CS 225

#39: Prim's Algorithm

December 4, 2017 · *Wade Fagen-Ulmschneider*

Kruskal's Algorithm

Pseudocode for Kruskal's MST Algorithm

```
1  KruskalMST(G):
2    DisjointSets forest
3    foreach (Vertex v : G):
4      forest.makeSet(v)
5
6    PriorityQueue Q    // min edge weight
7    foreach (Edge e : G):
8      Q.insert(e)
9
10   Graph T = (V, {})
11
12   while |T.edges()| < n-1:
13     Vertex (u, v) = Q.removeMin()
14     if forest.find(u) != forest.find(v):
15        T.addEdge(u, v)
16        forest.union( forest.find(u),
17                      forest.find(v) )
18
19   return T
```

Based on our algorithm choice:

Priority Queue Implementation:	Total Running Time
Heap	**O(n + m) + O(m lg(n))**
Sorted Array	**O(n + m lg(n)) + O(m)**

Why would we prefer a Heap?

Why would be prefer a Sorted Array?

This Week is CS Education Week

This week is the 111th birthday of Grace Hopper, a pioneer of the field of Computer Science! As a department, we're letting the world know about Computer Science all week.

Tonight, CS@Illinois and WCS is hosting an **Hour of Code** event.

Volunteer: Help others program their first line of code!
Tonight, Dec. 4, 6:00pm – 8:00pm
Basement of Siebel

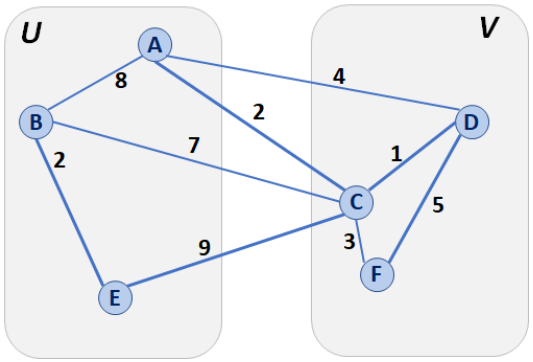
Partition Property

Consider an arbitrary partition of the vertices on **G** into two subsets **U** and **V**.

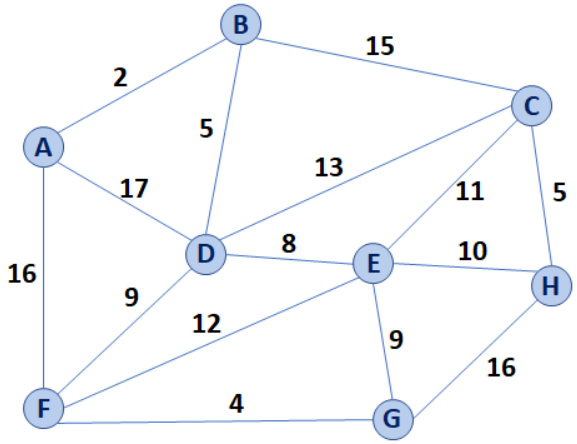
Let **e** be an edge of minimum weight across the partition.

Then **e** is part of some minimum spanning tree.

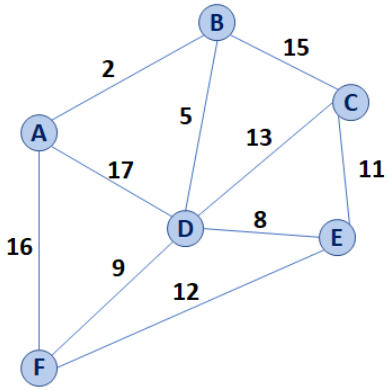
Proof in CS 374!



Partition Property Algorithm



Prim's Algorithm




```
Pseudocode for Prim's MST Algorithm
1  PrimMST(G, s):
2    Input: G, Graph;
3           s, vertex in G, starting vertex of algorithm
4    Output: T, a minimum spanning tree (MST) of G
5
6    foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9    d[s] = 0
10
11   PriorityQueue Q   // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T           // "labeled set"
14
15   repeat n times:
16     Vertex m = Q.removeMin()
17     T.add(m)
18     foreach (Vertex v : neighbors of m not in T):
19       if cost(v, m) < d[v]:
20         d[v] = cost(v, m)
21         p[v] = m
22
23   return T
```

	Adj. Matrix	Adj. List
Heap		
Unsorted Array		

Running Time of MST Algorithms

Kruskal's Algorithm:

Prim's Algorithm:

Q: What must be true about the connectivity of a graph when running an MST algorithm?

...what does this imply about the relationship between **n** and **m**?

CS 225 – Things To Be Doing:

1. Exam #12 (programming) starts today
2. MP7 extra credit deadline today (+14 EC)
3. Final lab out this week!
4. Multi-day "puzzle" POTDs available M/W/F