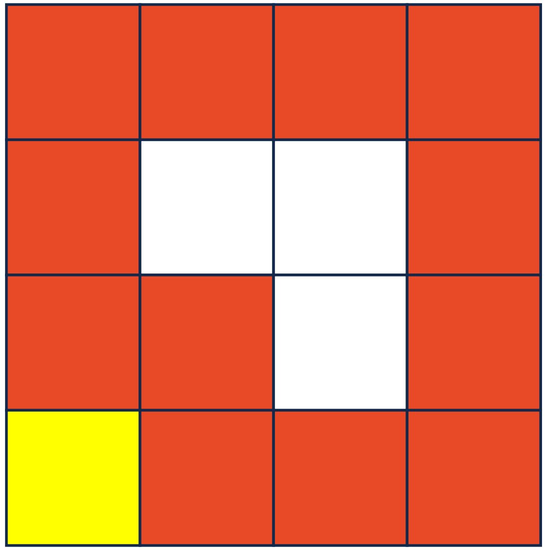
```
Point start(0,3);
```

# MP4



```
const PNG & png;
```

```
ImageTraversal *traversal = ● ;
```



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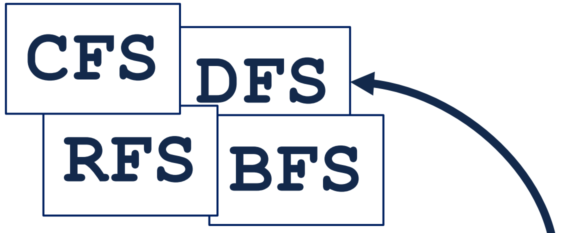
# MP4



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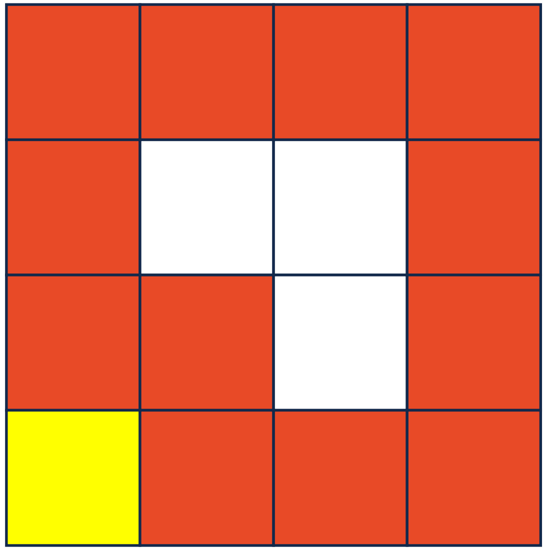

```
Point start(0,3);
```

```
ImageTraversal::Iterator
```

# Iterators

**Why do we care?**

# Iterators

## Why do we care?

```
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
  std::cout << (*it) << std::endl;
}
```

# Iterators

## Why do we care?

```
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
  std::cout << (*it) << std::endl;
}
```

```
DFS dfs(...);
for ( const Point & p : dfs ) {
  std::cout << p << std::endl;
}
```

# Iterators

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```
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for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
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```
DFS dfs(...);
for ( const Point & p : dfs ) {
  std::cout << p << std::endl;
}
```

```
ImageTraversal & traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

# Iterators

```
ImageTraversal *traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

# Iterators

```
std::list<Sphere> sphereList;
...
for (const Sphere & s : sphereList) {
   ...
}
```

```
std::vector<Sphere> sphereList;
...
for (const Sphere & s : sphereList) {
   ...
}
```

```
std::map<std::string, Sphere> sphereMap;
...
for (const std::pair<std::string, Sphere> & kv : sphereMap) {
   ...
}
```

# CS 225 – Things To Be Doing

## Exam 6 (Programming, Lists/Trees) is ongoing!

**More Info:** https://courses.engr.illinois.edu/cs225/fa2017/exams/

## MP4: Available now!

*Due: Monday, Oct. 23 at 11:59pm*

## Labs: lab_avl

*Implement an AVL tree in lab!*

## POTD

Every Monday-Friday – *Worth +1 Extra Credit /problem (up to +40 total)*