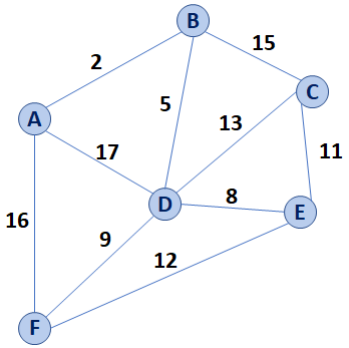
CS 225

#40: Dijkstra's Algorithm

December 6, 2017 · *Wade Fagen-Ulmschneider*

Prim's Algorithm (Minimum Spanning Tree)



Pseudocode for Prim's MST Algorithm

```
1  PrimMST(G, s):
2    Input: G, Graph;
3           s, vertex in G, starting vertex of algorithm
4    Output: T, a minimum spanning tree (MST) of G
5
6    foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9    d[s] = 0
10
11   PriorityQueue Q   // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T           // "labeled set"
14
15   repeat n times:
16     Vertex m = Q.removeMin()
17     T.add(m)
18     foreach (Vertex v : neighbors of m not in T):
19       if cost(v, m) < d[v]:
20         d[v] = cost(v, m)
21         p[v] = m
22
23   return T
```

	Adj. Matrix	Adj. List
Heap		
Unsorted Array		

Running Time of MST Algorithms

- Kruskal's Algorithm:
- Prim's Algorithm:

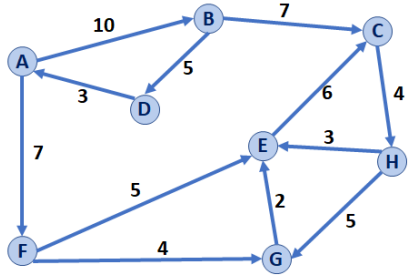
Q: What must be true about the connectivity of a graph when running an MST algorithm?

...what does this imply about the relationship between **n** and **m**?

Q: Suppose we built a new heap that optimized the decrease-key operation, where decreasing the value of a key in a heap updates the heap in amortized constant time, or O(1)*. How does that change Prim's Algorithm runtime?

Shortest Path Home:

Dijkstra's Algorithm (Single Source Shortest Path)



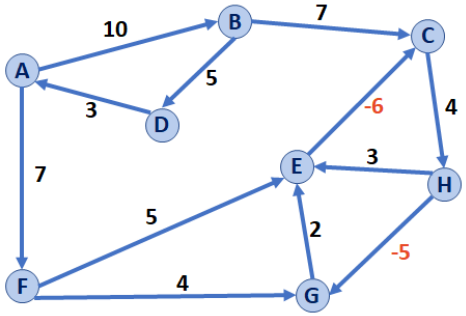
Dijkstra's Algorithm Overview:

- The overall logic is the same as Prim's Algorithm
- We will modify the code in only two places – both involving the update to the distance metric.
- The result is a directed acyclic graph or DAG

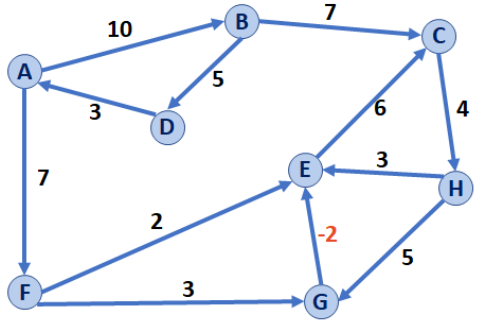
Pseudocode for Dijkstra's SSSP Algorithm

```
1  DijkstraSSSP(G, s):
2    Input: G, Graph;
3           s, vertex in G, starting vertex of algorithm
4    Output: T, DAG with shortest paths (and distances) to s
5
6    foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9    d[s] = 0
10
11   PriorityQueue Q   // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T           // "labeled set"
14
15   repeat n times:
16     Vertex m = Q.removeMin()
17     T.add(m)
18     foreach (Vertex v : neighbors of m not in T):
19       if _____________________ < d[v]:
20         d[v] = _____________________
21         p[v] = m
22
23   return T
```

Dijkstra: What if we have a negative-weight cycle?



Dijkstra: What if we have a minimum-weight edge, without having a negative-weight cycle?



Dijkstra makes an assumption:

Dijkstra: What is the running time?

CS 225 – Things To Be Doing:

1. Exam #12 (programming) continues today
2. MP7 due Monday, Dec. 11 at 11:59pm
3. lab_ml out today; due Sunday, Dec. 10 at 11:59pm
4. Multi-day "puzzle" POTDs available M/W/F