# Countability and Induction Questions

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## Problem 1

Rank the following sets by cardinality.

$$\{\mathbb{N},\emptyset\},\ \mathbb{N},\ \mathbb{Z},\ \mathbb{Q},\ \mathbb{C},\ \mathbb{Z}_{10},\ \emptyset,\ \mathbb{R},\ \mathbb{P}(\emptyset),\ \{1,2,3\},\ \mathbb{P}(\mathbb{N}),\ \mathbb{P}(\mathbb{R})$$

## Problem 2

**(i)** Is the set of all finite simple graphs a finite, countably infinite, or uncountable set?

**(ii)** An $m \times n$ picture is a finite array of size $m \times n$ where each element of the array contains 4 real numbers (red, green, blue, alpha) between 0 and 255. Is the set of all $10 \times 10$ pictures finite, countable, or uncountable?

**(iii)** Pick your favorite programming language. Is the set of all programs written in that language finite, countably infinite, or uncountable?

**(iv)** An RGB ring is a 3-cycle, each of whose nodes contains a color label (red, green, or blue) along with a rational number in the range $[0,1]$. Is the set of all RGB rings countable or uncountable?

## Problem 3

Use proof by induction to show that the following holds for all natural numbers $n$ and all real numbers $x \neq 1$:

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

## Problem 4

Use proof by induction to show that the following inequality holds for all positive integers $n \geq 2$:

$$2^n n! < (2n)!$$

## Problem 5

Suppose that we draw $n$ lines on a plane such that no lines are parallel and no point belongs to more than two lines. The lines divide up the plane into a set of regions. Prove the following claim, for any positive integer $n$:

Claim: we can color these regions with two colors such that adjacent regions do not have the same color.