Relations Discussion Questions

CS 173 Staff

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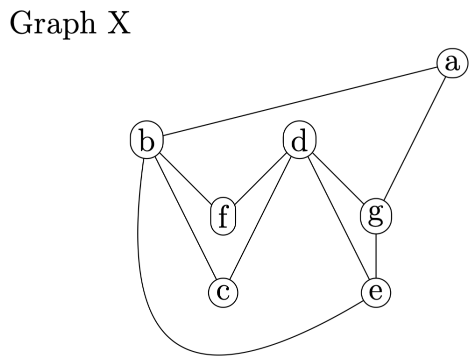
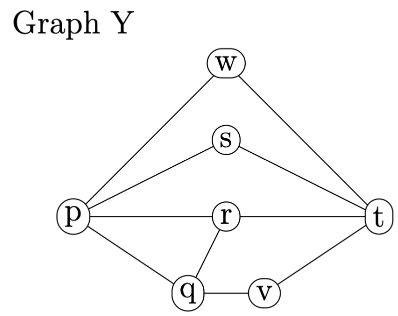
Problem 1

Are the two graphs below, Graph X and Graph Y, isomorphic? Justify your answer.





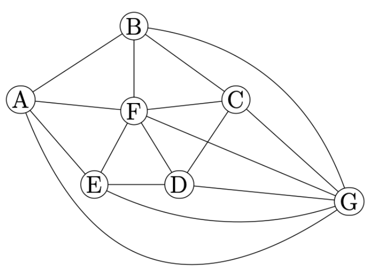
Bonus: Are either of these graphs bipartite? Explain why or why not.

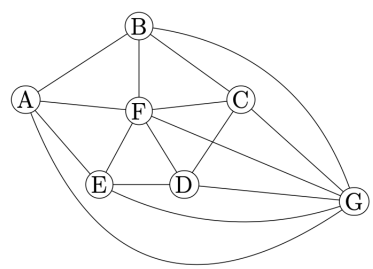
Problem 2

How many graphs can you draw (or describe), each with four nodes and four edges, none of which are isomorphic?

Problem 3

Recall that for a chromatic number proof, you must give an argument for the upper *and* lower bound. What is the chromatic number of the graph below? We provide a second copy of the graph so that you might try two different lower bound arguments (coloring by "forced choices", and by finding a particular subgraph).





Problem 4

Using the fact that $\sum_{i=0}^{k} i = \frac{(k+1)k}{2}$, find a closed form solution for $\sum_{i=3}^{k}(k+2i)$.

Recall that the closed form solution will only consist of constants, variables, and arithmetic operations.

Problem 5

Let $P(n)$ be a predicate / statement about the integers $n \in \mathbb{Z}$. Suppose that you have proven the following statements: $\forall n \in \mathbb{N}, P(n) \implies P(n+6)$ and $\forall n \in \mathbb{N}, P(n) \implies P(n-2)$. Suppose that you also have proven that $P(0)$ is true. Which of the following statements can be proven?

$P(3)$ $\quad$ $P(-2)$ $\quad$ $P(10)$ $\quad$ $P(-14)$ $\quad$ $P(-5)$

Problem 6

Prove the following claim using induction:

$$\text{For all integers } n \geq 1, \sum_{p=1}^{n} p \cdot 2^p = (n-1)2^{n+1} + 2.$$

When stating your inductive hypothesis, be explicit in what the hypothesis says rather than just referring to "the claim" or "this."