Thursday 7/24: NP

What are they?

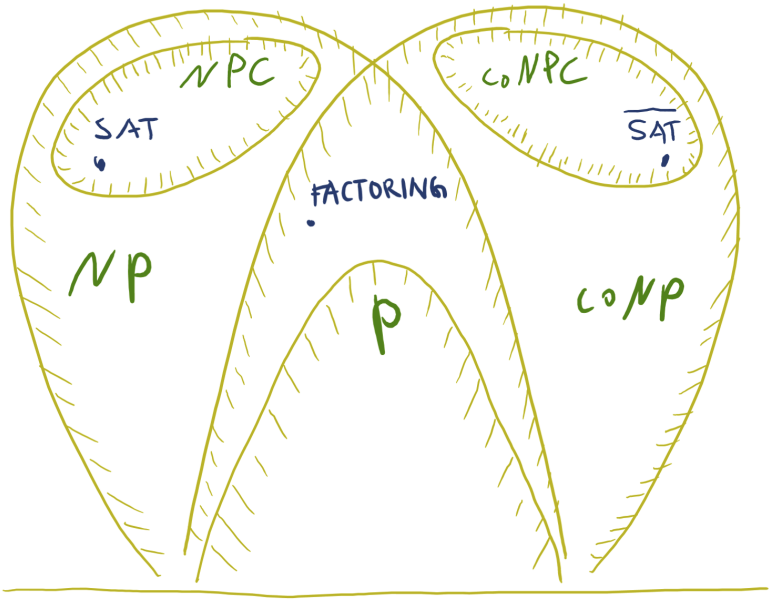
P: All problems that can be **solved** in polynomial time. The solution can be $O(1)$, $O(n)$, or even $O(n^{100000})$, as long as it is smaller than exponential.

NP: All problems that can be **verified yes** in polynomial time. It naturally contains all problems in P, and some harder problems whose best solutions so far take exponential time. It is unknown whether these problems have a chance of getting **solved** in polynomial time.

Co-NP: All problems that can be **verified no** in polynomial time. Similar to NP, co-NP also contains all of P and some harder problems whose best solutions so far take exponential time. It is unknown whether these problems have a chance of getting **solved** in polynomial time.

EXP (aka EXPTIME): All problems that **can be solved** in exponential time. Examples include Tower of Hanoi, generating all parse trees of a sentence, as well as all problems in NP and co-NP.

Up until this point, no one has proven that P=NP (or P≠NP). It is also unknown whether NP is equal to co-NP or not and whether NP is equal to EXP or not. The following figure depicts what people currently *speculate* the relationships should be like[1] (note that EXP should be the superset containing everything here). However, no one has solid answers yet.



NP-Complete

Note that P is a subset of both NP and co-NP, because if a problem can be solved in polynomial time, it can definitely be verified yes and no in polynomial time, too. Despite this subset relationship,

[1]Screenshot taken from https://courses.cs.cornell.edu/cs4814/2020sp/lectures/goodcharacterization.pdf

when we talk about NP, we often think about a family of some harder problems called **NP-Complete** that have the following characteristics.

- These problems can be verified in polynomial time, meaning that if you are given an answer, you can verify if this answer is correct in polynomial time.
- State-of-the-art solutions to these problems take exponential time. However, no one has proven that these problems **require** exponential time.
- These problems are essentially transformable between each other.
- Most importantly, it has been proven that *if* we can find a polynomial-time solution to any of these problems, it would *imply* that P=NP.

Members of this family include circuit satisfiability (SAT), graph colorability beyond 2 colors, propositional logic satisfiability, the Marker Making problem, vertex cover, the Traveling Salesman problem, etc. You do not need to know the details of these problems, but you should be able to recognize these names and know that they are NP-complete.