Other Topics Questions

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Problem 1

Prove the following claim by direct proof:

For any integers k and m, if $k \leq 7$ and $0 < m - 3 \leq \frac{k}{7}$, then $m^2 - 9 \leq k$.

Problem 2

Express the following set S as a finite set of elements:

$$S = \{ (a, b) \in \mathbb{N} \times \mathbb{Z} \mid 0 \leq a \leq 2 \text{ and } |a + b| \leq 1 \}$$

Problem 3

Define a relation R on $\mathbb{Z}^2$ by $(a, b)R(c, d)$ iff either $a < c$ or $a = c$ and $b \leq d$. Prove that R is transitive.

Problem 4

Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is known to be onto. Define $f : \mathbb{Z}^2 \to \mathbb{Z}$ by $f(x, y) = h(x) + h(y)$. Prove that f is onto.

Problem 5

Let V be a particular set of vertices of cardinality n. How many different (simple, undirected) graphs are there with these n vertices? Here we will assume that we are naming the edges of the graph using a pair of vertices (so you cannot get new graphs by simply renaming edges). Briefly justify your answer.

Problem 6

Find the closed form of the following recurrence by drawing the recursion tree.

$$T(1) = 5, \text{ and } T(n) = 3T(n/2) + 7 \text{ for } n \geq 2$$

Problem 7

Suppose f and g are increasing functions from reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then is $\log(f(x))$ $O(\log(g(x))$?

Problem 8

How many positive integer solutions are there to $a + b + c = 11$?