Discussion Problem Solutions for Examlet F

CS 173: Discrete Structures

Wednesday

Problem 17.1. in Discussion Manual

(a) $\{\emptyset, \{rain\}, \{snow\}, \{sun\}, \{rain, snow\}, \{rain, sun\}, \{snow, sun\}, \{rain, snow, sun\}\}$

(b) $\{\emptyset, \{(water, ice)\}\}$

(c) $\mathbb{P}(C) - D = \{\emptyset, \{ice\}, \{water\}, \{water, ice\}\} - \{\{water\}, \{milk\}\} = \{\emptyset, \{ice\}, \{water, ice\}\}$

(d) $\{\emptyset\}$

(e) $2^5 + 2^3 - 1 = 32 + 8 - 1 = 39$ *(The -1 is because the empty set is in both powersets.)*

Problem 17.3. in Discussion Manual

(a) $\mathbb{Z} - \{10, 37\}$. *(Every integer other than 37 appears in $f(37)$, and every integer other than 10 appears in $f(10)$; the intersection contains every integer which is in both, i.e. every integer other than 10 and 37.)*

(b) $\{3, 4\}$

(c) $\{4, 7\}$

Problem Partitions (a). from Collections of Sets

$M = \{p(2), p(5), p(7), p(8), p(13), p(21)\} = \{\{2, 8\}, \{5\}, \{7, 21\}, \{2, 8\}, \{13\}, \{7, 21\}\}$. We can remove the duplicates in this set to see that $M = \{\{2, 8\}, \{5\}, \{7, 21\}, \{13\}\}$.

This is a partition: by inspection, it follows all 3 rules (it covers all of A, it does not contain the empty set, and all the sets are disjoint). *(Note that the same function p used with a different base set - e.g. $A = \{2, 3, 6\}$ - might not produce a partition.)*

Thursday

Problem 7.5. in Discussion Manual

(c) Split the 10 nights into 5 pairs of consecutive nights (nights 1 and 2, nights 3 and 4, etc). Each of the 61 hours occurs during exactly one of these five pairs of nights. Since there are more than 12 times as many hours as pairs, there must be at least one of our pairs which ends up with 13 or more hours, which was what we needed to prove.

Commentary: Each hour is a pigeon and each pair *of nights is a pigeonhole. Note that we're using a slightly generalized version of the pigeonhole principle: there must always be at least one pigeonhole which contains an* average-or-greater *number of pigeons. In this case, the average number of pigeons is guaranteed to be 61/5=12.2, so some pigeonhole has at least 12.2 - and since we're working with integers, the only way to have at least 12.2 is to have at least 13.*

Problem 17.5. in Discussion Manual

(a) Use "combinations with repetition" formula with $k = 11$ objects and $n = 3$ types: $\binom{11+3-1}{11} = \binom{13}{11}$. (Or $\binom{13}{2}$, or $\frac{13\cdot 12}{2\cdot 1} = 78$.) *WARNING: Make sure you understand how to re-derive the formula for combinations with repetition, using the stars and dividers picture (section 18.6 in the textbook). Blindly memorizing the final formula leaves you open to a range of off-by-one errors.*

(e) We can sum the possible combinations of each odd-sized set: $\binom{10}{1} + \binom{10}{3} + \binom{10}{5} + \binom{10}{7} + \binom{10}{9}$

Problem 17.6. in Discussion Manual

(a) If we were to fully expand $(x + y + z)^{27}$ and not collect like terms yet, each of the 3^{27} terms would exactly correspond to one of the possible ways to choose an x, y, or z from each of the 27 trinomials. In particular, each instance of $x^3y^{14}z^{10}$ comes from choosing a total of 3 x's, 14 y's, and 10 z's.

There are $\binom{27}{3}$ ways to choose which trinomials provide the x's. After that there are $\binom{24}{14}$ ways to choose locations for the y's, and then the choices for z's are fully determined.

