Relations Discussion Questions CS 173 Staff
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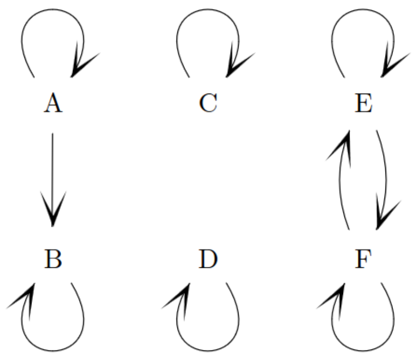
Problem 1



Consider the following relation R on the set $\{A, B, C, D, E, F\}$. Determine which of the following properties hold for R. If a property doesn't hold, explain why.

- Reflexive
- Irreflexive
- Symmetric
- Antisymmetric
- Transitive

Problem 2

Let R be a relation on $\mathbb{Z}$ such that $x\ R\ y$ for all x and y in $\mathbb{Z}$. Is R an equivalence relation? If so, what is $[7]$?

Problem 3

Define a relation T on $\mathbb{N}$ as follows:

$$a\ T\ b \text{ if and only if there exists } k \in \mathbb{N} \text{ such that } a = b + 2k.$$

Prove that T is antisymmetric and transitive.

Problem 4

One of the following statements is true, the other is false. Identify the true one and explain why the other is false.

1. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}, x^2 = y$
2. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{N}, x^2 = y$

Problem 5

Let $\mathbb{N}_{>1}$ be the set of natural numbers greater than 1. Consider the following function f:

$$f : \mathbb{N}_{>1} \to \mathbb{N}_{>1} \qquad f(n) = \min\{\, x \in \mathbb{N}_{>1} \mid x \text{ divides } n \,\}$$

Is f a function?

What are the domain and codomain of f?

What is the image of f?

What is the pre-image of 3?

Problem 6

Define a function f as follows:

$$f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}^2 \qquad f(x,y) = \left(\frac{x}{y},\ x+y\right)$$

Prove that f is one-to-one.

Problem 7

Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Define a function g as follows:

$$g : \mathbb{Z}^2 \to \mathbb{Z} \qquad g(x,y) = f(x-7) \cdot f(y)$$

Prove that g is onto.