## #34: Graph Implementations

November 15, 2017 · *Wade Fagen-Ulmschneider*

**Motivation:**
Graphs are awesome data structures that allow us to represent an enormous range of problems. To study these problems, we need:

1. A common vocabulary to talk about graphs
2. Implementation(s) of a graph
3. Traversals on graphs
4. Algorithms on graphs

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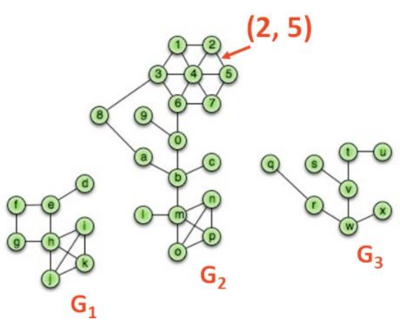
### Graph Vocabulary

Consider a graph **G** with vertices **V** and edges **E**, **G=(V,E)**.




Incident Edges:
**I(v) = { (x, v) in E }**

Degree(v): **|I|**

Adjacent Vertices:
**A(v) = { x : (x, v) in E }**

Path($G_2$): Sequence of vertices connected by edges

Cycle($G_1$): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

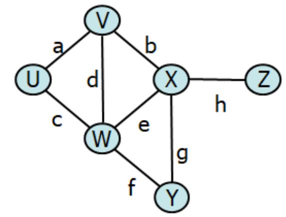
Subgraph(G): **G' = (V', E')**:
V' ∈ V, E' ∈ E, and (u, v) ∈ E → u ∈ V', v ∈ V'

Graphs that we will study this semester include:
Complete subgraph(G)
Connected subgraph(G)
Connected component(G)
Acyclic subgraph(G)
Spanning tree(G)

### Size and Running Times

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

For arbitrary graphs, the minimum number of edges given a graph that is:




*Not Connected:*

*Minimally Connected*:*

The maximum number of edges given a graph that is:

*Simple:*

*Not Simple:*

The relationship between the degree of the graph and the edges:

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### Proving the Size of a Minimally Connected Graph

**Theorem:** Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

**<u>Proof of Theorem</u>**
Consider an arbitrary, minimally connected graph **G=(V, E)**.

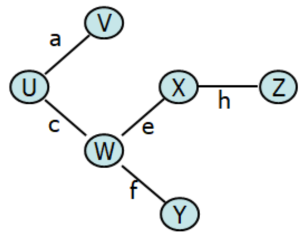
**Lemma 1:** Every connected subgraph of **G** is minimally connected. *(Easy proof by contradiction left for you.)*

**Inductive Hypothesis:** For any **j** < |**V**|, any minimally connected graph of **j** vertices has **j-1** edges.

**Suppose |V| = 1:**
**Definition:** A minimally connected graph of 1 vertex has 0 edges.
**Theorem:** |V|-1 edges ➔ 1-1 = 0.

**Suppose |V| > 1:**
Choose any vertex **u** and let **d** denote the degree of **u**.

Remove the incident edges of **u**, partitioning the graph into ____ components: $\mathbf{C_0 = (V_0, E_0)}, \ldots, \mathbf{C_d = (V_d, E_d)}$.




By Lemma 1, every component $\mathbf{C_k}$ is a minimally connected subgraph of **G**.

By our _________________________:

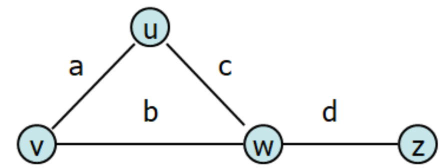
**Finally**, we count edges:

### Graph ADT

| Data | Functions |
|---|---|
| Vertices<br><br>Edges<br><br>Some data structure maintaining the structure between vertices and edges. | `insertVertex(K key);`<br>`insertEdge(Vertex v1, Vertex v2,`<br>`           K key);`<br><br>`removeVertex(Vertex v);`<br>`removeEdge(Vertex v1, Vertex v2);`<br><br>`incidentEdges(Vertex v);`<br>`areAdjacent(Vertex v1, Vertex v2);`<br><br>`origin(Edge e);`<br>`destination(Edge e);` |

### Graph Implementation #1: Edge List

| Vert. | Edges | | |
|---|---|---|---|
| u | | | a |
| v | | | b |
| w | | | c |
| z | | | d |




**Operations:**
insertVertex(K key):

removeVertex(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

incidentEdges(Vertex v):

### Graph Implementation #2: Adjacency Matrix

| Vert. | Edges | | |
|---|---|---|---|
| u | | | a |
| v | | | b |
| w | | | c |
| z | | | d |

**Adj. Matrix**

| | u | v | w | z |
|---|---|---|---|---|
| u | | | | |
| v | | | | |
| w | | | | |
| z | | | | |

**CS 225 – Things To Be Doing:**

1. Exam #10 (programming) is ongoing
2. MP6 due Friday, Nov. 17 (Friday before break starts)
3. lab_dict released today; due Wed. Nov. 29 @ 7pm
4. Daily POTDs