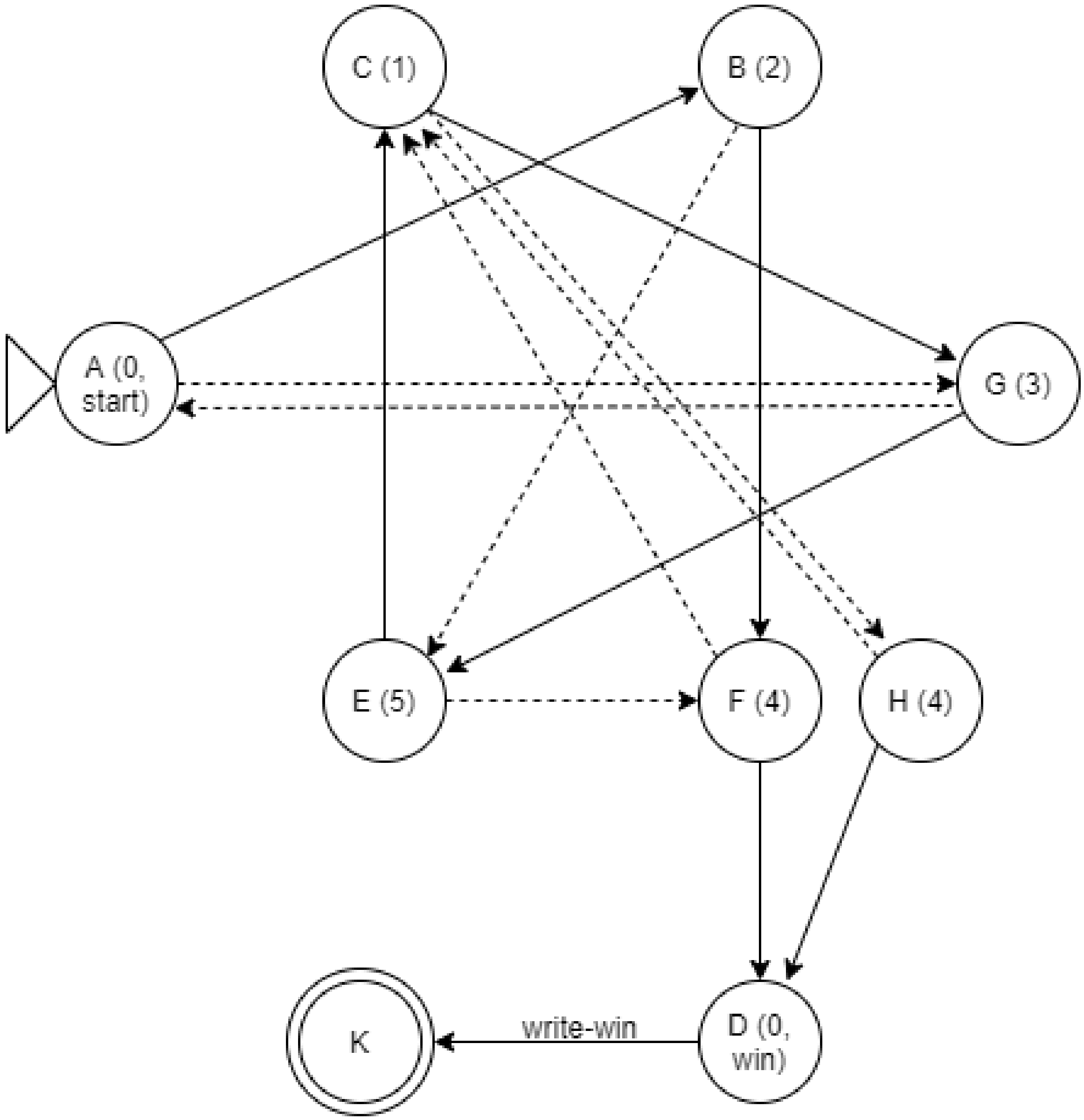
Total choices: $\binom{27}{3}\binom{24}{14} = \frac{27!}{24!3!}\frac{24!}{14!10!} = \frac{27!}{3!14!10!}$

(b) Using the same logic as part (a) and the fact that $27 = a + b + c$, we get $\binom{27}{a}\binom{27-a}{b} = \frac{27!}{a!(27-a)!}\frac{(27-a)!}{b!c!} = \frac{27!}{a!b!c!}$.

Friday

Problem 18.1. in Discussion Manual

(1) The original state diagram:

