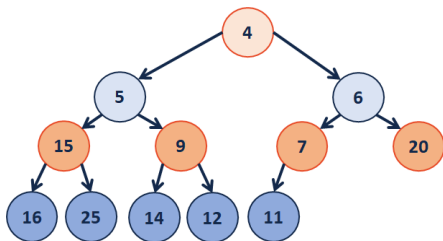


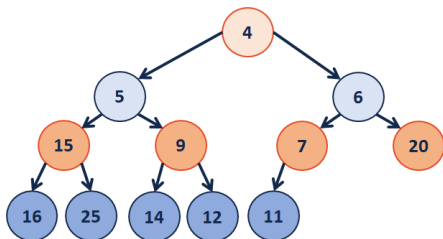
Heap Operation: insert / heapifyUp:



-	4	5	6	15	9	7	20	16	25	14	12	11			
---	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

Running Time?

Heap Operation: removeMin / heapifyDown:



-	4	5	6	15	9	7	20	16	25	14	12	11			
---	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

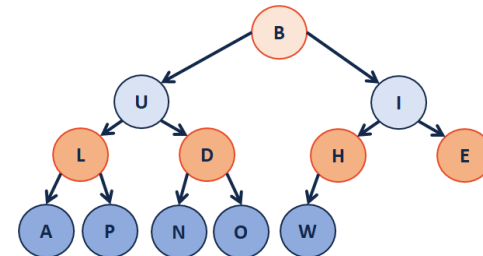
Heap.cpp (partial)

```

1  template <class T>
2  void Heap<T>::_removeMin() {
3      // Swap with the last value
4      T minValue = item_[1];
5      item_[1] = item_[size_];
6      size--;
7
8      // Restore the heap property
9      heapifyDown();
10
11     // Return the minimum value
12     return minValue;
13 }
14
15 template <class T>
16 void Heap<T>::_heapifyDown(int index) {
17     if ( !_isLeaf(index) ) {
18         T minChildIndex = _minChild(index);
19         if ( item_[index] > item_[minChildIndex] ) {
20             std::swap( item_[index], item_[minChildIndex] );
21             _heapifyDown( minChildIndex );
22         }
23     }
24 }
25 }

```

Q: How do we construct a heap given data?



-	B	U	I	L	D	H	E	A	P	N	O	W			
---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--

## An $O(n)$ approach to buildHeap:

```
Heap.cpp (partial)
1  template <class T>
2  void Heap<T>::buildHeap() {
3      for (unsigned i = parent(size); i > 0; i--) {
4          heapifyDown(i);
5      }
6  }
```

**Theorem:** The running time of buildHeap on array of size  $n$  is:

\_\_\_\_\_.

### Strategy:

- We know that constant work is done based on the distance a node is away from the root (eg: it's height).
- Therefore, the running time is proportional to the sum of the heights of the heights of all the nodes.
- We will work towards creating a proof around the sum of the heights of all the nodes.

### Define $S(h)$ :

Let  $S(h)$  denote the sum of the heights of all nodes in a complete tree of height  $h$ .

$S(0) =$

$S(1) =$

$S(h) =$

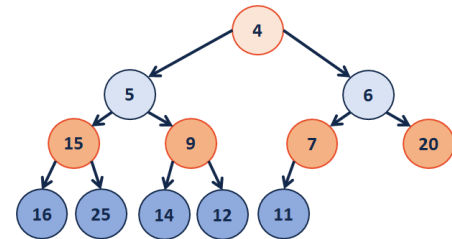
### Proof of $S(h)$ by Induction:

Base Case(s):

## General Case:

### Finally, finding the running time:

## Heap Sort



Algorithm:

- 1.
- 2.
- 3.

Running time?

Why do we care about another sort?

## CS 225 – Things To Be Doing:

1. Register for CS 225's Final Exam!
2. Exam #9 starts today
3. MP5 due tonight (grace period until tomorrow)
4. New lab on Wednesday
5. Daily POTDs