Monday

Math Review

Important sets:

- $\mathbb{N}$: natural numbers, $\{0, 1, 2...\}$
- $\mathbb{Z}$: integers, $\{0, -1, 1, -2, 2...\}$
- $\mathbb{Z}^+$: positive integers, $\{1, 2, 3...\}$
- $\mathbb{R}$: real numbers, $\{-2, 2.5, \pi...\}$
- $\mathbb{Q}$: rational numbers, numbers of the form $\frac{m}{n}$ where m and n are integers
- $\mathbb{C}$: complex numbers, $\{i, 2i + 1, \pi\}$

Important notation:

- $x \in R$ means x is an element of the reals
- $y \in (0, 5]$ for $y \in \mathbb{Z}$ means $y \in \{1, 2, 3, 4, 5\}$
- $(a, b) \in \mathbb{Z}^2$ means (a, b) is an ordered pair of integers, *i.e.,* $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, **NOT** squared integers

Exponents:

- $b^0 = 1$, $b^{0.5} = \sqrt{b}$, $b^{-1} = \frac{1}{b}$
- exponent rules: $b^x b^y = b^{x+y}$, $b^x a^x = (ba)^x$, $(b^x)^y = b^{xy}$

Because $y = b^x$ is equivalent to $x = \log_b y$ if $b > 1$ and $y > 0$, we also have the following logarithm rules:

- $b^{\log_b(x)}$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b(x^y) = y \log_b(x)$
- change of base formula: $\log_b(x) = \log_a(x) \log_b(a)$

Logarithm example: Simplify the following expression:

$$\log_2(13) \cdot \log_{13}(2048)$$

Recall the change of base formula: $\log_b a = \frac{\log_x(a)}{\log_x(b)}$.

Then, we can rewrite it as $\log_x b \cdot \log_b a = \log_x(a)$. We can now directly apply this formula, where $x = 2, b = 13$, and $a = 2048$.

$$\log_2(13) \cdot \log_{13}(2048) = \log_2(2048) = \log_2(2^{11}) = 11$$

Make sure you are also familiar with the other math review topics in the textbook: factorial, max, floor, ceiling, etc.

Logic

Propositional Logic

Proposition: a statement which is either true or false (not both, and not neither)
e.g., 5 is odd; I am in Europe right now

Complex proposition: a statement which combines one or more propositions with logical operators
operators: $\wedge$ (and), $\vee$ (or), $\implies$ (implies), $\iff$ (bi-directional implies), $\neg$ (not)
e.g., Naina is from Pennsylvania and Hongxuan is from Pennsylvania; the first proposition is true, and the second is false, so the entire statement is false

Truth tables: define under which conditions complex propositional statements are true and false.

Negation ($\neg$)

p	$\neg p$
T	F
F	T

Conjunction ($\wedge$)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction ($\vee$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication ($\implies$)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional ($\iff$)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical equivalence: two statements are logically equivalent if they evaluate to true and false under the same conditions.

e.g., Demorgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (try the truth table out on your own time)

Vacuous Truth Examples: Describe under which conditions each of the following statements are true and false.

(a) If it is raining when I wake up, then I will bring an umbrella to campus.

This statement tells me that rain existing when I wake up will implore me to be an umbrella to campus, otherwise the statement is false—if you catch me on campus without an umbrella on a rainy morning, I was lying.

However, I have not made any claims about what will happen if it is not raining when I wake up. I might bring an umbrella, and I might not. Either way, I wasn't lying to you when I made that statement.

This is a bit of a convention mathematically. In English you might assume that I'm also saying if it doesn't rain I won't bring an umbrella. That is perfectly reasonable but our mathematical convention is to not assume that.

(b) $\forall x \in \mathbb{Z}$, if $x^2 < 0$ then x is even.

$x^2 < 0$ will never be true for an integer x. Thus, the *hypothesis* of the *conditional* will never be executed, and we do not care what comes after.

Rules for Negation

- Negation of negation (double negation): $\neg(\neg p) \equiv p$
- Negation of conjunction (De Morgan's Law): $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Negation of disjunction (De Morgan's Law): $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- Negation of implication: $\neg(p \to q) \equiv p \wedge \neg q$

Negation example: Simplify the following expression so that all 'not's are on individual predicates:

$$\neg((p \wedge q) \to r)$$

The negated claim: $\neg((p \wedge q) \to r) \equiv (p \wedge q) \wedge \neg r \equiv p \wedge q \wedge \neg r$