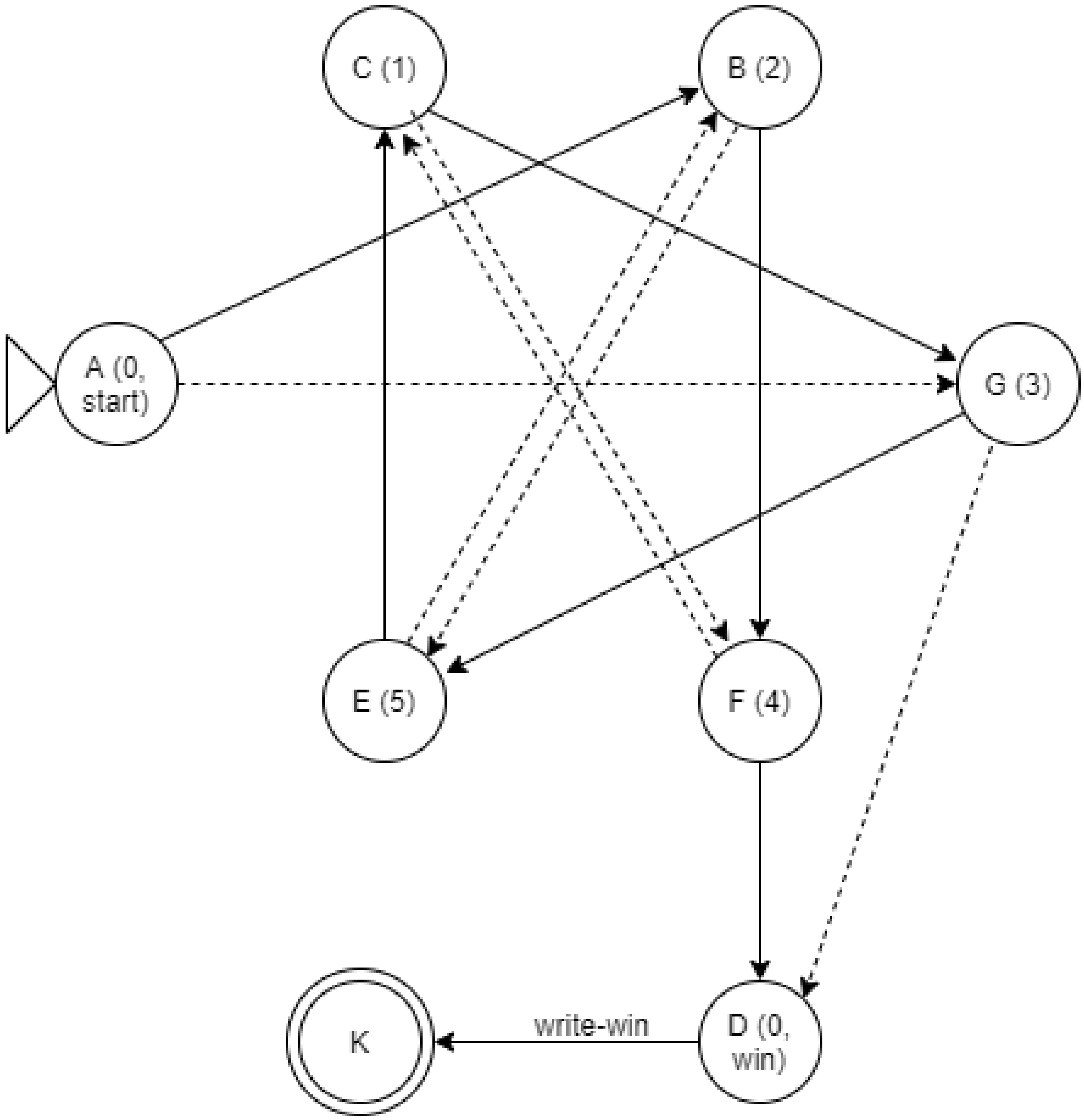


(For readability, unlabeled solid arrows are action "2", and dashed arrows are action "3". Each state has in parentheses the current remainder upon division by 6.)

(2) There are two incorrect transition arrows. $\delta(E, 3)$ should be $\{B\}$ because E is 5, B is 2, and $5 + 3 \equiv 2 \pmod 6$. $\delta(G, 3)$ should be $\{D\}$, since getting back to remainder zero should begin the end-game sequence.

(3) We don't need to keep both of H and F, since they both have the same outgoing transitions (because they both represent the remainder 4). *(Note that we* do *need to keep both A and D, since for remainder 0 we need to distinguish between whether the game has just begun (and so nobody has won), or the game is now ending.)*

(4) The diagram with those two arrows fixed and the extra state removed:



Monday

Problem 18.2. in Discussion Manual

(a) `states:   3          4              fail`
    `actions:    password   unauthorized`

(b) $S = \{start, 1, 2, 3, 4, 5, done, error, finished, fail\}$

$A = \{requestpage, login, shippage, notfound, password, unauthorized\}$

(c) $\delta(1, requestpage) = \emptyset$

$\delta(3, password) = \{4\}$

$\delta(start, requestpage) = \{1, 2, 5\}$

(d) $10 \cdot 6 = 60$

(e) $1, 2, 5$ can be combined into a single state. You could almost certainly combine "done" and "finished"; you could probably also combine "error" and "fail", though it depends on how this information is likely to be used (e.g. do we want later processing to be able to report different specific errors). *(This problem is a bit open-ended and open to interpretation.)*

Problem Magic word. from State Diagrams

All three actions are possible from any state, but for readability, all arrows returning to the start state have been omitted from the diagram below (so e.g. there is an implicit arrow with label "l" from "mam" back to the start state).