COS, Counting, and State Diagram Questions	CS 173 Staff
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Problem 1

(i) Compute the powerset $\mathbb{P}(A)$ for each set A in the following list of sets:

$$\emptyset, \{1\}, \{1,2\}, \{1,2,3\}$$

In general, what is the relation between the cardinality of a set A and the cardinality of its powerset $\mathbb{P}(A)$? Express $|\mathbb{P}(A)|$ in terms of $|A|$.

(ii) What is the definition of a partition? Give an example of a partition of the set $\{1,2,3,4,5\}$ as well as a non-partition. Also give an example of a partition of the natural numbers $\mathbb{N}$ as well as a non-partition.

Problem 2

State the Pigeonhole Principle and apply it to the following scenario:

Suppose we have a standard deck of 52 cards: 4 suits, each suit has 13 cards labelled Ace, $2, \ldots, 10$, Jack, Queen, and King (13 labels total). Note this means there are 4 Aces, 4 2s, etc. What is the minimum number of cards we need to draw in order to guarantee we have two cards of the same suit? What about two cards with the same label?

Problem 3

Suppose that set A has 6 elements and set B has 2 elements. How many different onto functions can be constructed from A to B?

Problem 4

(i) Suppose we have an alphabet Σ consisting of $|\Sigma| = s \geq 2$ characters. How many strings are there of length $\leq k$ where k is a natural number.

Hint: How many strings are there of length exactly k?

(ii) For $x, y, z \in \mathbb{N}$, how many solutions are there to the equation $x + y + z = 25$?

Hint: Formulate this as a combinations with repetitions problem and use the formula from the text.

Problem 5

A *triomino* is a triangular tile with a number on each edge. In our set of trinominos, the numbers on each edge can be labeled any integer from 0 to 5. Possible tiles are 5-3-4, 1-2-4, and 0-0-0. Tiles are the same if you turn it over, meaning 5-3-4 is the same as 4-3-5, or if you rotate it, so 5-3-4 is the same as 4-5-3. How many distinct trinominos are in our set?

Hint: Split into cases based on how the tiles are numbered. Even bigger hint: Actually draw some trinominos out, and their rotations!

Problem 6

Suppose we want a state machine that recognizes strings consisting of the letters a, b, and c.

Design and draw a ***deterministic*** state machine that accepts all strings that do ***not*** contain the substring ‘abc’. This means that every single state should have exactly one outgoing arrow per possible letter.

Problem 7

Suppose we want a state machine that recognizes certain base-3 numbers. It should read the numbers in left to right order. That is, if we've read the number x so far and read a new digit d, then our new number is $3x + d$ (*think about this in base 10 rather than 3 if that previous sentence is confusing*). The machine should be in an accepting state at the end of reading the input number if and only if the number is congruent to 2 (mod 5). Furthermore, your machine should be **deterministic** meaning that every state should have **exactly** one outgoing arrow per possible digit d.

Draw a state diagram to compute such a number.

Hint: we gave you a large portion of the transition function!